1. C
2. A
3. B
4. D
5. B
6. A
7. C
8. D
9. E
10. C
11. B
12. B
13. D
14. B
15. C
16. A
17. C
18. B
19. B
20. A
21. C
22. D
23. A
24. C
25. B
26. A
27. D
28. B
29. A
30. C

1. C Since $A = \frac{1}{2}ab\sin C$, $12 = \frac{1}{2} \cdot 8 \cdot \frac{1}{2} \cdot AN$, so $AN = 6$.
2. A Mitsuha and Taki form an $8-15-17$ triangle, with Mitsuha arriving in $\frac{15}{5} = 3$ hours. $\frac{8}{s} = 3$, so $s = \frac{8}{3}$.
3. B By inspection, the desired triangle is formed by connecting alternating vertices. By symmetry through reflecting the three regions inside the hexagon but not the triangle over the triangle, this triangle has half the area of the hexagon, or $\frac{1}{2} \cdot \frac{3 \cdot 2^2\sqrt{3}}{2} = 3\sqrt{3}$.
4. D Three vertices out of 6 must be chosen. $\binom{6}{3} = \frac{6!}{3!^2} = \frac{720}{36} = 20$.
5. B $ELDORA$ has $\frac{6 \cdot 3}{2} = 9$ diagonals, 3 of which pass through V for a fraction of $\frac{1}{3}$.
6. A The rotation produces a cone with radius 3 and height 4 and thus a volume of $\frac{\pi}{3} \cdot 3^2 \cdot 4 = 12\pi$.
7. C Since $A = rs$ and $s = 6$, $A = 6$.
8. D Let the triangle have vertices at $(0,0)$, $(0,8)$, and $(15,0)$. Then its centroid is at the average of these coordinates, or $\left(5, \frac{8}{3}\right)$. Its circumcircle is at the midpoint of its hypotenuse, or $\left(\frac{15}{2}, 4\right)$. The distance between these points is $\sqrt{\left(\frac{5}{2}\right)^2 + \left(\frac{4}{3}\right)^2} = \sqrt{\frac{289}{36}} = \frac{17}{6}$.
9. E Drawing the figure gives ADB as an equilateral triangle, so $AB = AD = BD = 1$. Since the triangle is a $30° - 60° - 90°$ right triangle, $BC = 2$, so $BE = CE = 1$. Since both D and E are on BC and $BD = BE$, $D = E$ and $DE = 0$.
10. C Since the circumradius is the intersection of the perpendicular bisectors, $AE = 6$, so the circumradius is $\sqrt{6^2 + 2^2} = 2\sqrt{10}$. Similarly, $AD^2 + 5^2 = 40$, so $AD = \sqrt{15}$ and thus $AB = 2\sqrt{15}$.
11. B Drawing altitude TA yields that $\angle LTA = 30°$, so $TL = 2AL$. Since HB is a median to TL, $TB = BL = \frac{TL}{2} = AL$. Since $BL = AL$, triangle BLA is isosceles with a vertex angle of $60°$; thus, it is not only isosceles but equilateral and $\frac{AB}{LT} = \frac{AB}{2BL} = \frac{1}{2}$.
12. B This is, famously, the $11 - 60 - 61$ right triangle. $\frac{11 \cdot 60}{2} = 330$.
13. D Triangle ERY is isosceles with a vertex angle of $30°$, so $\angle REY = \angle RYE = 75°$. Triangle ARY is isosceles, so $\angle ARY = \angle AYR = 75°$ and thus $\angle RAY = 30°$. Triangle AVE is isosceles, so $\angle VEA = \angle VAE = 30°$. $\angle AEY$ is a straight angle with $\angle AEV = 30°$ and $\angle YER = 75°$, so $\angle VER = 75°$.
14. B A $10 - 17 - 21$ triangle is a $6 - 8 - 10$ and a $8 - 15 - 17$ right triangle placed so the sides of length 8 coincide and the other legs are on the same line, so the altitude has length 8.
15. C Triangle WTX is a $30° - 60° - 90°$ right triangle with short leg 2, so its hypotenuse (XT) is 4. $XT = XY + YT$, so $YT = 2$.

