1. B $\frac{ln(x)}{1+ln(y)} = e \rightarrow \frac{1+ln(y)}{ln(x)} = \frac{1}{e} \rightarrow ln(y) = \frac{ln(x)}{e} - 1 \rightarrow y = (e^{ln(x)})^{\frac{1}{e}} * e^{-1}$ $= \frac{x^{\frac{1}{e}}}{e}$.

2. A Substitute $log_4(x) = \frac{log_2(x)}{2}$ for all occurrences of $log_4(x)$, multiply the equation by 2 to get rid of fractions, and combine like terms. This gives us $(log_2(x))^3 + 4(log_2(x))^2 - 52log_2(x) + 80 = 0$. This is simply a cubic in $log_2(x)$. Using the rational root theorem, we can determine that $log_2(x) = 4, 2, -10$, so $x = 16, 4, \frac{1}{1024}$. Thus the product of the largest and smallest solutions is $16 * \frac{1}{1024} = \frac{1}{64}$.

3. B First, consider $log_4(3) * log_9(7)$. When multiplying logarithms, we can switch the bases, so this equals $log_9(3) * log_4(7) = \frac{log_4(7)}{2} = \frac{log_2(7)}{4} = log_2(7^{\frac{1}{4}})$. Adding $log_2(49)$, we get $log_2(7^{\frac{1}{4}}) + log_2(49) = log_2(7^{\frac{1}{4}} * 7^2)$. Raising 2 to this power gives us $7^{\frac{9}{4}}$ or $49\sqrt[4]{7}$.

4. D The polynomial inside the logarithm is equal to $(2n-3)^4$, so our equation becomes $log_3((2n-3)^4) = 8 \rightarrow 4log_3(|2n-3|) = 8$. We need the absolute value since $2n-3$ may be positive or negative. Continuing, we have $log_3(|2n-3|) = 2 \rightarrow |2n-3| = 9$. Thus we have $2n-3 = 9 \rightarrow n = 6$ or $2n-3 = -9 \rightarrow n = -3$, which are both valid solutions.

5. A Using change of base, we have $log_2(k) = \frac{ln(k)}{ln(2)}$. Thus, we have $ln(ln(x)) + \frac{ln(ln(x))}{ln(2)} = \frac{ln(4e^2)}{ln(2)}$. Multiplying by $ln(2)$ and factoring the left side, we get $ln(ln(x))(1 + ln(2)) = ln(4e^2)$. Luckily, $ln(4e^2) = ln(4) + ln(e^2) = 2 + 2ln(2)$, so we have $ln(ln(x)) = 2 \rightarrow ln(x) = e^2 \rightarrow x = e^{e^2}$.

6. A This is equivalent to $2^{3\log_2 3} = 2^{\log_2 27} = 27$.

7. C Using the extended version of the binomial theorem, the fourth term is $C(\frac{2}{3}, 3) * m^{\frac{-7}{3}} * n^3$. $C(\frac{2}{3}, 3) = \frac{\frac{2}{3}*\frac{-1}{3}*\frac{-4}{3}}{3!} = \frac{4}{81}$, so our answer is $\frac{4}{81}m^{\frac{-7}{3}}n^3$.

8. B
   I. The right side is $log(6)$ while the left side is $log(1*2*3) = log(6)$. TRUE (Note: it is NOT true in general that $log(a+b) = log(a) + log(b)$.
   II. Using change of base, we get that $log(a) * ln(b) = \frac{\ln(a)}{\ln(10)} * ln(b) = ln(a) * \frac{\ln(b)}{\ln(10)} = ln(a) * log(b)$.TRUE
   III. $a = 10$ is an easy counterexample to this. FALSE

9. A 3^x has the range $(0,\infty)$. 2^{3^x}can be 2 to the power of any number in the range $(0,\infty)$, so its range is $(1,\infty)$. Finally $6^{2^{3^x}}$ can be 6 to the power of anything in the range $(1,\infty)$, so its range is $(6,\infty)$.

