Let \( L(x) = \begin{cases} 4 - x & \text{for } 1 < x \leq 4 \\ 3x^2 & \text{for } 0 \leq x \leq 1 \end{cases} \) and let \( U \) be the region bounded by the graph of \( L \), the x-axis, and the lines \( x=k \) and \( x = k+2 \), where \( 0 \leq k \leq 1 \). What value of “\( k \)” maximizes the area of \( U \)?
In triangle JPF, side JP=3, side JF=5, and the 
\[ \sin J = \frac{1}{3} \]. Find the product of all possible lengths 
of side PF.
Find the volume of the solid formed by rotating the region bounded by $y = 3x - 2$, $y = 2 - x^2$ and $x \geq 0$ about the line $x = -1$. 
Find the area of a non-degenerate triangle formed using the endpoints of a latus rectum and a focus for the given conic: $x^2 - 4y^2 + 10x + 24y + 25 = 0$
\[ \int_{1/2}^{1} \frac{dx}{2 \sqrt{x - x^2}} = \]
Trapezoid WXYZ has \( \overline{WZ} \parallel \overline{XY} \), \( XZ = 1, \ \angle ZXW = 23^\circ, \) and \( \angle XZY = 46^\circ \). The ratio of \( XY:WZ \) is 9:5. What is \( YZ \)?
Find the area of the region that lies within $r = 1 + 2 \cos \theta$ and outside $r = 2$. 
Mr. Lu plans to take the digits 8, 7, 4, 3, and 2 and put them in random order to make a 5-digit number. What is the probability that the resulting integer will be divisible by 11?
Find the volume of a solid with regular hexagonal cross-sections perpendicular to the x-axis and the longest diagonal of the hexagon lying in the region bounded by the curve: \( 9x^2 = 36 - 4y^2 \)
#10 Mu Ciphering
MAΘ National Convention 2021

Find the product of the solutions to:

\[ \log_4 k + \log_k \frac{1}{8} = 1. \]
A region is bound between a parabola and its latus rectum. If this region is revolved about the latus rectum, the resulting solid has a volume that can be represented as $k\pi p^3$, where $p$ is the distance from vertex to directrix. What is the value of $k$?
Can you factor!! Simplify
\[
\frac{(L^2 - 3^2 - U^2)^2 - 4(3U)^2}{(L^2 - U^2 - 6L + 9)(L^2 + 3L + 3U - U^2)}
\]