1. B The harmonic mean of three numbers x, y, and z is $\frac{3xyz}{xy+yz+xz}$. So for 6, 15, and 24, the harmonic mean would be $\frac{3*6*15*24}{6*15+6*24+15*24} = \frac{120}{11}$

2. D The geometric mean of a and b is $\sqrt{ab}$, so the geometric mean of 12 and 42 is $\sqrt{12*42} = \sqrt{4*3*3*14} = 6\sqrt{14}$

3. C
$$2a_{n-1} = 2a_n - 5$$
$$a_n = a_{n-1} + \frac{5}{2}$$
Each consecutive term increase by $\frac{5}{2}$
$$a_n = a_1 + (n-1)\frac{5}{2}$$
Using the given information: $a_1 = a_2 - \frac{5}{2} = \frac{195}{2}$
$$a_{131} = \frac{195}{2} + (131-1)\frac{5}{2}$$
$$a_{131} = \frac{195}{2} + 325 = \frac{845}{2}$$

4. A
$$\prod_{n=1}^{8}\begin{bmatrix} n & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1\times 2\times 3\times \ldots\times 8 & 0 \\ 0 & 1 \end{bmatrix}$$
$$\prod_{n=1}^{8}\begin{bmatrix} n & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 40320 & 0 \\ 0 & 1 \end{bmatrix}$$
The sum of the entries is $40320 + 1 = 40321$

5. B $0.\overline{21}_c = 2*c^{-1} + 2*c^{-3} + 2*c^{-5} \ldots + 1*\ c^{-2} + 1*c^{-4} + 1*c^{-6} \ldots$ So this becomes two infinite geometric sums with the same common ratio, but different first terms. We can now simplify the sum to $\frac{\frac{2}{c}}{1-\frac{1}{c^2}} + \frac{\frac{1}{c^2}}{1-\frac{1}{c^2}} = \frac{2c+1}{c^2-1} = \frac{25}{143}$. When you solve for c, the two values are $\frac{-14}{25}$ and 12. Because a base can only be positive, the correct answer is 12.

6. D The common difference in this sequence is 23. Therefore we subtract the first number from the last number, divide the difference by 23, and add 1 to get the number of terms in the sequence. This becomes $\frac{1408-5}{23} + 1 = 62$.

7. B The pattern eventual repeats, so it becomes $\sqrt{1+\sqrt{7+x}} = x$. Getting rid of the all the radicals, we get $x^4 - 2x^2 + 1 = 7 + x$. Solving for x, we get x = 2.

8. B The Least common multiple of 5 and 9 is 45. Any number divisible by 5 and 9, has a factor of 45.
$23(45) = 1035$ and $222(45) = 9990$
There are $222 - 23 + 1 = 200$ valid numbers on the interval.

9. B

$$\sum_{n=1}^{75}\left(\frac{n^2}{5}\right)=\frac{1}{5}\sum_{n=1}^{75}(n^2)$$

Common Summations → $\sum_{k=1}^{n}(k^2)=\frac{n(n+1)(2n+1)}{6}$

$$\sum_{n=1}^{35}\left(\frac{n^2}{5}\right)=\frac{1}{5}\left[\frac{35(35+1)(2(35)+1))}{6}\right]$$

$$\sum_{n=1}^{35}\left(\frac{n^2}{5}\right)=\frac{7(35+1)(2(35)+1))}{6}$$

$$\sum_{n=1}^{35}\left(\frac{n^2}{5}\right)=7(6)(71))$$

$$\sum_{n=1}^{35}\left(\frac{n^2}{5}\right)=42\times 71=2982$$

10. A We can rewrite the expression as $\frac{A}{k}+\frac{B}{k+2}$. So $A(k+2)+B(k)=1$. Therefore $A+B=0, and\ 2A=1, so\ A=\frac{1}{2}, and\ B=\frac{-1}{2}$. Therefore it becomes $\frac{\frac{1}{2}}{k}-\frac{\frac{1}{2}}{k+2}$. This becomes $\frac{\frac{1}{2}}{1}-\frac{\frac{1}{2}}{3}+\frac{\frac{1}{2}}{2}-\frac{\frac{1}{2}}{4}+\frac{\frac{1}{2}}{3}$. Eventually this becomes a telescoping series that cancels out except $\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$

