The abbreviation "NOTA" stands for "None of the Above". Good luck, and have fun!

1. Triangle $KAN$ has area 12. $\angle KAN = 30°$ and $KA = 8$. Find $AN$.

   A. 3 B. 4 C. 6 D. 8 E. NOTA

2. Mitsuha and Taki (who are currently 17 miles apart) are traveling on trains going due south and due west, respectively, towards the same train station. Taki is traveling at a speed of 5 miles per hour. Both of them are an integer number of miles away from the train station, with Taki being farther away. Find the speed of Mitsuha's train (in miles per hour) if both trains will arrive at the station at the same time.

   A. $\frac{8}{3}$ B. 3 C. $\frac{8}{5}$ D. 4 E. NOTA

Questions $3-5$ are about regular hexagon $ELDORA$ with side length 2 and center $V$.

3. Find the area of the largest triangle that can be formed by connecting the vertices of $ELDORA$.

   A. $2\sqrt{3}$ B. $3\sqrt{3}$ C. $4\sqrt{3}$ D. $6\sqrt{3}$ E. NOTA

4. How many triangles can be formed by connecting the vertices of $ELDORA$?

   A. 8 B. 12 C. 18 D. 20 E. NOTA

5. Find the fraction of the diagonals of $ELDORA$ that pass through $V$.

   A. $\frac{1}{5}$ B. $\frac{1}{3}$ C. $\frac{2}{3}$ D. $\frac{4}{5}$ E. NOTA

6. Points $A$, $B$, and $C$ are at $(4,0)$, $(0,3)$, and the origin, respectively. If triangle $ABC$ is rotated about the $x$-axis, what is the volume of the cone that is formed?

   A. $12\pi$ B. $16\pi$ C. $36\pi$ D. $48\pi$ E. NOTA

7. Triangle $CHP$ has an incircle of radius 1 and a perimeter of 12. Find the area of $CHP$.

A. 3 B. $2\sqrt{3}$ C. 6 D. $4\sqrt{3}$ E. NOTA

8. Triangle $WAT$ has side lengths 8, 15, and 17. Find the distance between the centroid and the circumcenter of $WAT$.

A. $\frac{5}{3}$ B. $\frac{11}{6}$ C. $\frac{13}{6}$ D. $\frac{17}{6}$ E. NOTA

9. Right triangle $A$ has a right angle at $A$. A segment is drawn from $A$ to point $D$ on $BC$ such that $\angle ABC = \angle ADB = 60°$. If $E$ is the midpoint of $BC$, find $DE$.

A. $\sqrt{2} - 1$ B. $\frac{\sqrt{2}}{2}$ C. $\sqrt{3} - 1$ D. $\frac{\sqrt{3}}{2}$ E. NOTA

10. $O$ is the circumcenter of triangle $ABC$. Points $D$ and $E$ lie on $AB$ and $AC$ respectively such that $OD \perp AB$ and $OE \perp AC$. If $AC = 12$, $OD = 5$ and $OE = 2$, find $AB$.

A. 7 B. $3\sqrt{6}$ C. $2\sqrt{15}$ D. 8 E. NOTA

11. In triangle $TLH$, $\angle T = 100°$ and $\angle L = 60°$. Points $A$ on $LH$ and $B$ on $TL$ are chosen such that $TA$ is an altitude to $LH$ and $HB$ is a median to $TL$. Find $\frac{AB}{LT}$.

A. $\frac{1}{3}$ B. $\frac{1}{2}$ C. $\frac{1}{\sqrt{3}}$ D. 1 E. NOTA

12. Right triangle $TOM$ has hypotenuse 61 and positive integral side lengths. Find the area of $TOM$.

A. 220 B. 330 C. 440 D. 660 E. NOTA

13. Triangle $ARY$ is isosceles with base $RY$. Points $V$ on $AR$ and $E$ on $AY$ are chosen such that $AV = EV$ and $RE = RY$. If $\angle ERY = 30°$, find $\angle VER$.

A. 30° B. 45° C. 60° D. 75° E. NOTA

14. A triangle has side lengths $10$, $17$, and $21$. Find the length of its only integer-length altitude.

A. $6$ B. $8$ C. $9$ D. $10$ E. NOTA

15. Equilateral triangle $WXY$ has side length $2$. $XY$ is extended to point $T$ such that triangle $WTX$ is a right triangle. Find $YT$.