10. B $5 \uparrow 2 = 5^2 = 25$. $5 \uparrow\uparrow 2 = 5 \uparrow 5 = 5^5 = 3125$. $25 + 3125 = 3150$

11. D $5 \uparrow\uparrow 3 = 5 \uparrow (5 \uparrow 5) = 5^{5^5}$. Since $5^5 = 3125$, this equals 5^{3125}. Taking the base 10 logarithm, we get $3125log(5)$. We are given that $log(5) =$

$.69897$, so$3125log(5) = 2184.28$ (approximately), meaning that $5^{3125} = 10^{2184.28}$. Thus our original number has 2185 digits.

12. A Since $2.\bar{3} = \frac{7}{3}$, we can split this up as $\ln(e^3) + \ln(7) - \ln(3) = 3 + b - a$.

13. A This is a simple infinite geometric series with the sum $\frac{\frac{2}{7}}{1-\frac{2}{7}} = \frac{2}{5}$.

14. C Taking the log mean of x, y, and z, we have $10^{\frac{log(x)+log(y)+log(z)}{3}} = 10^{\frac{log(xyz)}{3}} = (10^{log(xyz)})^{\frac{1}{3}} = \sqrt[3]{xyz}$, which is the same as the geometric mean of x, y, and z. Thus our ratio equals 1.

15. B $y = \sqrt{x + \sqrt{x + \sqrt{x+\ldots}}} \rightarrow y = \sqrt{x+y} \rightarrow y^2 - y - x = 0 \rightarrow y = \frac{1+\sqrt{1+4x}}{2}$($y$ must be positive, so we ignore the negative solution to the quadratic.

$y = \sqrt{x - \sqrt{x - \sqrt{x-\ldots}}} \rightarrow y = \sqrt{x-y} \rightarrow y^2 + y - x = 0 \rightarrow y = \frac{-1+\sqrt{1+4x}}{2}$.

Therefore, the difference is $\frac{1+\sqrt{1+4x}}{2} - \frac{-1+\sqrt{1+4x}}{2} = 1$.

16. B $C(10,3) * x^3(-2y)^7 = -15360x^3y^7$, so the coefficient is -15360.

17. B $25^3 = 15625$, so$15626^{\frac{1}{3}}$ is slightly greater than 25. $5^4 = 625$, so $650^{\frac{1}{4}}$ is slightly greater than 5. However the amount by which $650^{\frac{1}{4}}$ exceeds 5 is greater than the amount by which $15626^{\frac{1}{3}}$ exceeds 25. Thus the value inside the greatest integer function is slightly less than $25 - 5$, so the answer is 19.

18. C $f(1) = 3(1) + 1$. $f^2(1) = 3(3(1) + 1) + 1 = 3^2 + 3 + 1$. This can be extended as $f^n(x) = 3^n + 3^{n-1} + \ldots + 3 + 1$. Therefore, $f^6(x) = 3^6 + 3^5 + \ldots + 3 + 1 = \frac{3^7-1}{3-1} = 1093$.

19. C
$$log_{3\sqrt{3}}(3^{\frac{1}{5}} * (27^{\frac{1}{2}})^{\frac{1}{5}} * ((9^{\frac{1}{3}})^{\frac{1}{2}})^{\frac{1}{5}} * ((3^{\frac{1}{4}})^{\frac{1}{2}})^{\frac{1}{5}}) = log_{3\sqrt{3}}(3^{\frac{1}{5}} * 3^{\frac{3}{10}} * 3^{\frac{1}{15}} * 3^{\frac{1}{40}})$$
$$= log_{3\sqrt{3}}(3^{\frac{71}{120}}) = \frac{\frac{71}{120}}{\frac{3}{2}} = \frac{71}{180}.$$

20. E $84 + 16\sqrt{5} = 84 + 2\sqrt{320}$. We can find $\sqrt{84 + 2\sqrt{320}}$ by finding 2 numbers that sum to 84 and have a product of 320. Since 80 and 4 satisfy these criteria, $\sqrt{84 + 2\sqrt{320}} = \sqrt{4} + \sqrt{80} = 2 + 4\sqrt{5}$. Thus, $a + b + c = 2 + 4 + 5 = 11$.

