Good luck! :)

1. Terri and Avery are playing a game where each player takes turns flipping two fair coins on their turn. If Avery flips two heads on his turn, he wins, and if Terri flips at least one head on her turn, she wins. If Terri goes first in this game, what is the probability she wins?

    A. $\frac{5}{6}$ B. $\frac{12}{13}$ C. $\frac{6}{7}$ D. $\frac{1}{2}$ E. NOTA

2. Find $i^6 + i^7 + i^8 + \cdots + i^{2024}$.

    A. i B. $1 - i$ C. $-i$ D. i E. NOTA

3. Every two minutes, the number of words William writes per minute doubles. (It does not change otherwise.) If he starts out writing at 12 words per minute and writes continuously, how many words he would have written after 10 minutes?

    A. 724   B. 744   C. 880   D. 1200   E. NOTA

4. Define $a_n = a_{n-1} + a_{n-2} + 1$ for $n > 1$. If $a_0 = 2, a_1 = 3$, find $a_{101} - 2a_{99} - a_{98}$.

    A. 2   B. 64   C. 99   D. 198   E. NOTA

5. Let $a_n = 2a_{n-1} + 1$ for $n > 1$ with $a_0 = 1$. Which of the following correctly describes the sequence a_n?

    A. Arithmetic   B. Quadratic   C. Harmonic   D. Geometric   E. NOTA

6. A $f(x)$-sequence is defined as the sequence $\{f(1), f(2), \ldots\}$. For example, the x^2-sequence is $\{1, 4, 9, 16 \ldots\}$. A polynomial $f(x)$ has the $f(x)$-sequence starting with $\{0, 1, 12, 38, 84, 155, \ldots\}$. Find the minimum possible degree of $f(x)$.

    A. 2   B. 3   C. 4   D. 5   E. NOTA

7. Evaluate $4 + \cfrac{5}{4 + \cfrac{5}{4 + \ldots}}$.

    A. -1 B. 4   C. 5   D. 9   E. NOTA

8. An infinite geometric series has a first term of 1 and a common ratio of r. If the sum of the series is $4r^2 - 12r + 7$, what is the sum of the reciprocals of the possible values of r.

    A. 4   B. $\frac{19}{6}$ C. 2   D. 3   E. NOTA

9. Right triangle RKG has side lengths that form an arithmetic sequence. If the area of RKG is 96, what is the length of the hypotenuse?

    A. 16   B. 18   C. 20   D. 24   E. NOTA

10. Find the sum of the infinite geometric series with first term of $3 + 2i$ and second term of $1 + 5i$, where $i = \sqrt{-1}$.

    A. $\sqrt{13}$ B. $2 - 3i$ C. $2 + 3i$ D. Series Diverges   E. NOTA

11. Bob the Builder's crew of 10 are numbered from 1 to 10. Worker n builds $\frac{1}{n^2+n}$ block in 1 hours. If the workers need to build 30 blocks, how many hours will that take them?

    A. 11   B. 22   C. 33   D. $\frac{100}{3}$ E. NOTA

12. Albert writes a 8th degree function $f(x)$ where each term can be expressed as $a \cdot (2x)^n$ where $0 \le n \le 8$. If $f(1) = 2048 - b, f(-1) = -2 + b$, find the sum of the digits of b.

    A. 14   B. 11   C. 6   D. 4   E. NOTA

13. Jonathan writes out odd numbers in one long list of digits. (The list starts 135791113…) What is the 53rd digit he writes?

    A. 1   B. 3   C. 5   D. 7   E. NOTA

14. The sum of an infinite geometric series is 6. If Rubert squares every term of the series, the new sum is 144. Find the first term of the original series.

    A. $\frac{96}{13}$ B. 7   C. $\frac{66}{7}$ D. $\frac{48}{5}$ E. NOTA

Use the following information for questions 15 and 16:
The Wheel of Fortune has 20 sections numbered 1,2,3...20. Chris spins the wheel 3 times per round, each spin landing on a random DISTINCT section. His score is the sum of all the numbers the wheel lands on in that round.

