1. **D** Note that the smallest possible difference between the lengths of two sticks is 1, so no triangle with a side of length 1 will satisfy the triangle inequality. The next smallest candidate has sides 2, 3, and 4, which works and gives a perimeter of $\boxed{9}$.

2. **C** $AB = AC$ implies that $m\angle B = m\angle C = 40°$. This means that $m\angle A = 180° - 40° - 40° = \boxed{100°}$.

3. **B** Dropping an altitude from X to $\overline{AB}$ shows that ΔABX has base 6 and height 6, so its area is $\frac{1}{2}(6)(6) = \boxed{18}$.

4. **B** If two triangles are similar with ratio $a : b$, their areas are in the ratio $a^2 : b^2$. This means that ΔDEF is $\sqrt{\frac{9}{4}} = \frac{3}{2}$ the size of ΔABC, so the perimeter of ΔDEF is $\frac{3}{2}(12) = \boxed{18}$.

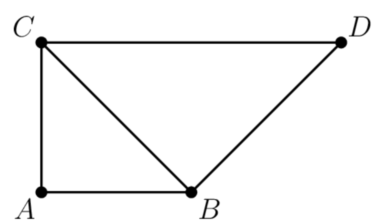
5. **C** Since the interior angles of a triangle sum to $180°$, we can see that

$$(x + 36) + (2x + 24) + (72 - x) = 2x + 132 = 180 \rightarrow x = \boxed{24}.$$

6. **A** In order for the two triangles to form a larger triangle rather than a quadrilateral, the right angles in each triangle must lie side to side. This means the larger triangle has side lengths 13, 15, and $5 + 9 = 14$, for a perimeter of $\boxed{42}$.

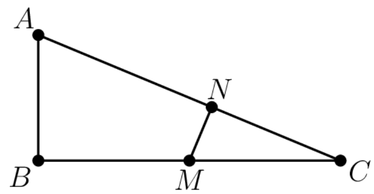
7. **C** First note that ΔABC has a right angle at B. Since the median to the hypotenuse of a right triangle has length half that of the hypotenuse, $BM = \frac{1}{2}(26) = \boxed{13}$.

8. **C**



Note ΔABC and ΔDBC are 45°-45°-90° right triangles, so $BC = BD = \sqrt{2}$ and $CD = 2$. Note $m\angle ACB = m\angle BCD = 45°$ so ΔABC has a right angle at C. By the Pythagorean Theorem, $AD = \sqrt{1^2 + 2^2} = \boxed{\sqrt{5}}$.

9. **C**



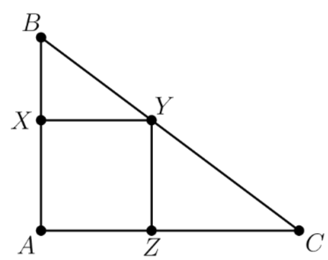
Since $m\angle B = m\angle N = 90°$, triangles ABC and MNC are similar by AA. This means

$$\frac{MN}{MC} = \frac{AB}{AC} \to MN = \frac{5 \cdot 6}{13} = \boxed{\frac{30}{13}}.$$

10. **B** By the Law of Cosines, $\cos(B) = \frac{a^2+c^2-b^2}{2ac} = \frac{7^2+5^2-8^2}{2 \cdot 5 \cdot 7} = \boxed{\frac{1}{7}}$.

11. **E** WLOG let c be the hypotenuse. We have that $a^2 + b^2 = c^2$ by the Pythagorean Theorem. Note that the square of an even integer is even, and the sum/difference of two even integers is even, so if two of a, b, c are even, so is the third. Therefore there are $\boxed{\text{no such triples}}$.

12. **A**



Note that $\Delta BXY \sim \Delta BAC$. Let the side length of the square be s. We can easily compute $BX = 3 - x$, so by the aforementioned similarity,

$$\frac{BX}{XY} = \frac{BA}{AC} \to \frac{3-s}{s} = \frac{3}{4} \to s = \boxed{\frac{12}{7}}.$$

13. **A** Note that $\overline{AD}$ is an angle bisector, so by the Angle Bisector Theorem $\frac{BD}{DC} = \frac{AB}{AC} = \frac{4}{5}$. This means that $DC = \frac{5}{9}(6) = \frac{10}{3}$. Since $\angle CAD \cong \angle ACD$, $AD = DC = \boxed{\frac{10}{3}}$.

