1. A
2. D
3. C
4. B
5. D
6. A
7. B
8. D
9. A
10. A
11. E
12. B
13. E
14. D
15. A
16. B
17. D
18. C
19. C
20. A
21. C
22. B
23. C
24. D
25. B
26. B
27. C
28. D
29. B
30. E

1) A We can use stars and bars to solve this problem. We know that each bag must contain at least one wand, so we can first put a wand in each bag, leaving us with 5 wands, which gives us 5 stars. Because there are 3 bags, we have 2 bars (2 bars will split it into 3 sections). With 5 stars and 2 bars, we get ${}_7C_2$ which is equal to 21.

2) D We are told that $K(x)$ is a cubic function so we can write it as $K(x) = ax^3 + bx^2 + cx + d$. We are also given four points on the function which means we can plug them in and solve a four-variable system of equations to get $a, b, c,$ and d. The equations we get are:

$$d = 1$$
$$a + b + c + d = 2$$
$$8a + 4b + 2c + d = 3$$
$$27a + 9b + 3c + d = 4$$

We can subtract the equations from each other and slowly eliminate the variables to get that $a = \frac{1}{6}, b = -\frac{1}{2}, c = \frac{4}{3}, d = 1$.
Now, we know that $K(x) = \frac{1}{6}x^3 - \frac{1}{2}x^2 + \frac{4}{3}x + 1$, and if we plug in $x = 4$, we get 9.

3) C For this question, we can take the harmonic mean of the speeds to get the average speed because they cover the same distance.
The harmonic mean formula for 2 numbers is $\frac{2}{\frac{1}{a}+\frac{1}{b}}$.
Plugging in 80 for a and 60 for b, we get $\frac{480}{7}$.

4) B We can see that we need to find the roots of $-4x^4 + 3x^3 + 8x^2 + 4x = 0$. We can take out an x to get $x(-4x^3 + 3x^2 + 8x + 4) = 0$.
Now, we can try to use synthetic division to find another factor. If we use the answer choices to guide our guess for what the root might be, we will see that $x = 2$ works. Therefore, the other root occurs at (2, 0), which is our answer.

5) D We know that the remainder must be in the form of $ax + b$. We can rewrite our division problem as multiplication instead:

$$x^4 + 3x^3 - 8x^2 + x - 1 = Q(x)(x^2 - 3x + 2) + (ax + b)$$

where $Q(x)$is the quotient we get when dividing the two polynomials.
We see that $x^2 - 3x + 2$ factors into $(x - 1)(x - 2)$, so we can try plugging in $x = 1$ and $x = 2$ into our equation.
Plugging in $x = 1$ yields $-4 = a + b$, and plugging in $x = 2$ yields $9 = 2a + b$.
Solving, we get $a = 13, b = -17$, so the remainder is $13x - 17$

6) A Using the properties of logarithms, we can split each term into a numerator and denominator:

$$\frac{log(625)}{log(49)} * \frac{log(9)}{log(5)} * \frac{log(\frac{1}{7})}{log(16)} * \frac{log(64)}{log(3)}.$$

Using more properties of logarithms, we can simplify this expression as we see that most of the numbers can be written as a^b:

$$\frac{4log(5)}{2log(7)} * \frac{2log(3)}{log(5)} * \frac{-log(7)}{4log(2)} * \frac{6log(2)}{log(3)}.$$

Now, we see that there are a lot of common factors that we can divide out to get the final answer of -6.

7) B Using properties of logarithms, we can rewrite our equation as

$$(log_2(x))^4 + (\frac{1}{log_2(x)})^4 = 47.$$

We can call $log_2(x) = a$ for convenience:

$$a^4 + \frac{1}{a^4} = 47.$$

We know that $(a^2 + \frac{1}{a^2})^2 = a^4 + \frac{1}{a^4} + 2$, and substituting in $a^4 + \frac{1}{a^4} = 47$ tells us that $a^2 + \frac{1}{a^2} = \pm 7$. However, we know that $a^2 + \frac{1}{a^2}$ must be positive so we can throw out -7.

Now, we have

$$a^2 + \frac{1}{a^2} = 7$$

We can use the same trick again with $(a + \frac{1}{a})^2 = a^2 + \frac{1}{a^2} + 2$, giving us

$$a + \frac{1}{a} = \pm 3.$$

If we multiply by a and use the quadratic formula, we see that $a = \frac{3 \pm \sqrt{5}}{2}, \frac{-3 \pm \sqrt{5}}{2}$.

Substituting a back into $log_2(x) = a$, we see that $x = 2^{\frac{3 \pm \sqrt{5}}{2}}, 2^{\frac{-3 \pm \sqrt{5}}{2}}$.We see that B is one of our solutions.

8) D We can use the continuous growth formula $A = Pe^{rt}$ to solve this problem.

In order for the bush to double in size, it takes $ln(3)$ minutes. That means that $2 = e^{r(ln(3))}$.

Solving for r, we get that $r = log_3(2)$.

Now, if we plug in 100 for A, (and 1 for P), we can solve for t, which comes out to be $log_2(3^{ln(100)})$, which simplifies to $2ln(10)log_2(3)$.

