

1. C There are 12 letters, but N is counted twice. So, there are $12!$ ways to order, but you must divide by 2 to account for the N's. So, the answer is C.
2. B After giving the hard-working student 3 problems and the other five students one problem. We must split 4 problems between 6 students. There are 9 stars and 5 bars, and $\binom{9}{5} = 126$. So, the answer is B.
3. A $\binom{12}{3} * (2x^3)^3 * \left(\frac{1}{x}\right)^9 = \binom{12}{3} * 8x^9 * \left(\frac{1}{x}\right)^9$, x^9 cancel, leaving $8 * \binom{12}{3}$ or 1760. So, the answer is A.
4. D Using stars and bars, we have to split 9 between 4. There are 3 bars and 9 stars, $\binom{12}{3} = 220$. So, the answer is D.
5. A There are $7!$ Ways to order the keys, but we must divide by 7 and 2 to account for rotations and flips. $7! / (7*2) = 360$. So, the answer is A.
6. E Using Pascal's rule, $\binom{81}{3} + \binom{81}{4} = \binom{82}{4}$. $85320 + 1663740 = 1749060$. So, the answer is E.
7. D The number of squares is $\frac{10*11*21}{6}$ which is 385. The number of rectangles is $\binom{11}{2}^2$, or 3025. $3025 + 385 = 3410$. So, the answer is D.
8. C Since there are 8 elements, and each can either be in the subset or out, there are 2^8 possible subsets. So, the answer is C.
9. D There are $\binom{50}{2}$ handshakes between the children and $\binom{40}{2}$ between the parents, $\binom{50}{2} + \binom{40}{2} = 2005$. So, the answer is D.
10. B We can use complementary counting by subtraction the total ways to arrange BALLOON by the ways that have the first and last letters the same. The total number is $7! / 2!2!$ or 1260, The number that have the first and last letters the same is $(2*5!) / 2!$ or 120. $1260 - 120 = 1140$. So, the answer is B.
11. A His alphabet has 20-3 or 17 consonants and 3 vowels. He has 17 choices for the first letter, 3 for the next, and 17 for the last. $17*17*3 = 867$. So, the answer is A.
12. B There is a total of $\binom{36}{3}$ ways to select a committee of 3, or 7140 ways. Then, we can complementary counting and subtract the total number of ways by the number of ways only 1 party is represented. Then later can be found by add $\binom{13}{3}, \binom{12}{3}, and \binom{11}{3}$. This is 671 and so the number of ways at least two parties are represented is $7140 - 671$. This is 6469. So, the answer is B.
13. E The six boys can be move into a super-person then there are 4 people, and the super-person has $6!$ Ways to be ordered, $6! * 4! = 17,280$. So, the answer is E.
14. A There is only one way to order any 6 distinct digits in decreasing order. $\binom{10}{6} = 210$. So, the answer is A.
15. B There is only one way to order any 3 distinct digits in strictly increasing order. However, 0 cannot be a digit. So, we do $\binom{9}{4}$ which is 126. So, the answer is B.
16. D There are 8 choices for bread, $\binom{5}{2}$ choices for meats and 4 choices for vegetables. $8*4*10 = 320$. So, the answer is D.
17. A There are 6 ways to get to (2,6) from (1,1). There are $\binom{8}{2}$ ways to get to (4,12) from (2,6). $\binom{6}{1} * \binom{8}{2} = 168$. So, the answer is A.

18. C Using the chicken McNuggets Theorem, the largest amount unobtainable is $m * n - m - n$, $9 * 7 - 9 - 7 = 47$. So, the answer is C.
19. B $30 = 2 * 3 * 5$. We must find how many 5's is in $2020!$ $(2020/5) + (2020/25) + (2020/125) + (2020/625) = 404 + 80 + 16 + 3 = 503$. So, the answer is B.
20. C The number of ways to choose a committee is $\binom{10}{4}$ or 210. If they are both in the committee, there are $\binom{8}{2}$ to assign the last two spots. If they are both out, there is $\binom{8}{4}$ ways to assign the 4 spots. $\binom{8}{2} + \binom{8}{4} = 98$. $98/210 = 7/15$. So, the answer is C.
21. A The last digit either has to be 5 or 0. If it is 5, we have $8 * 8 * 7$ numbers, and if it is 0, we have $9 * 8 * 7$ numbers. $9 * 8 * 7 + 8 * 8 * 7 = 952$. So, the answer is A.
22. B Every circle can intersect another circle twice, every line can intersect another line once, and every line can intersect every circle twice. There are $\binom{12}{2}$ pairs of circles, $\binom{9}{2}$ pairs of lines, and $9 * 12$ pairs of lines and circles. $2 * \binom{12}{2} + \binom{9}{2} + 2 * 9 * 12 = 216 + 132 + 36 = 384$. So, the answer is B.
23. E The x^3 term is $\binom{7}{6} * (3x)^6 * (1/x^3) = 7 * 3^6 * x^6 * 1/x^3 = 5103x^3$. So, the answer is E.
24. A Timmy and Jake can become a super-person and so can Edward and Samuel. There are $2 * 2$ ways to order the super-people, then there are only 5 people left to order. There are $4!$ Ways to order 5 people around a circle. $4! * 4 = 96$. So, the answer is A.
25. B There is $\binom{8}{4}$ ways to get 4 heads and 4 tails, or 70 ways. $256 - 70 = 186$. $186/2 = 93$. There are 93 cases with more tails than heads. $\binom{8}{6} = 28$, meaning that there are 28 cases with 6 tails. So, the answer is B.
26. D We can use the formula for principle of inclusion exclusion. $18 + 33 + 40 - 23 - 16 - 5 + 3 = 50$. So, the answer is D.
27. B We can do casework or $a + b + c = 9$, $a + b + c = 5$, all the way to $a + b + c = 0$. For $a + b + c = 9$, we can do stars and bars with a star and 2 bars, making $\binom{11}{2}$, for $a + b + c = 8$, we can do stars and bars with 8 stars and 2 bars, making $\binom{10}{2}$. We can do this continuously until $\binom{2}{2}$, and $\binom{11}{2} + \binom{10}{2} + \dots + \binom{2}{2} - \binom{12}{3}$ by the hockey stick identity. $\binom{12}{3} = 220$. So, the answer is B.
28. A The sum of the 60th row is 2^{60} . $2^{60} = 1 \pmod{61}$ because of Fermat's little theorem. So, the answer is A.
29. C Every match eliminates 1 player. The winner is determined when there is one player left. $64 - 1 = 63$. So, the answer is C.
30. A The only distinct ways to assign them are $(4,0,0,0)$, $(3,1,0,0)$, $(2,2,0,0)$, $(2,1,1,0)$, and $(1,1,1,1)$. So, the answer is A.