1. **A** The latus rectum is equal to $4p$, the distance between the focus and the directrix is equal to $2p$, and focal length is p. Therefore we have $\frac{4p+2p}{p} = 6$.
2. **B** $V = \frac{1}{2} \cdot \frac{4}{3}\pi(2)(2)(3) = 8\pi$
3. **A** Solving the system $\begin{matrix} 2x + 3y = -1 \\ 2x - 3y = 5 \end{matrix}$ we get $x = 1, y = -1$, which is the center of the hyperbola. Since the slope is $\pm\frac{2}{3}$, this makes $a^2 = 9$ and $b^2 = 4$. Thus the equation of the hyperbola is $\frac{(x-1)^2}{9} - \frac{(y+1)^2}{4} = 1$.
4. **D** Let $a = \sqrt{x}$. We now have $a^2 - 3a + 9$. This minimum is located when $a = -\frac{(-3)}{2(1)} = \frac{3}{2}$. Using our substitution, this makes $x = \frac{9}{4}$ and $f\left(\frac{9}{4}\right) = \frac{27}{4}$ or 6.75
5. **A** $x^2 - 5x + 6 < 0$ when x is between 2 and 3. Using these values as our bounds, $y = x^2 + 5x + 6$ takes on values from 20 to 30.
6. **B** There's only one point of integers that satisfy this equation and that is (6,5).
7. **D** We need to find where $b^2 - 4ac \geq 0$. $\left(4\sqrt{3}\right)^2 - 4(k)(k-1) \geq 0$. This simplifies to $k^2 - k - 12 \leq 0$. Solving for the critical values we get $k = -3$ and $k = 4$. 4 is the largest value.
8. **A** Solving the linear equation for y we get $y = 1 - x$. Substituting into the circle equation we get $(x-3)^2 + (6-x)^2 = 9$. This simplifies to $x^2 - 9x + 18 = 0$ which gives us $x = 3$ and $x = 6$. The intersection points are $(3, -2)$ and $(6, -5)$. The distance between these two points is $3\sqrt{2}$.
9. **B** Let take the parabola $y = x^2 - 2x - 8$. The vertex is $(1, -9)$ and a point on the parabola is $(4,0)$. Reflecting this parabola over the line $y = -9$, the vertex stays the same but the reflected point translate to $(4, -18)$. Using the vertex and the new point, the equation of the reflected parabola is $y = -x^2 + 2x - 10$. Adding the coefficients of the old and new parabola we get $1 - 2 - 8 - 1 + 2 - 10 = -18$. Therefore, the answer in question is $2k$.
10. **D** The vertex of $f(x)$ is $(-3, -4)$ and the vertex of $g(x)$ is $(1,2)$. The translation from f to g is right 4, up 6, hence the answer is D.
11. **D** $\frac{a}{b} + \frac{b}{a} = \frac{(a+b)^2 - 2ab}{ab}$. The sum of the roots is – 4 and the product of the roots is 1, therefore we have $\frac{(-4)^2 - 2(1)}{1} = 14$.
12. **E** $e = \frac{c}{a} \to \frac{1}{3} = \frac{1}{a} \to a = 3$ and $c = 1$. Therefore $b = \sqrt{3^2 - 1^2} = 2\sqrt{2}$. Double to find the length of the minor axis which is $4\sqrt{2}$.
13. **C** Using $B^2 - 4AC$ we have $3^2 - 4(1)(-4) = 13$. Since the value is positive and the conic is not degenerate, this is a hyperbola.
14. **C** The equation of the parabola is $8(y-3) = (x+1)^2$ with a latus rectum equal to 8. The circle in question has a radius of 2 and equation of $(x+1)^2 + (y-5)^2 = 4$. Therefore, the point $(0, 5 + \sqrt{3})$ lies on the circle.
15. **A** The center of this hyperbola will be the intersection of the vertical asymptote $(x = 1)$ and horizontal asymptote $(y = 4)$, thus the center is $(1,4)$.

16. **B** The width of the rectangle is $2c$. So, $c = \sqrt{13-9} = 2$ so $2c = 4$. The length of the rectangle is equal to the latus rectum of the ellipse which can be found by $\frac{2b^2}{a} = \frac{2(9)}{\sqrt{13}} = \frac{18\sqrt{13}}{13}$. The area is $4 \cdot \frac{18\sqrt{13}}{13} = \frac{72\sqrt{13}}{13}$.

17. **C** $(R^2 - r^2)\pi = 10\pi$ and $R + r = 5$. So we have $(R+r)(R-r)\pi = 10\pi \rightarrow (R-r)\cdot 5 = 10\pi$. So $R - r = 2$. Solving the system $\begin{matrix} R+r=5 \\ R-r=2 \end{matrix}$ for R, we get $R = \frac{7}{2}$.

18. **D** $x^2 + 4y^2 - 8y + k^2 = 0 \rightarrow x^2 + 4(y-1)^2 = -k^2 + 4$. This ellipse will be imaginary when$-k^2 + 4 < 0 \rightarrow k^2 > 4$ which is true when $k < -2$ or $k > 2$.