Everything we've done so far was in minutes and the question asks for hours, so we need to divide by 60 to get $\frac{ln(10)log_2(3)}{30}$.

9) A We can first start off by considering smaller cases. If we start with 1-9 coins, the troll clearly wins, as he can just take all of the coins on the first turn. However, if we start

with 10 coins, we see that we will win because the troll must take at least 1 coin, which leaves 1-9 coins in the pile for us to take. If we have 11-19 coins, the troll can take coins until there are 10 left in the pile because he knows that 10 coins will result in a loss for the person who takes from it first.
Now, we can begin to see a pattern. Every number that is congruent to 0(mod10) will guarantee that the first person who takes from it loses because the second person can just keep the pile at 0(mod10) until the last coin is taken. For example, if the troll takes 2 coins, you can take 8 to keep the pile at 0(mod10), if he takes 7 coins, you can take 3 and so on. Therefore, any number that is 0(mod10) will work, so 2020 is our answer.

10) A We know that all of the sides of the octagon and the two squares have length 1. If we look closely at triangle *BEF*, we can see that angle *BEF* is 45 degrees. So, we can drop an altitude from *F* to side *BE* and call the point where it intersects *M*. We can see that triangle *MEF* is a 45-45-90 right triangle, so *MF* has a length of $\frac{\sqrt{2}}{2}$. Using the area of a triangle formula ($base * height/2$), we get the area is $\frac{\sqrt{2}}{4}$.

11) E For this question, we can just extend the triangle inequality to the quadrilateral inequality, which states that any three sides of a quadrilateral must add up to be longer than the fourth side. So, we have 2 cases:
1) 19 is the longest side.
2) The missing side is the longest side.
For the first case, we can add 5 and 8 to get 13, so we see the last side must be at least 7 units long (because it has to have an integral length).
For the second case, we add 5, 8, and 19 to get 32, so the last side can be at most 31 units long.
There are 25 integers between 7 and 31 inclusive.

12) B For this question, we need to travel three units to the side and two units up in any order. Therefore, we can say that this problem simplifies to ${}_5C_2$ (because we move 5 times and choose 2 of them to be up and the rest to be to the right), which evaluates to 10.

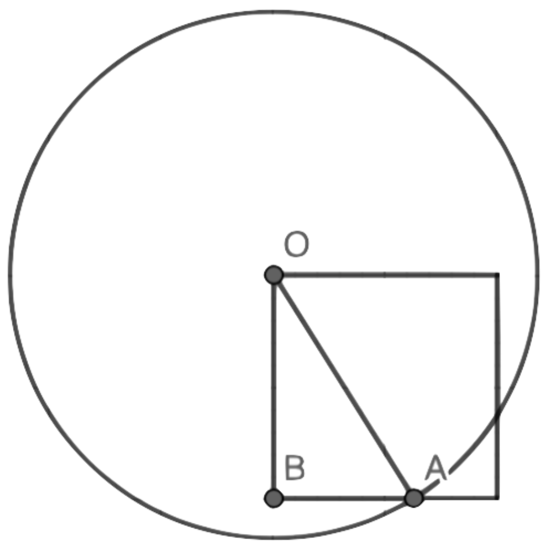
13) E I, II, and III are clearly all closed over real numbers because you cannot get a result that isn't real when performing these operations. IV is not closed because it is undefined when you divide by 0. V is not closed over reals because if you evaluate $(-1)^{\frac{1}{2}}$, you get an imaginary result.

14) D We need to count the total amount of red faces that we have because the cards with two red sides are twice as likely to be drawn with a red face than the cards with one green side and one red side. We see that we have 8 red sides from the 4 cards with red on both sides and 2 red sides from the other two cards. We only look at one side and see that it is red, so that means there is a $\frac{8}{10}$ or $\frac{4}{5}$ probability that we have drawn acard

with two red faces because the cards with two red faces contribute 8 red sides to the total of 10 red sides that are in the bag.

15) A We can see that, because the deck is shuffled, the probability that the last two cards are both aces is the same as the probability that the top two cards are both aces because we can just flip the deck over and the bottom cards will now be the top cards. Therefore, we just need to know the probability that the top two cards are both aces, which is $(\frac{4}{52})(\frac{3}{51})$, which comes out to be $\frac{1}{221}$.

16) B First, let us draw a picture.



We can call the pole that the rope is attached to O. Because the square is a hole, the rope can extend over it instead of stopping on the sides.
Circle O represents the area that Kira can roam without restrictions from the square. From the given information, we know that OA is 10, and because the square's area is 75 square feet, OB is $5\sqrt{3}$. We can now see that OAB is a 30-60-90 right triangle. That means that the portion of the circle the square is cutting out is two 30-60-90 right triangles along with a 30 degree sector of circle O. We can see that circle O has an area of 100π.The two triangles have a combined area of $2(\frac{1}{2})(5\sqrt{3})(5) = 25\sqrt{3}$. The sector of the circle has an area of $\frac{30}{360}(100\pi) = \frac{25\pi}{3}$. This leaves us with a final area of

$$100\pi - 25\sqrt{3} - \frac{25\pi}{3} = \frac{275\pi - 75\sqrt{3}}{3}.$$

17) D If we add all three of the equations, we get $4x + 4y + 4z = 36$, and dividing out a 4 gets us the answer: 9.