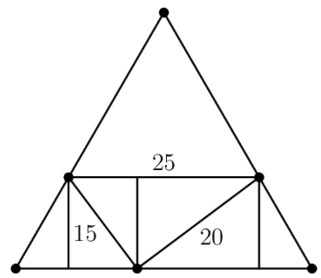
14. **D** Note that the area of ABC is given by $96 = \frac{1}{2}(AB)(AC)\sin(A)$. Similarly, the area of ADE is given by $\frac{1}{2}(AD)(AE)\sin(A)$ we can easily compute $AD = \frac{1}{3}AB$ and $AE = \frac{3}{4}AC$, so the area of ADE is $\frac{1}{2}\left(\frac{1}{3}AB\right)\left(\frac{3}{4}AC\right)\sin(A) = \frac{1}{4}(96) = \boxed{24}$.

15. **B** The side lengths of the resulting triangle are $2 + 3 = 5$, $2 + 10 = 12$, and $3 + 10 = 13$. This forms a right triangle with area $\frac{1}{2}(5)(12) = \boxed{30}$.

16. **C** The inradius of the triangle is given by $\frac{A}{P/2} = \frac{2A}{P}$, so the area of the incircle is $\pi\left(\frac{2A}{P}\right)^2 = \frac{4\pi A^2}{P^2}$, so the portion of the area taken up is $\left(\frac{4\pi A^2}{P^2}\right)/A = \boxed{\frac{4\pi A}{P^2}}$.

17. B Let the common vertex of the equilateral triangles be A. Using the sine area formula on triangles WAZ, ZAY, YAX gives $\frac{1}{2}(4)(4)(\sin(60°)) + \frac{1}{2}(4)(2)(\sin(60°)) + \frac{1}{2}(2)(2)(\sin(60°)) = 4\sqrt{3} + 2\sqrt{3} + \sqrt{3} = \boxed{7\sqrt{3}}$.

18. B

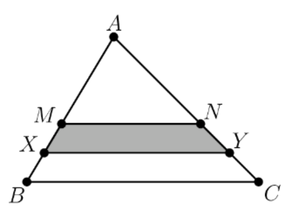


Drop perpendiculars from each vertex of the right triangle, and note they all have the same length since the hypotenuse is parallel to the side of the equilateral triangle. Using the area of the right triangle we can find this common length is $\frac{15\cdot 20}{25} = 12$. Noting the two outer right triangles are $30°$-$60°$-$90°$ triangles, we can divide the base of the equilateral triangle into a segment of length 25 and two segments of length $\frac{12}{\sqrt{3}} = 4\sqrt{3}$, for a total length of $\boxed{25 + 8\sqrt{3}}$.

19. A Drop into the coordinate plane and let $A(0,0)$ and $B(6,0)$. It follows that $X = (2,3)$ and $Y = (4,4)$, so line AX is given by $3x - 2y = 0$ and line BY is given by $2x + y = 12$. We can solve to get they intersect at $\left(\frac{24}{7}, \frac{36}{7}\right)$, so ΔABC has base 6 and height $\frac{36}{7}$ for an area of $\frac{1}{2} \cdot 6 \cdot \frac{36}{7} = \boxed{\frac{108}{7}}$.