15. What is the 20th least possible score Chris can get in a single round?
    A. 20   B. 22   C. 25   D. 30   E. NOTA

16. What is the sum of all possible scores Chris can spin?
    A. 630   B. 1638   C. 1827   D. 1950   E. NOTA

17. Evaluate:
$$3\sum_{n=0}^{\infty}\frac{n2^n+1}{4^n}$$
    A. 8   B. 10   C. 16   D. 20   E. NOTA

18. Equiangular hexagon RUIWEN has RU, UI, and IW in an arithmetic sequence in that order, with $RU = a$, and common difference of a, and WE, EN, and NR in a geometric sequence in that order, with $WE = b$, and common ratio of b. If the largest side of the hexagon is k, find $81k$..
    A. 41   B. 81   C. 360   D. 375   E. NOTA

19. Let r, s, t be the roots of $12x^3 - 6x^2 - 10x + 6 = 0$. Given that $|r|, |s|, |t| < 1$, find $(1 + r + r^2 + \cdots)(1 + s + s^2 + \cdots)(1 + t + t^2 \dots)$.
    A. $\frac{1}{3}$ B. $\frac{1}{2}$ C. 1   D.   2   E. NOTA

20. abc_n is a 3-digit number in base n. (Note that abc is the concatenation of the 3 digits, not the product of the 3 digits.) If the digits a, b, c form a geometric series with 3 distinct terms, find the minimum value of $a + b + c + n$.
    A. 5   B. 12   C. 15   D. 23   E. NOTA

21. A circle is inscribed in a square with side length of 2, and a square is inscribed in that circle. If this is repeated infinitely, what is the sum of the areas of all the circles?

    A. $\frac{\pi}{2}$ B. π C. 2π D. 4π E. NOTA

22. If 3 is the 3rd term of a geometric sequence and 9 is the 9th term, which of the following would be the 12th term?

    A. $9\sqrt{3}$ B. 27   C. $27\sqrt{3}$ D. 81   E. NOTA

23. Find the sum of the first 50 nonnegative integers.
    A. 1200   B. 1225   C. 1250   D. 1275   E. NOTA

24. The value of
$$\frac{2^2}{(2^2-1)} \cdot \frac{3^2}{(3^2-1)} \cdot \ldots \cdot \frac{100^2}{(100^2-1)} = \frac{m}{n}$$
    in simplest form. Find $m + n$.
    A. 201   B. 301   C. 401   D. 501   E. NOTA

25. The value of
$$\frac{2^2}{(2^2-1)} + \frac{3^2}{(3^2-1)} + \cdots + \frac{10^2}{(10^2-1)} = \frac{m}{n}$$
    in simplest form. Find $m + n$.
    A. 586   B. 591   C. 596   D. 603   E. NOTA

26. Find $\sum_{n=1}^{\infty} \frac{F_n}{2^n}$, where F_n represents the nth term of the Fibonacci sequence defined as $F_0 = 0, F_1 = 1, F_n = F_{n-1} + F_{n-2}$ for $n > 1$.
    A. 2   B. $\frac{19}{5}$ C. 4   D. $\frac{25}{6}$ E. NOTA

27. The first three terms of an arithmetic series sum to 15 while the first 9 terms sum to 36. Find the sum of the first 15 terms.
    A. 15   B. 30   C. 36   D. 45   E. NOTA

28. The sum of 12 consecutive even numbers is 372. Find the greatest of the 12 numbers.
    A. 20   B. 31   C. 42   D. 53   E. NOTA

29. Tomly's infinitely many friends are sharing a pie. The first friend takes $\frac{2}{5}$ of the pie, the second takes $\frac{1}{5}$, and so on such that each friend takes half of what the previous one took. If Tomly took the remainder of the pie for himself, what fraction of it did he eat?
    A. $\frac{1}{10}$ B. $\frac{1}{5}$ C. $\frac{2}{5}$ D. $\frac{1}{2}$ E. NOTA

30. The number 2024 is written as the sum of n consecutive integers. Find the number of possible values of n.
    A. 8   B. 12   C. 16   D. 32   E. NOTA