#1 Theta School Bowl
MAΘ National Convention 2021

Let $A =$ the area of a square whose diagonal length is the same as the length of the hypotenuse of a right triangle with legs 6 and 8.

Let $B =$ the total surface area of a right rectangular pyramid with height 24, base width 20, and base length 64.

Let $C =$ the value of half a dozen.

Find $A + B + C$.

The polynomial $x^3 + ax^2 + bx + c = 0$, where a, b, and c are rational numbers, has $\sqrt[3]{7} + \sqrt[3]{49}$ as one of its roots.

Let $A =$ the number of positive divisors of $|b|$.

Let $B =$ the number of positive divisors of $|c|$.

Find the sum of the positive divisors of the product AB.

#3 Theta School Bowl
MAΘ National Convention 2021

Let $A =$ the sum of the real values of x that satisfy $\left(x^2 - 5x + 5\right)^{x^2+4x-60} = 1$.

Let $B =$ the value of $a^3 + b^3 + c^3$ if a, b, and c are distinct nonzero real numbers such that

$$\frac{1-a^3}{a} = \frac{1-b^3}{b} = \frac{1-c^3}{c}.$$

Let $C =$ the value of $a^2 + b^2$ if $a + b = 12$ and $ab = 10$.

Find the value of $A + B + C$.

When the city of Vennville had 1003 inhabitants, it was found that 63 inhabitants did not have a radio or a car, but 794 had a radio and 187 had a car. Let $A =$ the number of Vennville's inhabitants that had both a radio and a car.

Vennville has grown to 10,000 families. Due to its growth, it has three newspapers: *Vennville View*, *Vennville Vibe*, and *Vennville Voice*. A recent survey found that 40% of the families subscribe to the *View*, 20% subscribe to the *Vibe*, and 10% subscribe to the *Voice*. Also, 5% subscribe to the *View* and to the *Vibe*, 3% subscribe to the *Vibe* and to the *Voice*, 4% subscribe to the *View* and to the *Voice*, and 2% subscribe to all three newspapers. Let $B =$ the number of families that subscribe to only the *Vennville View*.

Find the sum of the digits of the value of $B - A$.

The values of k for which $\left|\dfrac{x^2+kx+1}{x^2+x+1}\right|<2$ (for all real values of x) is $(m,\, p)$. Let $A=m+p$.

Exactly 15 integers satisfy $(x-4)(x+6)(x-5)(2x-B)\le 0,$ for $B>0$.

Find the smallest possible value of $A+B$.

For their birthdays, the farmer gave Mary and her little lamb \$1600 each.

Let $A =$ the positive difference, in dollars, between the simple interest and the compound interest accrued by \$1600 invested at 5% annual interest over two years.

Let $B =$ the percent increase after one year for \$1600 invested at 8% annual interest, compounded continuously. Let $e^{0.08} = 1.0832$.

Find the value of $100(B \div A \div A \div A)$.

Let $A =$ the value of x such that $(x+1)+(x+4)+(x+7)+...+(x+28)=155.$

Let $B =$ the number of sides of a convex polygon whose interior angles are in arithmetic progression with common difference $5°$ and whose smallest interior angle measures $120°$.

Let $C =$ the sum of the squares of two numbers whose harmonic mean is 4.2 and whose arithmetic mean a and geometric mean g satisfy $3a+g^2=36$.

Find the value of $3A+B+C$.

Let $A =$ the number of triangles that can be formed by joining the vertices of a regular decagon, if two of the sides of the triangle must be two sides of the decagon.

Let $B =$ the number of triangles that can be formed by joining the vertices of a regular decagon, if exactly one of the sides of the triangle must be a side of the decagon.

Let $C =$ the perimeter of a regular decagon of radius 14. Use $\cos 36^\circ = 0.8$.

Find the value of $\dfrac{BC^2}{A}$.