11. A $\frac{456*457}{2}-\frac{241*242}{2}=75035$. This is the answer.

12. C

$$m=\sum_{n=0}^{101}n!=1+1+2+6+24+120+720+\cdots$$

$n!$ will be a multiple of 10 for $n>4$

$1+1+2+6+4=14$ → The units digit is 4

13. B Anjana writes one digit for every number 1-9, two digits for every number 10-99, and three digits for every number 100-409. This becomes $1*(9-1+1)+2*(99-10+1)+3*(409-100+1)=1119$

14. E Consider $\frac{1}{i^{mn}}$ for values of m mod 4 as n ranges from 1 to 100.

0. mn is always 0 mod 4. So $\frac{1}{i^{mn}}$ is always 1, thus sum to 100.
1. mn cycles $1,2,3,0$ mod 4. So $\frac{1}{i^{mn}}$ cycles $-i,-1,i,1$, thus sum to 0.
2. mn cycles $2,0$ mod 4. So $\frac{1}{i^{mn}}$ cycles $-1,1$, thus sum to 0.
3. mn cycles $3,2,1,0$ mod 4. So $\frac{1}{i^{mn}}$ cycles $i,-1,-i,1$, thus sum to 0.

There are 25 values of m that are 0 mod 4, so the final sum is 2500.

15. B This is in a repeating form so it becomes $4-\frac{4}{x}=x$, so it becomes $x^2-4x+4=0. x=2$.

16. A She is choosing 3 cards from the deck, so the denominator is $52\ C\ 3$. There are 12 face cards, so the number of combinations to choose 3 face cards is $12\ C\ 3$. There are

26 red cards, so the number of combinations to choose 3 red cards is $26 \complement 3$. But because it says the word "or," we have to take into account the red face cards, which there are 6 of those. So the total combinations to choose 3 red face cards is $6 \complement 3$. So the final probability is $\frac{26 \complement 3 + 12 \complement 3 - 6 \complement 3}{52 \complement 3} = \frac{28}{221}$

17. A If 64a3b75 is divisible by 11, $6 + a + b + 5 - (4 + 3 + 7) = 11 + a + b - 14 = a + b - 3$. So the smallest value $a + b$ can be is if the original expression is 0, so $a + b = 3$. If we apply it to the other number $7 + a + b - (6 + 3 + 2) = 7 + a + b - 11 = a + b - 4 = 3 - 4 = -1$. Therefore the remainder is 1.

18. B We can split the expression up into $2\left(\frac{x}{2^x} + \frac{2}{2^x}\right)$. If we look at the first term $\frac{x}{2^x}$, we can write it as $\frac{1}{2^1} + \frac{2}{2^2} + \frac{3}{2^3} + \frac{4}{2^4} \ldots = S$. Therefore, $2S = 1 + \frac{2}{2^1} + \frac{3}{2^2} + \frac{4}{2^3} \ldots$ So subtracting the two equations, $S = 1 + \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} \ldots = \frac{1}{1-\frac{1}{2}} = 2$. So two times that becomes 4. Now looking at the second expression in the statement, $\frac{2}{1} + \frac{2}{2^1} + \frac{2}{2^2} + \frac{2}{2^3} \ldots = \frac{2}{1-\frac{1}{2}} = 4$. Two times that becomes 8. Therefore, the final sum is $4 + 8 = 12$.

19. E $(1 - i)^2 = -2i$. And $(1 + i)^2 = 2i$. So the expression becomes $(-2i)^{50} + (2i)^{50}$. This becomes -2^{51}.

20. B This becomes an infinite geometric series. In the first round of flying, the fly flies a net of 2 units to the right, then the next round 1 unit, then 0.5, and so on, so it becomes $\frac{2}{1-\frac{1}{2}} = 4$. The same thing happens for the vertical units, so the total distance from the origin is $\sqrt{4^2 + 4^2} = 4\sqrt{2}$

21. B If Devika goes first, the probability of her not winning on the first try and Navya winning on her first try is $\frac{1}{2} * \frac{1}{2}$. The probability of Devika not winning on her second try as well and Navya winning on her second try is $\frac{1}{2} * \frac{1}{2} * \frac{1}{2} * \frac{1}{2}$. This becomes an infinite geometric pattern with first term $\frac{1}{4}$ and common ratio $\frac{1}{4}$. So the total series, the probability of Navya winning becomes $\frac{\frac{1}{4}}{1-\frac{1}{4}} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$.

