1. A $4^2 - 2^4 = 0$
2. A $(x^2 - 2)^3 \rightarrow \pm\sqrt{2}$
3. C We can create the equation $600 * 0.85 = (600 + x) * 0.8$, where x is the number of gallons of lime juice Alice must add to her drink. This is because the amount of water stays the same the entire time, as she is only adding lime juice. Thus, the amount of water before she adds her lime juice $(600 * 0.85)$ must equal the amount of water after she adds her lime juice $((600 + x) * 0.8)$. Thus, we have:

$$600 * 0.05 = 0.8 * x$$
$$x = 37.5$$

4. D The asymptotes are when $y \rightarrow \pm\infty$ or $x \rightarrow \pm\infty$. Plugging all these values give $x = 1, y = 1, y = -1$.
5. A Let the rectangle be $ABCD$ where $AB = 6$ and $BC = 3$. Suppose we reflect D across diagonal AC to the point D'. $AD' = 3$ and $CD' = 6$. Suppose CD' intersects AB at point E. Our desired area is the area of triangle $AD'C$ minus the area of $AD'E$. The area of $AD'C$ is simply $\frac{3*6}{2} = 9$. To find the area of $AD'E$, we must find the length of $D'E$. Let $D'E = k$. BE also equals k, and $CE = 6 - k$. This gives us right triangle BCE on which we can apply the Pythagorean Theorem with $BE = k, BC = 3$, and CE (the hypotenuse) $= 6 - k$. Thus, we have $k^2 + 3^2 = (6 - k)^2 \rightarrow k = \frac{9}{4}$. This means that the area of $AD'E$ is $\frac{3*\frac{9}{4}}{2} = \frac{27}{8}$, so the desired area is $9 - \frac{27}{8} = \frac{45}{8}$.

6. A We can reverse the draw sequence of the coins. Thus, we want to find the probability that the first coin Alex takes out is a penny, which is just $\frac{2}{19}$.

7. D We see that triangles FRE and FAZ are similar, so the ratio between RE and AZ is equal to the ratio between FE and FR. Thus, calling $FE = x$, $\frac{4}{24} = \frac{1}{6} = \frac{x}{x+9}$, giving us $x = FE = \frac{9}{5}$.

8. B We see that there's something particular to the points that are given—the slope between $(-6,\ 3)$ and $(-3, -2)$ is $-\frac{5}{3}$ while the slope between $(2,\ 1)$ and $(-3, -2)$ is $\frac{3}{5}$, meaning the two are perpendicular, meaning the triangle made from these points is a right triangle, with a right angle at point $(-3, -2)$. Thus, we know that the center of the circle that passes through these points is the midpoint of the hypotenuse of the triangle, or $(\frac{-6+2}{2}, \frac{3+1}{2}) = (-2, 2)$. The radius of the circle is the distance between $(2, 1)$ and $(-2, 2)$, which is equal to $\sqrt{(2+2)^2 + (1-2)^2} = \sqrt{16+1} = \sqrt{17}$. Thus, our circle is centered at $(-2, 2)$ and has a radius of $\sqrt{17}$, giving us our equation of $(x+2)^2 + (y-2)^2 = 17$. That means $h = -2, k = 2$, and $r^2 = 17$. Thus, $h + k + r^2 = -2 + 2 + 17 = 17$.

9. D Let a, b, and c be the roots of an equation $x^3 + mx^2 + px + q = 0$. By Vieta's, we know that this equation is $x^3 - x^2 + 2x - 3 = 0$. The desired value factors to $(1-a)(1-b)(1-c)(-1-a)(-1-b)(-1-c) = f(1)f(-1) = -1 \cdot -7 = 7$.

10. B The distance from the center of one side of the base to the center of the base is $2 + 2\sqrt{2}$. This is one leg of a right triangle whose hypotenuse is the slant height and whose other leg is the height. Solving $\left(2+2\sqrt{2}\right)^2 + h^2 = 30$ gives $h^2 = 18 - 8\sqrt{2}$ and $h = 4 - \sqrt{2}$.

11. B By definition of a hyperbola, $2a = 3 - 2 = 1$. The other point $(x, 0)$ satisfies $|x-3| - \sqrt{x^2+4} = -1$. Solving for x gives $x = \frac{3}{2}$.

12. E $n^2 - 5n + 6 = 0$ at $n = 2{,}3$

13. B $<$; The right hand side simplifies to $\binom{10}{6}$ since choosing the team of 6 leaves a team of 4, so we no longer need to worry about choosing the team of 4. However, the left hand side simplifies to $\binom{10}{5} * \frac{1}{2}$ because choosing the initial team of 5 is the same as choosing the complementary team of 5, so we must account for overcounting. Thus, $\binom{10}{5} * \frac{1}{2} < \binom{10}{6}$ reduces to $3 < 5$.

14. B $<$; We can label the three individuals A, B, and C. For both scenarios, there are 365^3 ways to designate birthdays to the group. However, there is only 1 way such that A, B, and C are all born on January 1st, while there's 3! ways such that 1 of A, B, and C was born on each of January 1st, 2nd, and 3rd. Thus, we have $\frac{1}{365^3} < \frac{3!}{365^3}$.

15. C $>$; Let's set H to represent heads and T to represent tails. Considering the first toss, if it's T, we're guaranteed to see TH before we see HH. If it's H, we could still either see TH or HH first. Thus, the probability of HH occurring later than TH is greater than $\frac{1}{2}$.

