The answer choice E. NOTA denotes that 'none of these answers' are correct. Diagrams *are* to scale. Problems are not necessarily in increasing order of difficulty, so don't be afraid to skip around. Good luck and have fun!

1. Legosi has six sticks of bamboo of lengths 1, 2, 3, 4, 5, and 6. He selects three of the sticks and arranges them into a non-degenerate triangle. What is the smallest possible perimeter of his triangle?
   A. 7 B. 6 C. 8 D. 9 E. NOTA

2. Triangle ABC is isosceles with $AB = AC$, if $m\angle C = 40°$, find $m\angle A$.
   A. $40°$ B. $70°$ C. $100°$ D. $80°$ E. NOTA

3. Square $ABCD$ has side length 6. Point X lies on $\overline{CD}$ such that $CX = 2$. Find the area of triangle ABX.
   A. 36 B. 18 C. 24 D. 12 E. NOTA

4. Triangles ABC and DEF are similar. If the area of triangle ABC is 4, the area of triangle DEF is 9, and the perimeter of triangle ABC is 12, find the perimeter of triangle DEF.
   A. 24 B. 18 C. 16 D. 27 E. NOTA

5. A triangle has interior angles with measures $(x + 36)°$, $(2x + 24)°$ and $(72 - x)°$. Find x.
   A. 12 B. 18 C. 24 D. 16 E. NOTA

6. A 5-12-13 right triangle and a 9-12-15 right triangle are glued together on their sides of length 12 to form a larger triangle. Find the perimeter of the resulting triangle. (Here an a-b-c right triangle is a right triangle with side lengths a, b, and c.)
   A. 42 B. 66 C. 54 D. 52 E. NOTA

7. Triangle ABC has $AB = 10$, $BC = 24$, and $AC = 26$. If M is the midpoint of $\overline{AC}$, find BM.
   A. 15 B. 12 C. 13 D. 12.5 E. NOTA

8. Triangle ABC has $AB = AC = 1$ and $m\angle A = 90°$. Point D lies in the same plane as triangle ABC and satisfies the property that $BD = BC$, $m\angle DBC = 90°$, and triangles ABC and DBC share no interior points. Find AD.
   A. 2 B. $\sqrt{3}$ C. $\sqrt{5}$ D. $\sqrt{6}$ E. NOTA

9. Triangle ABC has $AB = 5$, $BC = 12$, and a right angle at B. Let M be the midpoint of $\overline{BC}$, and let N lie on $\overline{AC}$ such that $\overline{MN}$ is perpendicular to $\overline{AC}$. Find MN.

A. $\frac{5}{2}$ B. $\frac{12}{5}$ C. $\frac{30}{13}$ D. $\frac{25}{12}$ E. NOTA

10. Triangle ABC has $AB = 5$, $BC = 7$, and $AC = 8$. Compute $\cos(B)$.

A. $\frac{11}{14}$ B. $\frac{1}{7}$ C. $\frac{1}{2}$ D. $\frac{2}{5}$ E. NOTA

11. Suppose that positive integers a, b, and c are the side lengths of a right triangle. If exactly two of a, b and c are even, find the smallest possible value of $a + b + c$.

A. 41 B. 67 C. 57 D. 81 E. NOTA

12. Triangle ABC has $AB = 3$, $AC = 4$, and a right angle at A. Points X, Y, and Z lie on $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ such that $AXYZ$ is a square. Find the side length of this square.

A. $\frac{12}{7}$ B. $\frac{12}{5}$ C. $\frac{15}{8}$ D. $\frac{20}{9}$ E. NOTA

13. Triangle ABC has $AB = 4$, $BC = 6$, and $AC = 5$. Point D lies on $\overline{BC}$ such that $\angle BAD \cong \angle CAD \cong \angle ACD$. Find AD.

A. $\frac{10}{3}$ B. $\frac{8}{3}$ C. 3 D. $\frac{24}{11}$ E. NOTA

14. Triangle ABC has area 96. Point D lies on $\overline{AB}$ such that $AD : DB = 1 : 2$. Point E lies on $\overline{AC}$ such that $AE : EC = 3 : 1$. Find the area of triangle ADE.

A. 8 B. 32 C. 16 D. 24 E. NOTA

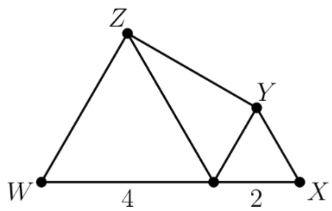
15. Three circles of radius 2, 3, and 10 are pairwise externally tangent. Find the area of the triangle formed by connecting their centers.

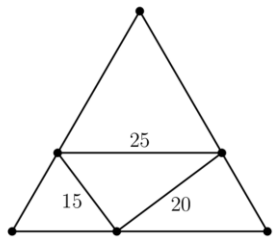
A. 24 B. 30 C. 27 D. 36 E. NOTA

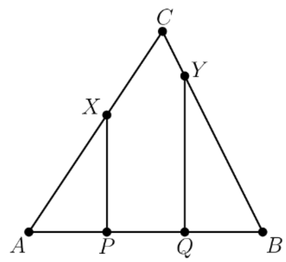
16. A triangle has area A and perimeter P. Express the fraction of the area of the triangle that lies inside the incircle of the triangle in terms of A and P.

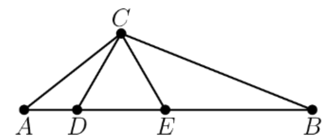
A. $\frac{\pi A}{P^2}$ B. $\frac{\pi A}{2P^2}$ C. $\frac{4\pi A}{P^2}$ D. $\frac{2\pi A}{P^2}$ E. NOTA

Use the following four diagrams for problems 17-20, from left to right.