A. $\frac{4\sqrt{3}}{3} - 2$ B. $\sqrt{3} - 1$ C. $2$ D. $3$ E. NOTA

16. Triangle $MRZ$ has a right angle at $M$. Let $L$ and $U$ be the midpoints of $MR$ and $RZ$, respectively. If $MU^2 = 1312$ and $ZL^2 = 1819$, find $ZM$.

A. $26$ B. $28$ C. $30$ D. $34$ E. NOTA

17. In triangle $HAN$, $HA = 8$ and $AN = 7$. If $\angle H = 60°$, find the sum of all possible side lengths of $NH$.

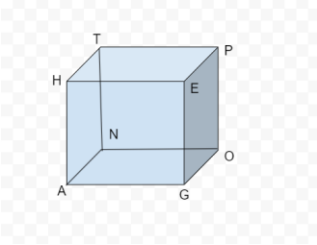
A. $5$ B. $7$ C. $8$ D. $9$ E. NOTA

18. Right triangle $RAD$ has leg lengths of $20$ and $24$. Triangle $R'A'D'$ is formed by connecting the midpoints of the sides of $RAD$. You throws a dart that lands at a random point inside $RAD$. If the dart also lands inside $R'A'D'$, you win! Find the probability you win.

A. $\frac{1}{6}$ B. $\frac{1}{4}$ C. $\frac{1}{3}$ D. $\frac{11}{4\sqrt{61}}$ E. NOTA

19. Three points are chosen on a circle. What is the probability that the center of the circle is in the interior of the triangle formed by connecting those points?

A. $\frac{1}{6}$ B. $\frac{1}{4}$ C. $\frac{1}{3}$ D. $\frac{1}{2}$ E. NOTA




20. Cube $HEPTAGON$ (above) has sides of length 1. Find the area of triangle $TOG$ divided by the area of $TAO$.

A. $\frac{\sqrt{6}}{3}$ B. $\frac{\sqrt{3}}{2}$ C. $1$ D. $\frac{\sqrt{6}}{2}$ E. NOTA

21. Find the area of a triangle in the coordinate plane with vertices at $(0,0)$, $(5,4)$, and $(9,-6)$.

A. $27$ B. $30$ C. $33$ D. $36$ E. NOTA

22. Regular hexagon $HEXGON$ has side length 4. Find the area of triangle $EXO$.

A. $8$ B. $6\sqrt{3}$ C. $12$ D. $8\sqrt{3}$ E. NOTA

23. Ben's home is located at $(5,3)$. He is currently at $(-4,9)$. Ben needs to visit a river (the $x$-axis) before he goes home. What is the minimum distance he needs to travel?

A. $15$ B. $12\sqrt{2}$ C. $12\sqrt{3}$ D. $21$ E. NOTA

24. $2024$ points are evenly spread around the circumference of a circle. Three distinct points are chosen at random. What is the probability that these points can be connected to form a right triangle?

A. $\frac{1}{1012}$ B. $\frac{3}{2024}$ C. $\frac{3}{2023}$ D. $\frac{1}{674}$ E. NOTA

25. Convex quadrilateral $ABCD$ has its vertices at lattice points in the $xy$-plane. Its side lengths are $AB = 12$, $BC = 5$, $CD = 2\sqrt{5}$, and $DA = 13$. Find the area of $ABCD$.

A. $35$ B. $40$ C. $45$ D. $55$ E. NOTA

26. A right triangle with hypotenuse 1 also contains an angle of measure $15°$. Find the area of that triangle.

A. $\frac{1}{8}$ B. $\frac{\sqrt{6}-\sqrt{2}}{8}$ C. $\frac{1}{4}$ D. $\frac{\sqrt{6}-\sqrt{2}}{4}$ E. NOTA

27. Let $A$ be the area of a triangle with side lengths $51$, $52$, and $53$. Find the number of positive integer factors of $A$.

A. 12 B. 16 C. 18 D. 24 E. NOTA

28. Triangle $MAO$ has point $M$ at the origin and points $A$ and $O$ on the line $x = 24$ in the first quadrant. If $MA = 25$ and $MO = 26$, find $AO$.

A. 2 B. 3 C. 4 D. 5 E. NOTA

29. Find the radius of the circle that circumscribes a triangle with side lengths $13$, $14$, and $15$.

A. $\frac{65}{8}$ B. $\frac{65}{7}$ C. $\frac{65}{6}$ D. 13 E. NOTA

30. Find the area of a regular octagon with side length 2.

A. $4\left(1+\sqrt{2}\right)$ B. $4\left(2+\sqrt{2}\right)$ C. $8\left(1+\sqrt{2}\right)$ D. $8\left(2+\sqrt{2}\right)$ E. NOTA