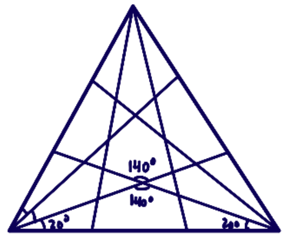
20. B By Heron's formula, the ΔABC has area $\sqrt{\frac{15}{2} \cdot \frac{9}{2} \cdot \frac{5}{2} \cdot \frac{1}{2}} = \frac{15\sqrt{3}}{4}$. This means the common altitude of the ΔABC and ΔCDE has length $\frac{2(15\sqrt{3}/4)}{7} = \frac{15\sqrt{3}}{14}$, and thus the side length of the ΔCDE is $\frac{15\sqrt{3}}{14} \cdot \frac{2}{\sqrt{3}} = \frac{15}{7}$. It follows that $AD + EB = AB - DE = 7 - \frac{15}{7} = \boxed{\frac{34}{7}}$.

21. C



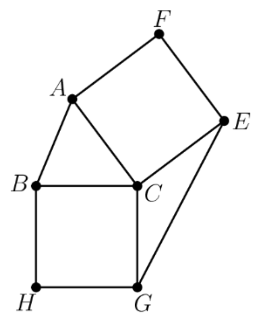
Note that $\Delta AMN \sim \Delta AXY \sim \Delta ABC$ by AA, and by the given lengths we have similarity ratios $3 : 4 : 5$. Note that $MNYX$ is the difference of AXY and AMN, so its area is $\left(\frac{4}{5}\right)^2 \cdot 50 - \left(\frac{3}{5}\right)^2 \cdot 50 = \boxed{14}$.

22. B



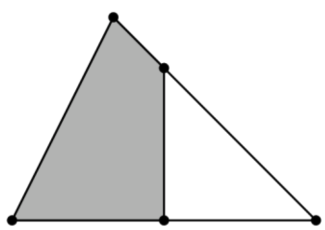
The angle trisectors split each angle into three $20°$ angles. Considering the triangle at the bottom of the equilateral triangle and using vertical angles gives that one of the angles has measure $180° - 20° - 20° = 140°$. Since the interior angles of a hexagon sum to $720°$, the other angle measure is $\frac{1}{3} \cdot 720° - 140° = 100°$, so the desired difference is $|140 - 100| = \boxed{40}$.

23. D



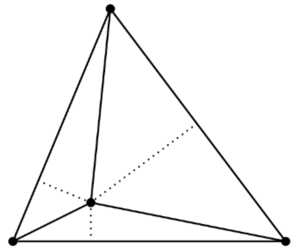
Split the desired hexagon into triangle ABC, squares $ACEF$ and $BCGH$, and triangle CEG. By Heron's (or the decomposition from problem 6), the area of ΔABC is 84, and we can easily compute the areas of $ACEF$ and $BCGH$ to be $15^2 = 225$ and $14^2 = 196$ respectively. Noting that $\sin(\angle ECG) = \sin(180° - \angle ACB) = \sin(\angle ACB)$, we can note that triangles ABC and CEG have the same area using the formula $\frac{1}{2}ab \sin \mathrm{C}$, so the total area is $84 + 225 + 196 + 84 = \boxed{589}$.

24. A



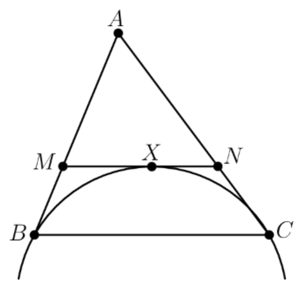
Note that the set of all points closer to A than B is the set of all points to the left of the perpendicular bisector of $\overline{AB}$. The set of the points to the right of this line is a right triangle with legs of 3 and 3, so its area is $\frac{9}{2}$. The total area of the triangle is 12, so the desired probability is $1 - \frac{\frac{9}{2}}{12} = \boxed{\frac{5}{8}}$.

25. C



Draw in $\overline{PA}$, $\overline{PB}$, and $\overline{PC}$, dividing the triangle into ΔPAB, ΔPBC, and ΔPCA. We can compute the areas of ΔPAB and ΔPBC to be $\frac{1}{2}(13)(2) = 13$ and $\frac{1}{2}(14)(3) = 21$ respectively. The area of ΔABC is 84 (which can be computed using Heron's or recalling the division in problem 6), so the area of ΔPCA is $84 - 21 - 13 = 50$, so the distance from P to $\overline{AC}$ is $\frac{2 \cdot 50}{15} = \boxed{\frac{20}{3}}$.