21. B $\frac{8!}{1!*4!*2!*1!}(1)^1(x)^4(y)^2(z)^1$. Thus the coefficient is $\frac{40320}{24*2} = 840$.

22. C This is one of the cube roots of unity, meaning that $(\frac{-1}{2} + \frac{\sqrt{3}}{2}i)^3 = 1$. Thus raising it to the fourth power gives us 1*$(\frac{-1}{2} + \frac{\sqrt{3}}{2}i) = \frac{-1}{2} + \frac{\sqrt{3}}{2}i$. If you did not notice this, you could have simply expanded it using the binomial theorem.

23. A We will start with the key realization of this problem. Clearly $ln(x) * ln(5) = ln(x) * ln(5)$. However we can further manipulate this equation as $ln(5^{ln(x)}) = ln(x^{1n(5)}) \rightarrow 5^{ln(x)} = x^{ln(5)}$. Using this insight, we can rewrite our problem's equation as $(5^{ln(x)})^2 - 5^{ln(ex)} - 6 \rightarrow (5^{ln(x)})^2 - 5 * 5^{ln(x)} - 6 \rightarrow (5^{ln(x)} -$

6)$(5^{ln(x)}+1)$, which means $5^{ln(x)}=6$ since $5^{ln(x)}=-1$ is impossible. Thus, $ln(x)=log_5(6)\rightarrow ln(x)=\frac{ln(6)}{ln(5)}\rightarrow x=6^{\frac{1}{ln(5)}}$.

24. A $x=\sqrt{6+2\sqrt{5}}-\sqrt{6-2\sqrt{5}}\rightarrow x^2=6+2\sqrt{5}+6-2\sqrt{5}-2\sqrt{16}=4\rightarrow x=2$.

25. A $log_9(a^2+2ab+b^2)=log_3(a+b)$ since a and b are both positive. For our problem's equation, we have $log_3(a^3+b^3)-log_3(a+b)-2log_3(b)=2\rightarrow log_3(\frac{a^3+b^3}{a+b})-log_3(b^2)=2\rightarrow log_3(\frac{a^2-ab+b^2}{b^2})=2\rightarrow(\frac{a}{b})^2-\frac{a}{b}+1=9\rightarrow(\frac{a}{b})^2-\frac{a}{b}-8=0$. This is a quadratic in $\frac{a}{b}$, so $\frac{a}{b}=\frac{1+\sqrt{33}}{2}$ (we ignore the negative solution since a and b are positive.)

26. E $65536=2^{16}$ and $16\sqrt{2}=2^{\frac{9}{2}}$, so our answer is $\frac{16}{\frac{9}{2}}=\frac{32}{9}$.

27. B Since the characteristic of $log_2(9)$ is 3 and the characteristic of $log_4(7)$ is 1, the ratio of the mantissas is $\frac{log_2(9)-3}{log_4(7)-1}=\frac{log_2(\frac{9}{8})}{log_4(\frac{7}{4})}=\frac{log_4(\frac{81}{64})}{log_4(\frac{7}{4})}=log_{\frac{7}{4}}(\frac{81}{64})$.

28. C $\frac{3}{log_2(7)}+2log_{0.5}(4)=3log_7(2)-4=log_7(8)-log_7(2401)=log_7\left(\frac{8}{2401}\right)$.

29. D $(3^x)^k=3^y$, so $k=\frac{y}{x}$.

30. C Daniel will make a total of $klog_3(k)$ cuts in the log. Since cutting a log into 19 pieces requires 18 cuts, we have $klog_3(k)=18$. Guessing and checking, we find that $k=9$.