16. B With 12 total letters, the word BILLIEEILISH has repeat letters of 4 I's, 3 L's, and 2 E's, giving us our answer of $\frac{12!}{4!3!2!} = 1663200$.

17. B Since lines AM and AY are both radii of circle A, they have the same length. Thus, triangle AMY is an isosceles triangle, making angles AMY and AYM equal, with angles A, M, and Y having to sum to 180. Thus, since angles AMY and AYM are both 56°, angle YAM must be 68°.

18. A A Pythagorean triple is well-known to have components $(m^2 - n^2, 2mn, m^2 + n^2)$. Solving for $m^2 + n^2 = 89$ gives $m = 8, n = 5$. Thus, the Pythagorean triple is $(39, 80, 89)$

19. D $(1+2i)(2+i) = 5i$, so the expression equals $(5i)^4(2+i)^2 = 625(3+4i) = 1875 + 2500i$. $1875 + 2500 = 4375$.

20. C Let the three terms of the arithmetic sequence be (a-d), a, and (a+d). Then, we have the following three equations: $75 + r = (a - d)^2$, $171 + r = a^2$, $299 + r = (a + d)^2$. Solve the system to see that (*a*, *d*, *r*) = (14, 4, 25). Thus, the sum of the digits of *r* is 2+5 = 7.

21. D Augmenting to the identity matrix, we have

$$\begin{bmatrix} 1 & 4 & 0 & 1 & 0 & 0 \\ 2 & 3 & -1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}$$

Then, performing elementary row operations, we get

$$\begin{bmatrix} 1 & 4 & 0 & 1 & 0 & 0 \\ 0 & -5 & -1 & -2 & 1 & 0 \\ 0 & -4 & -1 & -1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & -1 \\ 0 & -4 & -1 & -1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 4 & 1 & 1 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -3 & 4 & -4 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -3 & 4 & -5 \end{bmatrix}$$

to arrive at our answer of

$$\begin{bmatrix} -3 & 4 & -4 \\ 1 & -1 & 1 \\ -3 & 4 & -5 \end{bmatrix}.$$

The sum of the entries here is -6.

22. C After the first roll, there is only one number in the sequence. The probability that the next number chosen is not already in the sequence is, 5/6; therefore, the expected number of rolls to achieve a number different from the first is 6/5. Similarly, the probability that a 3rd new number is chosen is 4/6, and the expected number of rolls to achieve the 3rd new number is 6/4. Thus, the final expected value is: 1 + 6/5 + 6/4 + 6/3 + 6/2 + 6/1 = 14.7 which rounds to 15.

23. D Let the variable *S* represent the sum $a_1 + a_3 + a_5 + \cdots + a_{2023}$ .The sum of the coefficients of the binomial expansion of $(2x + 1)^{2023}$ can be found by plugging in x = 1. Thus, $a_1 + a_2 + a_3 + \cdots + a_{2024} = 3^{2023}$. Plugging in x = -1 will negate any term where the exponent of x is odd. Therefore, $-a_1 + a_2 - a_3 + \cdots - a_{2023} + a_{2024} = (-1)^{2023} = -1$. Subtract the two equations to get $2S = 3^{2023} + 1$. The units digits of this is 8.

24. D Let's call the number of pages in this Chinese Romance Novel *x*. Thus, Mr. Lu has written *x* * *.7* * *.64* = .448*x* pages, and his friend has written *x* ****.7*.36* = .***252x*. His friend needs to overall hit .5*x*, so he needs .5*x* -.252*x*=.248*x* of the remaining 30% of the book, so he must write .248x/.3x = 82 2/3 % of the remaining book.

25. C We can cut the area common to all three circles into three rounded areas as well as one equilateral triangle, where the former is simply 1/6th of the area of the circle minus the area of the equilateral triangle, and the latter is simply the area of an equilateral triangle with side length 6 (as the radii of the circles are 6). Thus, we can sum this to be $9\sqrt{3} + 3\left(6\pi - 9\sqrt{3}\right) = 18\pi - 18\sqrt{3}$.

26. D There are 99^2 possibilities, which is our denominator for the probability. If a and b are both powers of 2, powers of 3, powers of 5, powers of 6, powers of 7, or powers of 10, the logarithm will be rational. Furthermore, if a and b do not fall into any of those categories, but they are equal, the logarithm will still be rational. There are 6 powers of 2 in the set, 4 powers of 3, 2 powers of 5, 2 powers of 6, 2 powers of 7, and 2 powers of 10, which makes $6^2 + 4^2 + 2^2 + 2^2 + 2^2 + 2^2 = 68$ possibilities. Additionally, there are $99 - 6 - 4 - 2 - 2 - 2 - 2 = 81$ other numbers for a total of $68 + 81 = 149$ total possibilities. Thus the final probability is $\frac{149}{99^2} = \frac{149}{9801}$.

27. B The cycle reforms every 8. (1-2-3-4-5-4-3-2). Taking 2023 mod 8 gives 7, which is equivalent to the index finger.

28. A $(30 + 46 + 71) - (16 + 19 + 18) + 6 = 100$. $102 - 100 = 2$.

29. A The number of occurrences where William's age is an integer multiple of Rishi's age is the number of factors of *n*. (Check smaller values such as 8 or 4). The smallest value of n with 12 integer factors is 60.

30. C The constant term is $\binom{8}{2}(5x^3)^2\left(\frac{1}{x}\right)^6 = 28 * 25x^6 * \frac{1}{x^6} = 28 * 25 = 700$.