22. C $1^2\ to\ 3^2$ is 3 numbers with 1 digit each. $4^2\ to\ 9^2$ is 6 numbers with two digits each. $10^2\ to\ 31^2$ is 22 numbers with 3 digits. $32^2\ to\ 50^2$ is 19 numbers with 4 digits each. The total amount of digits becomes $1 * 3 + 2 * 6 + 3 * 22 + 4 * 19 = 3 + 12 + 66 + 76 = 157$.

23. B Repeating form, so it becomes $\sqrt{6 + y} = y^2$. Squaring both sides, we get $y^2 - y - 6 = (y + 2)(y - 3)$. Since this sum must be positive, the correct answer is 3.

24. D If we divide the second term by the first term we get $\frac{9+i}{2+i} = 4 - i$. The third term is then $(9 + 2i)(4 - i) = 38 - i$. Fourth term is $(38 - i)(4 - i) = 151 - 42i$. The sum of the four terms is $200 - 40i$.

25. D Rewriting the terms in the sequence, we have $7^{\frac{1}{8}}, 7^{\frac{1}{12}}, 7^{\frac{1}{24}}$. We notice that it can also be written as $7^{\frac{3}{24}}, 7^{\frac{2}{24}}, 7^{\frac{1}{24}}$. So the common ratio is $7^{\frac{-1}{24}}$. So the next term in the sequence is $7^0 = 1$.

26. D The perfect square numbers from 1 to 6 are $1\ and\ 4$. So the probability of rolling a perfect square is $\frac{1}{3}$. The prime numbers from 1 to 6 are 2, 3, 5. So the probability of rolling a prime number is $\frac{1}{2}$. So the expected value would be $\frac{1}{3} * 20 - \frac{1}{2} * 10 = \frac{5}{3}$. This would be for one roll, so for ten rolls, the expected value is $\frac{50}{3}$

27. B $4x^3 + 6x^2 - bx - 7.5$ shows that the sum of the roots is $\frac{-3}{2}$ and $\frac{-15}{8}$. If we notice they're in arithmetic progression, we see that the three roots would be $\frac{-5}{2}, \frac{-1}{2}, \frac{3}{2}$. The middle root would then be $\frac{-1}{2}$.

28. E Let $a_{i+1} - a_i = d$. Then $\frac{1}{a_i a_{i+1}} = \frac{a_{i+1} - a_i}{d a_i a_{i+1}} = \frac{1}{d}\left(\frac{1}{a_i} - \frac{1}{a_{i+1}}\right)$. The sum therefore telescopes and is equal to $\frac{1}{d}\left(\frac{1}{a_1} - \frac{1}{a_{177}}\right) = \frac{1}{d}\left(\frac{a_{177} - a_1}{a_1 a_{177}}\right) = \frac{1}{d}\left(\frac{176d}{a_1 a_{177}}\right) = \frac{176}{3 \cdot 2024} = \frac{2}{69}$.

29. C The formula for the sum of squares from 1 to n is $\frac{n(n+1)(2n+1)}{6} = \frac{(2022)(2023)(4045)}{6}$ when you prime factorize it becomes $5 * 7 * 17^2 * 337 * 809$. So the number of factors is found by adding 1 to the exponents and multiplying them together, making $2 * 2 * 3 * 2 * 2 = 48$.

30. C If we look at the series, it is $1 * \frac{2}{3} + 2 * \left(\frac{2}{3}\right)^2 + 3 * \left(\frac{2}{3}\right)^3 + 4 * \left(\frac{2}{3}\right)^4 = S$. $\frac{3}{2}S = 1 + 2 * \left(\frac{2}{3}\right)^1 + 3 * \left(\frac{2}{3}\right)^2 + 4 * \left(\frac{2}{3}\right)^3$ ... If we subtract the two equations, we get $\frac{S}{2} = 1 + \left(\frac{2}{3}\right) + (\frac{2}{3})^2$ ... = $\frac{1}{1-\frac{2}{3}} = 3$. So $S = 6$.