16. A By similar triangles, the median of the hypotenuse lies on the perpendicular bisectors of both legs. Let $ZM = 2a$ and $MR = 2b$. Then $4a^2 + b^2 = 1819$ and $a^2 + b^2 = 1312$, so $3a^2 = 507$, $a^2 = 169$, and $a = 13$. Thus, $ZM = 26$.

17. C Let $NH = x$. By the Law of Cosines, $\cos 60° = \frac{x^2+64-49}{2\cdot 8\cdot x}$, so $\frac{1}{2} = \frac{x^2+15}{16x}$ and $x^2 + 15 = 8x$. Solving $x^2 - 8x + 15 = 0$ gives $x = 3$ and $x = 5$, both valid. $3 + 5 = 8$.

18. B The hypotenuse of the median triangle is parallel to the hypotenuse of RAD, so the median triangle is similar to RAD with a scaling factor of $\frac{1}{2}$. Thus, its area is $\frac{1}{4}$ that of RAD, which is also the probability of you winning.

19. B Let X be the largest angle of the triangle, so $60° \leq X \leq 180°$. The triangle must be acute to contain the center, so we desire the range $60° \leq X < 90°$, which takes up a proportion of the range of $\frac{30}{120} = \frac{1}{4}$.

20. A The side lengths of triangle TOG are 1, $\sqrt{2}$, and $\sqrt{3}$, so it is a right triangle with area $\frac{\sqrt{2}}{2}$. The side lengths of triangle TAO are all $\sqrt{2}$, so its area is $\frac{\sqrt{3}}{2}$. $\frac{\sqrt{2}/2}{\sqrt{3}/2} = \frac{\sqrt{6}}{3}$.

21. C Using matrices, $\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 9 & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix} = \frac{1}{2}(36 + 30) = 33$.

22. D EXO is a right triangle. It is given that $EX = 4$. EO bisects $\angle HEX$, so $\angle OEX = 60°$ and $XO = 4\sqrt{3}$. The area of the triangle is $\frac{1}{2} \cdot 4 \cdot 4\sqrt{3} = 8\sqrt{3}$.

23. A Reflect $(-4,9)$ over the x-axis, and by symmetry, Ben is traveling in a straight line from $(5,3)$ to this new point, $(-4,-9)$. This is a distance of 9 horizontally and 12 vertically for a total distance of 15.

24. C For each of the 1012, diameters, there are 2022 other points that can be selected to form a right triangle. There are $\binom{2024}{3} = \frac{2024\cdot 2023\cdot 2022}{3\cdot 2\cdot 1} = 1012 \cdot 2023 \cdot 674$ ways to choose three points. The probability is $\frac{1012\cdot 2022}{1012\cdot 2023\cdot 674} = \frac{3}{2023}$.

25. B Choose $A = (0,0)$ and $B = (12,0)$. Inspired by the numbers present, choose $D = (12,5)$. By inspection, $C = (16,3)$. Dividing the quadrilateral into two pieces by its vertical diagonal, its area is $\frac{1}{2} \cdot 12 \cdot 5 + \frac{1}{2} \cdot 5 \cdot 4 = 30 + 10 = 40$.

26. A The length of the adjacent side is $\cos 15°$, so the area is $\frac{1}{2} \cdot 1 \cdot \cos 15° \cdot \sin 15°$. By the sine double angle formula, $\frac{\sin 15° \cos 15°}{2} = \frac{\sin 30°}{4} = \frac{1}{8}$.

27. D The semiperimeter is 78, so $A = \sqrt{78 \cdot 27 \cdot 26 \cdot 25} = 2 \cdot 3^2 \cdot 5 \cdot 13$, which has $2 \cdot 3 \cdot 2 \cdot 2 = 24$ positive integer factors.

28. B Let $B = (24,0)$. MBA is a $7 - 24 - 25$ right triangle, and MBO is a $10 - 24 - 26$ right triangle, so $AO = 10 - 7 = 3$.

29. A This is a triangle with a well-known area of 84. Using $R = \frac{abc}{4A}$, the circumradius is $\frac{13\cdot 14\cdot 15}{4\cdot 84} = \frac{65}{8}$.

30. C Plugging in $s = 2$ to $A = 2s^2(1 + \sqrt{2})$ gives $8(1 + \sqrt{2})$.