For all questions, answer choice "E. NOTA" means none of the above answers is correct.

1. Evaluate $(2^2)^2 - 2^{(2^2)}$.

   A. 0 B. 8 C. 16 D. 240 E. NOTA

2. Find the number of distinct solutions of the equation $x^6 - 6x^4 + 12x^2 - 8 = 0$.

   A. 2 B. 3 C. 4 D. 5 E. NOTA

3. Alice's daily pre-made 600-gallon drink is 15% lime juice. However, she's feeling a little more sour today and a little more thirsty, so how many gallons of lime juice must she add to make her drink 20% lime juice?

   A. 17.5 B. 28.2 C. 37.5 D. 40.2 E. NOTA

4. Find the number of asymptotes of the function $f(x) = \frac{x+2}{|x-1|}$

   A. 0 B. 1 C. 2 D. 3 E. NOTA

5. A rectangular piece of paper with side lengths 3 and 6 is folded along one of its diagonals. What is the area of the region in which the paper overlaps with itself?

   A. $\frac{45}{8}$ B. $\frac{27}{8}$ C. $\frac{9}{2}$ D. 9 E. NOTA

6. Alex has 3 quarters, 5 dimes, 9 nickels, and 2 pennies in his pocket. He starts randomly taking them out one by one, then places them on the table after taking each out. What is the probability the last coin Alex takes out is a penny?

   A. $\frac{2}{19}$ B. $\frac{1}{86}$ C. $\frac{2}{171}$ D. $\frac{1}{1361360}$ E. NOTA

7. $R$ and $E$ are on sides $FA$ and $FZ$, respectively, of triangle $FAZ$ such that $RE \parallel AZ$. Given $RE = 4, EZ = 9$, and $AZ = 24$, find the length of $FE$.

A. $\frac{21}{2}$ B. $\frac{29}{3}$ C. $\frac{5}{9}$ D. $\frac{9}{5}$ E. NOTA

8. The equation of the circle that passes through the points $(-6,\ 3), (2,\ 1)$, and $(-3, -2)$ is $(x-h)^2 + (y-k)^2 = r^2$. What is $h + k + r^2$?

A. $15$ B. $17$ C. $21$ D. $25$ E. NOTA

9. *a*, *b*, and *c* are complex numbers that satisfy the following set of equations:

$$a + b + c = 1$$
$$ab + bc + ca = 2$$
$$abc = 3$$

Find the value of $(a^2 - 1)(b^2 - 1)(c^2 - 1)$.

A. $-7$ B. $-1$ C. $1$ D. $7$ E. NOTA

10. A right octagonal pyramid has a base in the shape of a regular octagon with side length 4. The prism has a slant height (from the midpoint of one of the sides to the tip of the pyramid) of $\sqrt{30}$. Find its height.

A. $4 - 2\sqrt{2}$ B. $4 - \sqrt{2}$ C. $2 + \sqrt{2}$ D. $2 + 2\sqrt{2}$ E. NOTA

11. A hyperbola has foci $(0,2)$ and $(3,0)$. It has two $x$-intercepts, and one of them is the origin. The other has coordinates $\left(\frac{m}{n}, 0\right)$ in simplest form. Find the value of $m + n$.

A. $4$ B. $5$ C. $6$ D. $7$ E. NOTA

12. Find the value of $\sum_{n=1}^{\infty} \frac{6}{n^2 - 5n + 6}$.

A. $-3$ B. $-4$ C. $-\frac{9}{2}$ D. $-6$ E. NOTA

For questions 13-15, decide whether the blank should be filled in with $=$, $<$, or $>$. In question 13, order doesn't matter.

13. (number of ways to break 10 people into 2 teams of 5) _____ (number of ways to break 10 people into a team of 6 and a team of 4)

A. $=$ B. $<$ C. $>$ D. Not enough info E. NOTA

14. (probability that all 3 people in a group of 3 were born on March 1) _____ (probability that in a group of 3 people, 1 was born on each of March 1, 2, and 3)
*Note: You may assume that it is equally likely with nonzero probability to be born on any of the three dates.*

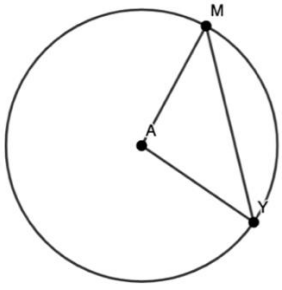
A. $=$ B. $<$ C. $>$ D. Not enough info E. NOTA

15. Helena and Kira are incredibly bored one day, so they decide to toss a fair coin until either two consecutive heads are tossed or a tail immediately followed by a head is tossed. Kira wins the game if and only if two consecutive heads occurs before a tail immediately followed by a head occurs. If not. Helena wins the game. Determine what the blank should be filled with: (probability that Helena wins) ____ (probability that Kira wins)