18) C First, we can calculate how far each person/birb traveled. Kira obviously traveled 100 feet, you were 5 feet behind her, so you traveled 95 feet, and the guards were 10 feet behind you but they also started 20 feet behind so they traveled 105 feet.
We can call the rates of Kira, you, and the guards $K, Y,$ and G respectively. We can now set up three equations using Distance = Rate $*$ Time because all of the times are the same (so we can call it t):

$$100 = Kt$$
$$95 = Yt$$
$$105 = Gt.$$

If we divide the third equation by the second equation, cancel out the t's, and multiply both sides by $Y,$ we get

$$\frac{21}{19}R = G.$$

This means that the guards can run $\frac{21}{19}$ feet for every foot you run. Because you have 5 feet left to run, the guards will run for $\frac{105}{19}$ feet, and because they were 15 feet away from the end of the corridor, they will be $\frac{180}{19}$ feet behind when you reach the end.

19) C We can calculate the number of liters of each of the three components in the cauldron. We see that we have 2L of magic, 6L of health, and 12L of water currently in the cauldron.
Our final mixture needs equal amounts of magic and health, so we can add some of the liquid from the 80% magic 20% water bottle. For every 5 liters of this liquid that we add, we are getting 4 liters of magic and 1 liter of water. We see that we need exactly 4 more liters of magic, so we can add 5L from this bottle, bringing our numbers in the cauldron up to 6L of magic, 6L of health, and 13L of water.
Now, we just need to add a little bit from the 50% magic 50% health bottle. We see that if we add 1 liter from that bottle, we get 6.5L of magic, 6.5L of health, and 13L of water in the cauldron, which are the correct concentrations.
We don't want to add anything more because we want the least possible amounts added, so our final answer is 5 + 1 = 6L

20) A We can factor $2x^5 - 3x^4 - x^3 - 27x^2 - x + 30 = 0$ as

$$(x-1)(x+1)(x-3)(2x^2+3x+10).$$

We can see that our only real solutions are 1, $-1,$ and 3. Reciprocating and adding, we get $\frac{1}{3}$.

21) C If we take Kira's advice, we can eventually notice that this is an infinite-infinite geometric series. We can rewrite our given fractions as $\frac{1}{2}, \frac{2}{6}, \frac{3}{18}, \frac{4}{54},$ and $\frac{5}{162}$. We see that we have a common ratio of $\frac{1}{3}$ between the denominators and the numerators are increasing by 1. Using the formula for an infinite-infinite geometric series $(\frac{a}{(1-r)^2})$, we see that our final answer is $\frac{9}{8}$.

22) B We know that the shortest distance between two points is a straight line. However, we must go to the road along the way. We see that we can reflect our starting point over the line to get a straight line distance between the points. (-6, 2) reflected over $y = x - 2$ is (4, -8). Using the distance formula between (4, -8) and (2, 10), we see that the shortest distance is $2\sqrt{82}$.

23) C The volume of a frustum is $\frac{1}{3}\pi h(r^2 + rR + R^2)$. Our slant height is 5, and the difference between the radii is 4, so our height is 3 by the Pythagorean Theorem. If we plug in our values, we get that the volume is 196π.

24) D We can notice that the triangle is actually a right triangle with hypotenuse $2\sqrt{13}$. Knowing that, the area is then $\frac{1}{2}(7)(\sqrt{3})$, and the circumradius is just half the hypotenuse, which is $\sqrt{13}$. Summing those, we get $\frac{2\sqrt{13}+7\sqrt{3}}{2}$.

25) B To find the space diagonal, we can just extend the pythagorean theorem to three dimensions to get $\sqrt{3^2 + 4^2 + 12^2}$ which evaluates to 13.

26) B We can apply the hockey stick theorem here. We can see that this is equal to $_4C_9$ by the hockey stick theorem, but we are missing $_3C_3$. So, our answer is $_4C_9 - {}_3C_3$, which comes out to be 125.

27) C First, we can set up our system of equations where C is chikadees and H is horsees:

$$C + H = 8$$
$$2C + 4H = 28$$

Dividing out a 2 from the second equation and solving gives us $C = 2, H = 6$. So, our answer is 6.

28) D This is a derangement question. We can use the principle of inclusion/exclusion (or PIE), but we can also use the approximation for derangements, which is $\frac{n!}{e}$. In this case, n is 5, so our approximation is $\frac{5!}{e} = \frac{120}{e}$. If we say that e is approximately 2.7, we can evaluate the approximation as about 44.4. Rounding down, we get 44.

29) B We can see that the third step is not a logical continuation of the second step because by order of operations, we must operate inside the square root before we can do anything else with it, and if we split the square root, we are changing the order of operations into an invalid order.

30) E We can see that if you take 2 apples, then you will have 2 apples.