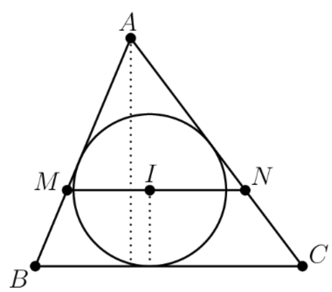
26. A



Let $\overline{MN}$ be tangent to Ω at X. Since tangents from a point to a circle are congruent, $MX = MB$ and $NX = NC$ (the diagram above is not to scale, but if I drew it correctly it would look like that). The perimeter of ΔAMN is equal to

$$AM + MX + XN + NA = AM + MB + CN + NA = AB + AC = \boxed{34}.$$

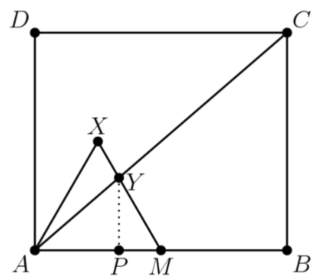
27. D



Note that $\Delta AMN \sim \Delta ABC$ by AA (parallel lines tend to do that). We can easily compute that the altitude from A has length $\frac{2 \cdot 144}{18} = 16$. We can see the scale factor from between ΔAMN and ΔABC is $\frac{\text{altitude}-\text{inradius}}{\text{altitude}} = \frac{16-4}{16} = \frac{3}{4}$ (diagram not to scale), so the area of ΔAMN is $\left(\frac{3}{4}\right)^2 \cdot 144 = \boxed{81}$.

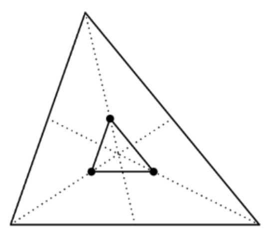
28. **C** Let the vertices of the triangle in question be $(0,0)$, (a,a), and $(b,4b)$ for some real a,b. The centroid of this triangle is $\left(\frac{0+a+b}{3},\frac{0+a+4b}{3}\right)$. In order for this to lie on $y=3x$ we need $\frac{a+4b}{a+b}=3 \rightarrow a+4b=3a+3b \rightarrow b=2a$. This means the triangle has vertices $(0,0)$, (a,a), and $(2a,8a)$ so the last line has slope $\frac{8a-a}{2a-a}=\boxed{7}$.

29. **D**



Drop a perpendicular from Y to $\overline{AB}$ and let it meet $\overline{AB}$ at P. Note that $\Delta APY \sim \Delta ABC$ with similarity ratio $\frac{1}{3}$. Note that Y is $\frac{1}{3}$ of the way along $\overline{AC}$, so we can easily compute that Y is also $\frac{1}{3}$ of the way along XM (consider x-coordinates). With this in mind, we can find that $PY=\frac{2}{3}XM=\frac{2}{3}\left(\frac{\sqrt{3}}{2}\right)(2)=\frac{2\sqrt{3}}{3}$. It then follows that $BC=3PY=2\sqrt{3}$, so the area of $ABCD$ is $4\left(2\sqrt{3}\right)=\boxed{8\sqrt{3}}$.

30. **A**



Draw the three medians of ΔABC. The centroid G of ΔABC is $\frac{2}{3}$ of the way along each median from the vertex, while each midpoint is $\frac{1}{2}$ of the way along each median from the vertex. This means that each midpoint is $\frac{2/3-1/2}{2/3}=\frac{1}{4}$ of the way from the centroid to each vertex, so our desired "median midpoint triangle" is similar to ΔABC with a factor of $\frac{1}{4}$ and has area $\frac{1}{16}$ that of ΔABC. By Heron's formula, the area of ΔABC is $\sqrt{15\cdot 6\cdot 5\cdot 4}=30\sqrt{2}$ this gives a final answer of $\frac{30\sqrt{2}}{16}=\boxed{\frac{15\sqrt{2}}{8}}$.