For all questions $\log x$ written without a base denotes the base 10 logarithm of x while $\ln x$ denotes the natural logarithm of x.

1. If $\frac{\ln x}{1+\ln y} = e$, solve for y in terms of x.

   A. $e^e x^e$ B. $\frac{x^{\frac{1}{e}}}{e}$ C. $\frac{\ln x}{e} - 1$ D. $\ln x^e$ E. NOTA

2. Find the absolute value of the product of the largest and smallest solution to the equation $(\log_2 x)^2 \log_4 x + 5(\log_2 x)^2 - 6\log_2 x \log_4 x - 26\log_2 x = -40$.

   A. $\frac{1}{64}$ B. 40 C. 64 D. $\frac{1}{256}$ E. NOTA

3. Simplify $2^{(\log_4 3)(\log_9 7)+\log_2 49}$.

   A. $7\sqrt{7}$ B. $49\sqrt[4]{7}$ C. $\sqrt[4]{343}$ D. $49\sqrt{7}$ E. NOTA

4. Solve for n over real numbers: $\log_3(16n^4 - 96n^3 + 216n^2 - 216n + 81) = 8$

   A. 6 B. $-3, 3$ C. 3 D. $-3, 6$ E. NOTA

5. Solve for x: $\ln(\ln x) + \log_2(\ln x) = \log_2(4e^2)$.

   A. e^{e^2} B. e^{2e} C. e^e D. e^{e+2} E. NOTA

6. Find the value of $8^{\log_2 3}$.

   A. 27 B. 9 C. 4 D. 3 E. NOTA

7. What is the fourth term in the expansion of $(m+n)^{\frac{2}{3}}$?

   A. $\frac{4}{81}m^{\frac{-10}{3}}n^4$ B. $\frac{-7}{243}m^{\frac{-10}{3}}n^4$ C. $\frac{4}{81}m^{\frac{-7}{3}}n^3$ D. $\frac{-7}{243}m^{\frac{-7}{3}}n^3$ E. NOTA

8. Which of the following are true? (correct answer must include all true statements)

I. $\log(1+2+3) = \log 1 + \log 2 + \log 3$

II. $(\log a)(\ln b) = (\ln a)(\log b)$

III. $\log(\ln a) = \ln(\log a)$

A. II B. I and II C. II and III D. All 3 E. NOTA

9. What is the range of $f(x) = 6^{2^{3^x}}$?

A. $(6, \infty)$ B. $[6, \infty)$ C. $(36, \infty)$ D. $[36, \infty)$ E. NOTA

The next 2 problems involve Knuth's up-arrow notation, which is used to denote operations of a higher order than exponentiation.

A single arrow ($\uparrow$) denotes exponentiation.
For example, $3 \uparrow 4 = 3^4 = 81$.

Two arrows ($\uparrow\uparrow$) denotes tetration, which is iterated exponentiation
For example, $3 \uparrow\uparrow 4 = 3 \uparrow (3 \uparrow (3 \uparrow 3)) = 3^{3^{3^3}} = 3^{3^{27}}$, which is a very large number.

Three arrows ($\uparrow\uparrow\uparrow$) denotes pentation, which is iterated tetration.
For example, $3 \uparrow\uparrow\uparrow 4 = 3 \uparrow\uparrow (3 \uparrow\uparrow (3 \uparrow\uparrow 3))$, which is an absurdly large number

In general, the operation with n arrows is the iteration of the operation with $n-1$ arrows.

It is important to note that iteration begins from the right side (note the placement of the parentheses in the examples).

10. Evaluate $5 \uparrow 2 + 5 \uparrow\uparrow 2$.

A. 50 B. 3150 C. 78125 D. 5^{27} E. NOTA

11. Given that, to 5 decimals places, $\log 5 = 0.69897$, how many digits does $5 \uparrow\uparrow 3$ have?

A. 87 B. 88 C. 2184 D. 2185 E. NOTA

12. If $a = \ln(3)$ and $b = \ln(7)$, what is $\ln\left(e^3 \cdot (2.\overline{3})\right)$ in terms of a and b?

A. $3 + b - a$ B. $\frac{3a}{b}$ C. $3 + \frac{a}{b}$ D. $a + b - 3$ E. NOTA

13. Evaluate $\sum_{n=1}^{\infty}(\frac{2}{7})^n$

A. $\frac{2}{5}$ B. $\frac{7}{5}$ C. $\frac{7}{2}$ D. 1 E. NOTA

14. Define the "log mean" of a set of numbers to be 10 raised to the power of the average of the base 10 logarithms of the numbers. For example, The log mean of 4, 5, and 6 would be $10^{\frac{\log 4+\log 5+\log 6}{3}}$.What is the ratio of the log mean of 3, 6, and 12 to the geometric mean of 3, 6, and 12?