A. $=$ B. $<$ C. $>$ D. Not enough info E. NOTA

16. How many distinct arrangements are there for the letters in BILLIEEILISH?

A. 138600 B. 1663200 C. 3326400 D. 6652800 E. NOTA




17. Point $A$ is the center of the circle in the diagram above. Find $\angle YAM$ if $\angle AMY = 56°$.

A. 56 B. 68 C. 124 D. 140 E. NOTA

18. Find the smallest component of the Pythagorean triple that has 89 as the length of the hypotenuse.

A. 39 B. 59 C. 80 D. 88 E. NOTA

19. If $(1+2i)^4(2+i)^6 = A + Bi$ for real $A, B$, find $A + B$.

A. $-4375$ B. $-625$ C. 625 D. 4375 E. NOTA

20. If the positive integer *r* is added to each of the numbers 75, 171, and 299, one obtains the squares of three consecutive terms of an arithmetic sequence of positive integers. Find the sum of the digits of *r*.

A. 5 B. 6 C. 7 D. 8 E. NOTA

21. Find the sum of the entries of the inverse of the following matrix: $\begin{bmatrix} 1 & 4 & 0 \\ 2 & 3 & -1 \\ 1 & 0 & -1 \end{bmatrix}$.

A. 1 B. $-2$ C. 3 D. $-6$ E. NOTA

22. Ruiwen has a fair, six-sided die whose faces are numbered 1,2,3,4,5,6. He creates a sequence by rolling the die and recording the number on its top face. Ruiwen stops rolling when all six numbers (1, 2, 3, 4, 5, and 6) appear in his sequence. Let *x* be the expected value of the number of times Ruiwen will have to roll the die. Find *x* rounded to the nearest integer.

A. 6 B. 36 C. 15 D. 20 E. NOTA

23. The expansion of $(2x+1)^{2023}$ is written as

$$a_1x^{2023} + a_2x^{2022} + a_3x^{2021} + \cdots + a_{2023}x + a_{2024}$$

Find the units digit of $2(a_1 + a_3 + a_5 + \cdots + a_{2023})$.

A. 5 B. 6 C. 7 D. 8 E. NOTA

24. Mr. Lu and his friend are writing a Chinese Romance Novel, and they are 70% of the way through. Of that 70%, Mr. Lu has written 64% of the novel, and his friend has written 36% of the novel. At least what percentage of the remaining book must Mr. Lu's friend to write as much as Mr. Lu at the end when the book is finished?

A. 77% B. $76\frac{2}{3}\%$ C. 83% D. $82\frac{2}{3}\%$ E. NOTA

25. Three circles of area $36\pi$ lie in a plane such that each passes through the center of the other two. Find the area common to all three circles.

A. $8\pi$ B. $12\pi - 18\sqrt{3}$ C. $18\pi - 18\sqrt{3}$ D. $18\pi - 27\sqrt{3}$ E. NOTA

26. Set $S$ contains all positive integers from 2 to 100 inclusive. $a$ and $b$ are randomly selected from $S$ with replacement. What is the probability that $\log_a b$ is rational?

A. $\frac{159}{9801}$ B. $\frac{135}{9801}$ C. $\frac{145}{9801}$ D. $\frac{149}{9801}$ E. NOTA

27. Eddie is counting to 2023. Starting from his thumb, he goes to his little finger, then comes back and repeats this process. (thumb → index finger → middle finger → ring finger → little finger → ring finger → ...), repeating this process until he gets to 2023. If he starts counting from 0, which finger will he end on?

A. thumb B. index finger C. middle finger D. ring finger E. little finger

28. At Buccholz University, there are only three classes offered, one taught by Xecc, Jecc, and Becc. The 102 students that are enrolled at Buccholz University. If there are 30 students taking Xecc's class, 46 students taking Jecc's class, 71 students taking Becc's class, 16 students taking both Xecc and Becc's class, 19 students taking both Becc and Jecc's class, 18 students taking both Jecc and Xecc's class, and 6 students taking all three classes offered at the university. How many students aren't taking any class?

A. 2 B. 3 C. 4 D. 5 E. NOTA

29. William is exactly $n$ years older than his only grandchild, Rishi. There are twelve occurrences during Rishi's life where William's age will be an integer multiple of Rishi's age. Find the smallest value of *n*. Assume that age is $\lfloor number\ of\ years\ alive \rfloor$.

A. 60 B. 72 C. 96 D. 84 E. NOTA

30. What is the constant term in the expansion $\left(5x^3 + \frac{1}{x}\right)^8$?

A. 0 B. 25 C. 700 D. 437500 E. NOTA