17. Two equilateral triangles of side lengths 4 and 2 are placed side to side such that their bases lie on the same line. Their top vertices are connected, forming quadrilateral $WXYZ$. Find the area of $WXYZ$.
    A. $9\sqrt{3}$ B. $7\sqrt{3}$ C. $6\sqrt{3}$ D. $8\sqrt{3}$ E. NOTA

18. A 15-20-25 right triangle is inscribed in an equilateral triangle such that its side of length 25 is parallel to the base of the equilateral triangle. Find the side length of the equilateral triangle.
    A. $25+9\sqrt{3}$ B. $25+8\sqrt{3}$ C. $25+7\sqrt{3}$ D. $25+6\sqrt{3}$ E. NOTA

19. In triangle ABC points P and Q lie on $\overline{AB}$ such that $AP = PQ = QB = 2$. Points X and Y lie on $\overline{AC}$ and $\overline{BC}$ respectively such that $\overline{XP}$ and $\overline{YQ}$ are perpendicular to $\overline{AB}$. If $XP = 3$ and $YQ = 4$, find the area of triangle ABC.
    A. $\frac{108}{7}$ B. $\frac{90}{7}$ C. $\frac{72}{7}$ D. $\frac{81}{7}$ E. NOTA

20. Triangle ABC has $AC = 3$, $BC = 5$, and $AB = 7$. Points D and E lie on $\overline{AB}$ such that triangle CDE is equilateral. Find $AD + EB$.
    A. $\frac{83}{14}$ B. $\frac{34}{7}$ C. $\frac{44}{7}$ D. $\frac{93}{14}$ E. NOTA

21. In triangle ABC, points M and X lie on $\overline{AB}$ and points N and Y lie on $\overline{AC}$ such that M and N are closer to A than X and Y, respectively, and $\overline{MN} \parallel \overline{XY} \parallel \overline{BC}$. If the distance from A to $\overline{MN}$ is 3, the distance between $\overline{MN}$ and $\overline{XY}$ is 1, the distance from $\overline{XY}$ to $\overline{BC}$ is 1, and the area of triangle ABC is 50, find the area of trapezoid $MNYX$.
    A. 20 B. 16 C. 14 D. 18 E. NOTA

22. In each interior angle of an equilateral triangle, two segments are drawn to the opposite side of the triangle that split the angle into three congruent pieces (these segments are known as *angle trisectors*). The six angle trisectors of this triangle determine a hexagon with angle measures that alternate between $a°$ and $b°$ for real numbers a and b. Compute $|a - b|$.
    A. 20 B. 40 C. 60 D. 80 E. NOTA

23. Triangle ABC has $AB = 13$, $BC = 14$, and $AC = 15$. Squares $ACEF$ and $BCGH$ are constructed such that they share no interior points with triangle ABC. Compute the area of hexagon $ABHGEF$.

A. 568 B. 505 C. 617 D. 589 E. NOTA

24. Triangle ABC has vertices at $A(0,0)$, $B(6,0)$, and $C(2,4)$. If a point is selected uniformly at random inside triangle ABC, find the probability it is closer to vertex A than it is to vertex B.

A. $\frac{5}{8}$ B. $\frac{1}{2}$ C. $\frac{2}{3}$ D. $\frac{3}{4}$ E. NOTA

25. Triangle ABC has $AB = 13$, $BC = 14$, and $AC = 15$. Point P lies inside triangle ABC such that the distance from P to $\overline{AB}$ is 2 and the distance from P to $\overline{BC}$ is 3. Find the distance from P to $\overline{AC}$.

A. $\frac{7}{2}$ B. $\frac{10}{3}$ C. $\frac{20}{3}$ D. 7 E. NOTA

26. Triangle ABC has $AB = 17$, $BC = 18$, and $AC = 17$. Circle Ω is tangent to $\overline{AB}$ at B and to $\overline{AC}$ at C. Points M and N lie on $\overline{AB}$ and $\overline{AC}$ respectively such that $\overline{MN}$ is parallel to $\overline{BC}$ and tangent to Ω. Find the perimeter of triangle AMN.

A. 34 B. 26 C. 37 D. 33 E. NOTA

27. Triangle ABC has area 144, inradius 4, and $BC = 18$. The line parallel to $\overline{BC}$ passing through the incenter I of triangle ABC intersects $\overline{AB}$ at M and $\overline{AC}$ at N. Find the area of triangle AMN.

A. 96 B. 64 C. 108 D. 81 E. NOTA

28. A triangle is bounded by the lines $y = x$, $y = 4x$, and $y = ax + b$ for some real numbers a and b with $b \neq 0$. If the centroid of this triangle lies on the line $y = 3x$, find a.

A. 5 B. 8 C. 7 D. 2 E. NOTA

29. Rectangle $ABCD$ has $AB = 4$. Let M be the midpoint of $\overline{AB}$, let X lie inside $ABCD$ such that ΔAMX is equilateral, and let Y be the intersection of $\overline{AC}$ and $\overline{MX}$. If $CY = 2AY$, find the area of $ABCD$.

A. $12\sqrt{3}$ B. $4\sqrt{3}$ C. $6\sqrt{3}$ D. $8\sqrt{3}$ E. NOTA

30. Triangle ABC has $AB = 9$, $BC = 10$, and $AC = 11$. Let P, M, and N denote the midpoints of the medians of ΔABC from A, B, and C, respectively. Find the area of ΔPMN.

A. $\frac{15\sqrt{2}}{8}$ B. $\frac{5\sqrt{2}}{6}$ C. $\frac{5\sqrt{2}}{3}$ D. $\frac{10\sqrt{2}}{3}$ E. NOTA