19. **D** $R = \frac{abc}{4(Area)} = \frac{7\cdot 8\cdot 9}{4\left(\sqrt{12\cdot 3\cdot 4\cdot 5}\right)} = \frac{21\sqrt{5}}{10}$

20. **D** The conic in question is a hyperbola therefore the number of linear permutations is $9! = 362880$

21. **C** Using the system $y = ax^2 + bx + c$, we get the system $\begin{matrix} a+b=5 \\ 4a+2b=12 \end{matrix}$. Solving gives the parabola $x = y^2 + 4y - 5$. The y −coordinate of the vertex is $y = -\frac{4}{2(1)} = -2$.

22. **C** Perimeter = $2r + \frac{\theta}{2\pi}(2\pi r) = 2r + \theta r = 10$ and the area = $\frac{\theta}{2\pi}(\pi r^2) = \frac{1}{2}\theta r^2 = 4$. Therefore, $\theta = \frac{8}{r^2}$. Making the substitution we have $2r + \frac{8}{r} = 10$. Solving for r we get $r = 4,1$. When $r = 4$, $\theta = \frac{1}{2}$ which works but when $r = 1, \theta = 8$, which we reject because $8 > 2\pi$.

23. **C** $\begin{matrix} 6xy + 2y^2 - 5y - 13 = 0 \\ 4xy + 2y^2 - 3y - 12 = 0 \end{matrix} \rightarrow 2y^2 + y - 10 = 0 \rightarrow y = -\frac{5}{2}, 2$. This gives $x = \frac{4}{5}, \frac{5}{4}$. Therefore, $AC + BD = \left(\frac{4}{5}\right)\left(\frac{5}{4}\right) + \left(-\frac{5}{2}\right)(2) = -4$.

24. **A** Since point Q must be on the same chord as point P and the focus $\left(0, \frac{1}{4}\right)$, then $m_{PF} = m_{FQ}$. Let $Q(x, y)$.

$$m_{PF} = m_{FQ}$$
$$\frac{9 - \frac{1}{4}}{-3} = \frac{\frac{1}{4} - y}{-x}$$
$$-\frac{35}{4}x = -\frac{3}{4} + 3y$$
$$-\frac{35}{4}x = -\frac{3}{4} + 3x^2$$
$$12x^2 + 35x - 3 = 0$$
$$(12x - 1)(x + 3) = 0$$
$$x = \frac{1}{12}, -3$$

25. **D** Slope of the radius through $(0,6)$ and the center of the circle $(3,2)$ is $m = \frac{2-6}{3-0} = -\frac{4}{3}$. We need the perpendicular slope so $m = \frac{3}{4}$. Therefore our tangent line is $y = \frac{3}{4}x + 6$ which becomes $3x - 4y + 24 = 0$.

26. **D** The equation of the ellipse is $\frac{x^2}{576}+\frac{y^2}{400}=1$. Plug in the height of 10 we get $\frac{x^2}{576}+\frac{100}{400}=1 \rightarrow \frac{x^2}{576}=\frac{3}{4} \rightarrow x^2=\frac{(576)(3)}{4} \rightarrow x=12\sqrt{3}$. But we need to double to get the full width.

27. **C** The ellipse becomes $\frac{(x+3)^2}{64}+\frac{(y-4)^2}{100}=1$, with center $(-3,4)$ and $a=10, b=8, c=6$. To find the directrices of the ellipse, we use $y=k\pm\frac{a^2}{c} \rightarrow y=4\pm\frac{10^2}{6}=4\pm\frac{50}{3}$. Since we need a positive value, $y=4+\frac{50}{3}=\frac{62}{3}$

28. **E** $x^2-y^2-10x-14y-24=0 \rightarrow (x-5)^2=(y+7)^2 \rightarrow x-5=y+7$. This is two lines which is a degenerate hyperbola.

29. **C** The only way this is possible is in the points lie on the same line. $\frac{4-2k}{3k-1}=\frac{6k-4}{5-3k} \rightarrow 6k^2-22k+20=18k^2-18k+4 \rightarrow 3k^2+k-4=0 \rightarrow (3k+4)(k-1)$. Solving gives $k=-\frac{4}{3}$, 1.

30. **C** $e=\frac{c}{a} \rightarrow a=\frac{c}{e}$. Since $e=\frac{7}{10}$, $a=\frac{10}{7}c \rightarrow a^2=\frac{100}{49}c^2$. The latus rectum is equal to $\frac{2b^2}{a}=51$. So, $51a=2b^2 \rightarrow 51a=2(a^2-c^2) \rightarrow 51\left(\frac{10}{7}c\right)=2\left(\frac{100}{49}c^2-c^2\right)$. Simplifying the equation we get $\frac{102}{49}c^2-\frac{510}{7c}=0$. Solving for c we get $c=0, 35$, so $c=35$ and $a=50$. Solving for b: $b=\sqrt{a^2-c^2}=\sqrt{50^2-35^2}=\sqrt{1275}=5\sqrt{51}$. Therefore, the area of the ellipse is $A=\pi(50)\left(5\sqrt{51}\right)=250\pi\sqrt{51}$.