Let $A =$ the degree of $f(x)$, if $f(x) = \begin{vmatrix} 1+a^2x & \left(1+b^2\right)x & \left(1+c^2\right)x \\ \left(1+a^2\right)x & 1+b^2x & \left(1+c^2\right)x \\ \left(1+a^2\right)x & \left(1+b^2\right)x & 1+c^2x \end{vmatrix}$ and $a^2+b^2+c^2=-2$.

Let $B = \det\left(\begin{bmatrix} 7 & 2 \\ 8 & -4 \end{bmatrix}^2\right)$.

Let $C = \det\left(4\begin{bmatrix} 7 & 2 \\ 8 & -4 \end{bmatrix}\right)$.

Find the value of $A+B+C$.

Let $A = h + k$, where (h, k) is the center of the graph generated by $9x^2 - 16y^2 - 72x + 96y - 144 = 0$.

Let $B =$ the sum of the coordinates of all x- and y-intercepts of the asymptotes of $9x^2 - 16y^2 - 72x - 96y - 144 = 0$.

Let $C =$ the sum of the coordinates of all x- and y-intercepts of $9x^2 - 16y^2 + 72x - 96y - 144 = 0$.

Let $D =$ the eccentricity of $9x^2 + 16y^2 + 72x - 96y - 144 = 0$.

Find the value of $\dfrac{B^{2B}D^2}{A} + C^2$.

Fred can paint his house in 10 days, while Barney can paint his house—which is identical to Fred's—in 5 days. Let $A =$ the number in days that it takes them to paint an identical house, working together.

In $\triangle MEH$, the bisectors of angles E and H meet at point I. If $m\angle M = 54^\circ$, Let $B =$ the degree measure of $\angle EIH$.

Hexagon SCRIBE is inscribed in a circle. Let $C =$ the degree measure of $m\angle CSE + m\angle EBI + m\angle IRC$.

The speed of a boat in still water is 25 mph. It takes the boat 1.5 times longer to travel 120 miles upstream (against the current) than it does to travel 120 miles downstream (with the current). Let $D =$ the speed of the stream in miles per hour.

Find the value of $AB + CD$.

#12 Theta School Bowl
MAΘ National Convention 2021

Let $A = 8^8 \bmod 15$.

Let $B =$ the number of nonnegative ordered pairs of integers that satisfy $20x + 16y = 500$.

Let $C =$ the sum of all x- and y-values in the ordered pairs found in part B.

Let $D =$ the number of three-digit positive integers that contain at least one 5.

Find the value of $\dfrac{D}{B} + A + C$.

#13 Theta School Bowl
MAΘ National Convention 2021

Let $A =$ the sum of the characteristics of $\log 0.009423$ and $\log_3 2499$.

Let $B =$ the sum of all possible values of $b \geq 10$ for which $(166)(56) = 8590$ in base b.

Let $C =$ the value of a such that $y = 6x$ intersects $y = x^2 + a$ exactly once.

Let $D =$ the value of $\dfrac{1}{\log_8 24} + \dfrac{1}{3\log_{64} 2 + \log_{64} 3} + \dfrac{3}{\log_3 24}$.

Find the value of $(ABCD) \bmod 17$.

Let $h(x)=5-x$. Use h and the table to evaluate each expression.

x	-1	2	3	4	5	6
$f(x)$	8	-1	0	1	2	3
$g(x)$	1	3	5	-6	7	0

$A=\left(g^{-1}\circ f^{-1}\right)(2)$

$B=\left(g^{-1}\circ h\right)(4)$

$C=\left(h^{-1}\circ f\circ g^{-1}\right)(3)$

$D=\left(g\circ f^{-1}\right)(-1)$

$E=\left(f^{-1}\circ g^{-1}\right)(3)$

$F=\left(h^{-1}\circ g^{-1}\circ f\right)(6)$

A triangle plotted on the Argand plane has vertices $A+Bi, C+Di, E+Fi$. Find the area of the triangle.