A. $\frac{\sqrt[3]{6}}{6}$ B. $\frac{7}{6}$ C. 1 D. $\frac{6}{7}$ E. NOTA

15. Given that x is a positive integer, simplify $\sqrt{x + \sqrt{x + \sqrt{x+\ldots}}} - \sqrt{x - \sqrt{x - \sqrt{x-\ldots}}}$.

A. $\sqrt{4x+1}$ B. 1 C. $\sqrt{4x-1}$ D. 0 E. NOTA

16. What is the coefficient of the eighth term in the expansion of $(x - 2y)^{10}$?

A. 11520 B. -15360 C. 45 D. -120 E. NOTA

17. What is the greatest integer less than $15626^{\frac{1}{3}} - 650^{\frac{1}{4}}$?

A. 20 B. 19 C. 0 D. -1 E. NOTA

18. Let $f(x) = 3x + 1$. Assuming $f^n(x)$ denotes the function f iterated n times, what is $f^6(1)$?

A. 364 B. 3280 C. 1093 D. 2188 E. NOTA

19. $\log_{3\sqrt{3}}\left(\sqrt[5]{3\cdot\sqrt{27\cdot\sqrt[3]{9}\cdot\sqrt[4]{3}}}\right)$

A. $\frac{157}{60}$ B. $\frac{71}{120}$ C. $\frac{71}{180}$ D. $\frac{157}{90}$ E. NOTA

20. $\sqrt{84\ +\ 16\sqrt{5}} = a + b\sqrt{c}$ where a, b, and c are positive integers and c is not divisible by the square of any integer greater than 1. What is $a + b + c$?

A. 7 B. 8 C. 9 D. 10 E. NOTA

21. What is the coefficient of x^4y^2z in the expansion of $(1 + x + y + z)^8$?

A. 420 B. 840 C. 35 D. 140 E. NOTA

22. Evaluate $\left(\frac{-1}{2} + \frac{\sqrt{3}}{2}i\right)^4$

A. $\frac{1}{2} + \frac{\sqrt{3}}{2}i$ B. $\frac{-1}{2} - \frac{\sqrt{3}}{2}i$ C. $\frac{-1}{2} + \frac{\sqrt{3}}{2}i$ D. 1 E. NOTA

23. Solve the equation $5^{\ln x}\cdot x^{\ln 5} - 5^{\ln(ex)} - 6 = 0$

A. $6^{\frac{1}{ln(5)}}$ B. $5^{\frac{1}{ln(6)}}$ C. $\frac{1}{6^{ln(5)}}$ D. $\frac{1}{5^{ln(6)}}$ E. NOTA

24. Compute $\sqrt{6 + 2\sqrt{5}} - \sqrt{6 - 2\sqrt{5}}$

A. 2 B. 4 C. $2\sqrt{2}$ D. $2\sqrt{5}$ E. NOTA

25. If $\log_3(a^3 + b^3) - 2\log_3 b = \log_9(a^2 + 2ab + b^2) + 2$, find $\frac{a}{b}$ given that $a, b > 0$.

A. $\frac{1+\sqrt{33}}{2}$ B. $\frac{1\pm\sqrt{33}}{2}$ C. $\frac{-1+\sqrt{33}}{2}$ D. $\frac{-1\pm\sqrt{33}}{2}$ E. NOTA

26. Evaluate $\log_{16\sqrt{2}} 65536$.

A. 4 B. $\frac{28}{9}$ C. $\frac{9}{32}$ D. $\frac{10}{3}$ E. NOTA

27. Find the ratio of the mantissa of $\log_2 9$ to the mantissa of $\log_4 7$.

A. 3 B. $\log_{\frac{7}{4}}\left(\frac{81}{64}\right)$ C. $\log_{\sqrt{7}} 9$ D. $\log_{\frac{81}{64}}\left(\frac{7}{4}\right)$ E. NOTA

28. Express as a single logarithm: $\frac{3}{\log_2 7} + 2\log_{0.5} 4$

A. $\log_2\left(\frac{343}{16}\right)$ B. $\log_7\left(\frac{8}{7}\right)$ C. $\log_7\left(\frac{8}{2401}\right)$ D. $\log_2\left(\frac{343}{2}\right)$ E. NOTA

29. If x and y are real numbers such that $a = 3^x$ and $b = 3^y$, what is the value of $\log_a b$?

A. x^y B. xy C. $\frac{x}{y}$ D. $\frac{y}{x}$ E. NOTA

30. This last question will be a log question. Daniel can make $\log_3 k$ cuts into a log per hour, with each cut cutting the section the cut is on into two. Mr. Frazer tasks Daniel with cutting a giant log into 19 pieces in k hours, where k is a positive integer, which Daniel should be able to complete just in time. What is the value of k?

A. 7 B. 8 C. 9 D. 10 E. NOTA