

# #1 Mu Ciphering

## MAO National Convention 2021

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Let  $L(x) = \begin{cases} 4 - x & \text{for } 1 < x \leq 4 \\ 3x^2 & \text{for } 0 \leq x \leq 1 \end{cases}$  and let  $U$  be the

region bounded by the graph of  $L$ , the  $x$ -axis, and the lines  $x=k$  and  $x = k+2$ , where  $0 \leq k \leq 1$ . What value of “ $k$ ” maximizes the area of  $U$ ?

## #2 Mu Ciphering

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In triangle JPF, side JP=3, side JF=5, and the

$\sin J = \frac{1}{3}$ . Find the product of all possible lengths  
of side PF.

### #3 Mu Ciphering

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Find the volume of the solid formed by rotating the region bounded by  $y = 3x - 2$ ,  $y = 2 - x^2$  and  $x \geq 0$  about the line  $x = -1$ .

## #4 Mu Ciphering

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Find the area of a non-degenerate triangle formed using the endpoints of a latus rectum and a focus for the given conic:  $x^2 - 4y^2 + 10x + 24y + 25 = 0$

## #5 Mu CIPHERING

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$$\int_{\frac{1}{2}}^1 \frac{dx}{2\sqrt{x-x^2}} =$$

## #6 Mu Ciphering

### MA $\Theta$ National Convention 2021

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Trapezoid  $WXYZ$  has  $\overline{WZ}$  parallel to  $\overline{XY}$ ,  $XZ = 1$ ,  $\angle ZXW = 23^\circ$ , and  $\angle XZY = 46^\circ$ . The ratio of  $XY:WZ$  is  $9:5$ . What is  $YZ$ ?

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Find the area of the region that lies within  $r = 1 + 2 \cos \theta$  and outside  $r = 2$ .

## #8 Mu Ciphering

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Mr. Lu plans to take the digits 8, 7, 4, 3, and 2 and put them in random order to make a 5-digit number. What is the probability that the resulting integer will be divisible by 11?

## #9 Mu Ciphering

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Find the volume of a solid with regular hexagonal cross-sections perpendicular to the  $x$ -axis and the longest diagonal of the hexagon lying in the region bounded by the curve:  $9x^2 = 36 - 4y^2$

## #10 Mu Ciphering

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Find the product of the solutions to:

$$\log_4 k + \log_{k^2} \frac{1}{8} = 1.$$

## #11 Mu Ciphering

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A region is bound between a parabola and its latus rectum. If this region is revolved about the latus rectum, the resulting solid has a volume that can be represented as  $k\pi p^3$ , where  $p$  is the distance from vertex to directrix. What is the value of  $k$ ?

## #12 Mu Ciphering

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Can you factor!! Simplify

$$\frac{(L^2 - 3^2 - U^2)^2 - 4(3U)^2}{(L^2 - U^2 - 6L + 9)(L^2 + 3L + 3U - U^2)}$$