Unless otherwise denoted, $i = \sqrt{-1}$.

1. Calculate the harmonic mean of 6, 15, and 24.

   A. $\frac{60}{11}$ B. $\frac{120}{11}$ C. $\frac{240}{11}$ D. $\frac{30}{11}$ E. NOTA

2. Calculate the geometric mean of 12 and 42.

   A. $3\sqrt{7}$ B. $3\sqrt{14}$ C. $6\sqrt{7}$ D. $6\sqrt{14}$ E. NOTA

3. What is a_{131} in a sequence recursively defined as $a_{n-1} = \frac{2a_n - 5}{2}$, where $a_2 = 100$?

   A. 222.5 B. 420 C. 422.5 D. 425 E. NOTA

4. Find the sum of the entries in *m*:

$$m = \prod_{n=1}^{8} \begin{bmatrix} n & 0 \\ 0 & 1 \end{bmatrix}$$

   A. 40321 B. 39521 C. 5041 D. 37 E. NOTA

5. For some positive integer c, the repeating base-c representation of the base-ten fraction $\frac{25}{143}$ is $0.\overline{21}_c$. Find c.

   A. 10 B. 12 C. 18 D. 24 E. NOTA

6. How many terms does the arithmetic sequence 5, 28, 51, 74, … 1408 have?

   A. 59 B. 60 C. 61 D. 62 E. NOTA

7. Solve the equation for x: $x = \sqrt{1+\sqrt{7+\sqrt{1+\sqrt{7+\sqrt{1+\cdots}}}}}$

A. 1 B. 2 C. $\frac{3}{2}$ D. 3 E. NOTA

8. How many integers in the interval $[1000,10000)$ are divisible by 5 and 9?

A. 201 B. 200 C. 199 D. 198 E. NOTA

9. Evaluate:

$$\sum_{n=1}^{35} \frac{n^2}{5}$$

A. 3024 B. 2982 C. 1554 D. 1512 E. NOTA

10. Compute:

$$\sum_{k=1}^{\infty} \frac{1}{k^2+2k}$$

A. $\frac{3}{4}$ B. $\frac{1}{2}$ C. 1 D. 2 E. NOTA

11. What is the sum of the integers from 242 to 456 inclusive?

A. 75035 B. 77025 C. 77035 D. 77085 E. NOTA

12. Find the units digit of m.

$$m = \sum_{n=0}^{101} n!$$

A. 2 B. 3 C. 4 D. 6 E. NOTA

13. Anjana writes out all the integers from 1 to 409 inclusive. How many digits did she write?

A. 1110 B. 1119 C. 1120 D. 1121 E. NOTA

14. Compute:

$$\sum_{n=1}^{100}\sum_{m=1}^{100}\frac{1}{i^{mn}}$$

A. $-i$ B. i C. 0 D. -1 E. NOTA

15. $4-\frac{4}{4-\frac{4}{4-\frac{4}{4-\cdots}}}=a$. Find a.

A. 1 B. 2 C. 3 D. $\frac{7}{2}$ E. NOTA

16. If Casie has a deck of cards and draws 3 cards without replacement, what's the probability she draws all face cards or she draws all red cards?

A. $\frac{28}{221}$ B. $\frac{42}{221}$ C. $\frac{28}{1105}$ D. $\frac{141}{1105}$ E. NOTA

17. If the number 64a3b75 is divisible by 11 and the sum $a+b$ is as small as possible, what is the remainder when 76a3b2 is divided by 11?

A. 1 B. 7 C. 9 D. 10 E. NOTA

18.

$$\sum_{x=0}^{\infty}\frac{2x+4}{2^x}$$

A. 10 B. 12 C. 14 D. 16 E. NOTA

19. Compute $(1-i)^{100}+(1+i)^{100}$.

A. 0 B. -2^{100} C. -2^{50} D. 2^{51} E. NOTA

20. A fly starts at the origin and travel in a repetitive motion: right, up, left, down. It first travels 4 units right, 3 units up, 2 units left, 1 unit down. Then half as far in each direction – 2 units right, 1.5 units up, 1 units left, 0.5 units down. If the pattern repeats infinitely, what is the distance it ends up from the origin?

A. 4 B. $4\sqrt{2}$ C. 8 D. $8\sqrt{2}$ E. NOTA

21. Devika and Navya are playing a game, taking turns flipping a coin. The way for Devika to win is for her to flip heads, and the way for Navya to flip tails. Whoever flips their respective winning side first wins the game, and the game ends. The game continues to the next person's flip otherwise. Devika goes first. What's the probability Navya wins?

A. $\frac{1}{4}$ B. $\frac{1}{3}$ C. $\frac{1}{2}$ D. $\frac{2}{3}$ E. NOTA

22. If I write out all the perfect squares from 1 to 2500 inclusive, how many digits will I write?

A. 149 B. 154 C. 157 D. 161 E. NOTA

23. $\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\cdots}}}} = y$. Find y.

A. 2 B. 3 C. 4 D. 6 E. NOTA

24. A geometric sequence has a first term $2 + i$ and the second term $9 + 2i$. Find the sum of the first four terms in the sequence.

A. $154 - 54i$ B. $151 - 42i$ C. $200 - 54i$ D. $200 - 40i$ E. NOTA

25. The first three terms in a geometric sequence are $\sqrt[8]{7}$, $\sqrt[12]{7}$, $\sqrt[24]{7}$. What is the next term?

A. $\sqrt[32]{7}$ B. $\sqrt[40]{7}$ C. $\sqrt[48]{7}$ D. 1 E. NOTA

26. Hannah is playing a game with a fair six-sided die. If Hannah rolls a perfect square, she earns 20 dollars, but if she rolls a prime number, she loses 10 dollars. If she rolls anything else, she earns zero dollars. After 10 rolls, what is her total expected value for her earnings?

A. $\frac{5}{3}$ B. 5 C. 12.5 D. $\frac{50}{3}$ E. NOTA

27. The polynomial $4x^3 + 6x^2 - bx - 7.5$ has roots in arithmetic progression. What is the middle root?

A. $-\frac{5}{2}$ B. $\frac{-1}{2}$ C. $\frac{3}{2}$ D. $\frac{5}{2}$ E. NOTA

28. $\{a_n\}_{1\leq n\leq 177}$ is an arithmetic sequence with $a_1 = 3$ and $a_{177} = 2024$. Evaluate:

$$\sum_{i=1}^{176} \frac{1}{a_i a_{i+1}}$$

A. $\frac{1}{69}$ B. $\frac{3}{138}$ C. $\frac{1}{23}$ D. $\frac{2}{23}$ E. NOTA

29. Let S be the sum of the first 2022 perfect squares (where 1 is the first). How many total factors does S have?

A. 16 B. 24 C. 48 D. 96 E. NOTA

30. You've reached the end of the test! To reward you, please solve the following series.

$$\sum_{k=1}^{\infty} k\left(\frac{2}{3}\right)^k$$

A. $\frac{3}{2}$ B. 4 C. 6 D. $\frac{15}{2}$ E. NOTA