

For all questions, answer choice “E” – NOTA, means none of the above answers is correct.
Good Luck and Have Fun!

1. Isosceles trapezoid ABCD with $AB \parallel CD$ and $AD = BC = \sqrt{5}$ is inscribed in circle ω . If $AC = 3$ and $CD = \sqrt{14}$, find the area of ω .
A. 7π B. 3.5π C. $\frac{15\pi}{56}$ D. 2.5π E. NOTA
2. Which of the following lines is an asymptote of the graph of $f(x) = \frac{x^2 - 2x - 3}{x^3 - 6x^2 + 5x + 12}$?
A. $x = 3$ B. $x = -1$ C. $y = 1$ D. $y = 0$ E. NOTA
3. How many perfect squares are strictly in between 8^4 and 4^8 ?
A. 191 B. 193 C. 291 D. 293 E. NOTA
4. The area of the circle circumscribed about the triangle formed by the intersection points of lines $x = 5$, $y = 0$, and $y = \frac{x\sqrt{6}}{2}$ can be written as n . Find $[n]$ where $[x]$ is the greatest integer less than x .
A. $\frac{125\pi}{8}$ B. $\frac{125\pi}{2}$ C. $\frac{625\pi}{8}$ D. $\frac{625\pi}{4}$ E. NOTA
5. Angela, Aditi, and Billy are playing a game of Monkey in the Middle! When Angela tosses the ball, it follows a perfectly parabolic path to Aditi, who is 50 feet away from Angela. During this trajectory, assume the starting position (when the ball leaves Angela’s hand) and ending position (when Aditi catches the ball) of the ball is always 6 ft above the ground. Billy is a fifth of the way from Angela to Aditi, and when the ball is directly over him, it is 2 feet above his reach of 6 ft. What is the maximum height (in feet) the ball reaches from the ground?
A. $\frac{23}{8}$ B. 25 C. $\frac{73}{8}$ D. 8 E. NOTA
6. Find the sum of $5^3 + 6^3 + 7^3 + \dots + 14^3 + 15^3 + 16^3$.
A. 18376 B. 18396 C. 18496 D. 18956 E. NOTA

7. How many positive integral factors does the number 2024 have?
A. 8 B. 12 C. 18 D. 24 E. NOTA
8. $\triangle XYZ$ is isosceles with $XY = YZ$. Point D is on the line segment YZ such that $XD = XZ$ and $YD = DX$. Find the ratio: $\frac{YD}{DZ}$.
A. $\frac{1}{2}$ B. $\frac{1+\sqrt{5}}{2}$ C. 2 D. $\frac{\sqrt{5}-1}{2}$ E. NOTA
9. Let the answer to question #8 be denoted by the variable r . A regular pentagon $ABCDE$. If the side length of the pentagon is $4r$, calculate $AB + AC + AD + AE$ in terms of r . (Of note: $\triangle DBE$ is similar to $\triangle XYZ$ as defined in question #8.)
A. $8r + 8$ B. $10r + 8$ C. $16r + 16$ D. $10r + 12$ E. NOTA
10. On a coordinate plane, the point (0, 5) is labelled A and the point (0, 15) is labelled B. A locus is constructed of every point P in the plane such that $PA + PB = 12$. What is the length of the latus rectum of the conic described by this locus?
A. 20 B. $\frac{50}{3}$ C. $\frac{11}{3}$ D. 10 E. NOTA
11. If matrix $M = \begin{bmatrix} 4 & 5 \\ 2 & 3 \end{bmatrix}$, what is $\det(M^T + M^{-1})$?
A. $\frac{5}{2}$ B. 31 C. $\frac{59}{2}$ D. 8 E. NOTA
12. Simplify $3 + \frac{1}{3 + \frac{1}{3 + \dots}}$.
A. $\frac{3+\sqrt{13}}{2}$ B. $\frac{3-\sqrt{13}}{2}$ C. $3 + \sqrt{13}$ D. $3 - \sqrt{13}$ E. NOTA
13. Express the number 42313_5 in base 8.
A. 2436 B. 2637 C. 4152 D. 5421 E. NOTA

14. Evaluate the following sum:

$$\sum_{i=0}^{10} \sum_{j=0}^i \binom{i}{j}$$

- A. 1024 B. 1023 C. 2048 D. 2047 E. NOTA
15. A right circular cone with diameter of 8 and height of 3 is circumscribed about a sphere ω . Find the surface area of ω .
- A. $\frac{64\pi}{9}$ B. $\frac{16\pi}{9}$ C. $\frac{25\pi}{6}$ D. $\frac{8\pi}{3}$ E. NOTA
16. The greatest common divisor of 360 and k is 40. How many k exist in the interval from 40 to 360 inclusive?
- A. 10 B. 12 C. 6 D. 5 E. NOTA
17. Mr. Lu lives near a river (with a current) and kayaks to school and back home every day! Today, Mr. Lu began kayaking down the stream to school at 8:00 AM. However, when Mr. Lu reached school (which is 15 yards away from his house), he realized he left his watch at home. He immediately kayaked up the stream to get home and arrived at his house at 8:08 AM. If Mr. Lu rows at the constant speed of 4 yards/min, what is the rate of the current (in feet/min) of the river?
- A. 1 B. 3 C. 12 D. 15 E. NOTA
18. What is the centroid of the triangle enclosed by the graphs of $y = x$, $y = -1$, and $y = -7x + 6$?
- A. $\left(\frac{4}{9}, -\frac{2}{9}\right)$ B. $\left(\frac{1}{4}, -\frac{1}{6}\right)$ C. $\left(\frac{1}{4}, -\frac{5}{12}\right)$ D. $\left(\frac{4}{9}, \frac{4}{9}\right)$ E. NOTA
19. Find the units digit of the value of x that satisfies the following logarithmic equation:
- $$\log_3(\log_{81} x) = \log_9(\log_9 x)$$
- A. 9 B. 7 C. 1 D. 8 E. NOTA

20. A quadrilateral $MAOT$ is circumscribed about a circle. Given $MA = 7$, $AO = 6$, and $OT = 4$. What is the length of MT ?
- A. 4 B. $\frac{16}{3}$ C. 5 D. 12 E. NOTA
21. What is the shape of the graph of $x^2 - 9y^2 + 24y - 25 = -9$?
- A. Parabola B. Circle C. Ellipse D. Hyperbola E. NOTA
22. How many integers between 1 and 2024 inclusive are divisible by none of 2, 3, and 7?
- A. 576 B. 578 C. 577 D. 580 E. NOTA
23. Circle w with radius 4 is circumscribed about rectangle $LMNO$. If the measure of angle LMO is 60 degrees, what is the area of rectangle $LMNO$?
- A. $8\sqrt{3}$ B. $4\sqrt{3}$ C. $16\sqrt{3}$ D. $32\sqrt{3}$ E. NOTA
24. Assume that variables o , p , and s are nonzero, real numbers that sum to zero. Which of the following is the sum of the distinct value(s) that the following expression can attain?
- $$\frac{|op|}{op} + \frac{|os|}{os} + \frac{|ps|}{ps} - \left(\frac{|s|}{s} + \frac{|p|}{p} + \frac{|o|}{o} \right)$$
- A. -6 B. -21 C. -2 D. 0 E. NOTA
25. A regular tetrahedron has a volume of $18\sqrt{2} \text{ cm}^3$. What is its height in centimeters?
- A. $2\sqrt{6}$ B. $\frac{3\sqrt{6}}{2}$ C. $\frac{2\sqrt{6}}{3}$ D. 6 E. NOTA
26. Richard puts a rope through a collar on his ZhuZhu pet guinea pig such that the guinea pig can run anywhere along the length of the 50 cm rope. The ends of the rope are tied down at points 40 cm apart. What is the area, in square centimeters of the area where Richard's guinea pig can play in?
- A. 375π B. 400π C. 500π D. 750π E. NOTA

27. A sequence is defined as follows:

$$f_n = \begin{cases} \frac{f_{n-1}}{2} & n \text{ is even} \\ n & n \text{ is odd} \end{cases}$$

Evaluate

$$\sum_{n=0}^{20} f_n$$

- A. $299/2$ B. 150 C. $399/2$ D. 200 E. NOTA

28. The value of $1234_7 \times 4_7 + 2345_7 \times 2_7$ can be written as $ABCDE_7$ where A, B, C, D, and E are digits from 0 to 6. Find $A - B + C - D + E$.

- A. 4 B. 10 C. -4 D. 3 E. NOTA

29. When $0.\bar{n}_{n+1}$ is written as a simplified base 10 fraction, it can be expressed as $\frac{a_n}{b_n}$ where a and b are relatively prime positive integers. Evaluate

$$\sum_{i=1}^8 (a_i + b_i)$$

- A. 63 B. 72 C. 44 D. 16 E. NOTA

30. If $\log_6 12 = x$, find the value of $\log_2 6$ in terms of x .

- A. $\frac{1}{x}$ B. $1 + x$ C. $1 + \frac{1}{x}$ D. $\frac{1}{x-1}$ E. NOTA