

Theta

Numbers and Probability

Test #413

Directions:

1. Fill out the top section of the Round 2 Google Form answer sheet and select **Theta-Numbers and Probability** as the test. Do not abbreviate your school name. Enter an email address that will accept outside emails (some school email addresses do not).
2. Scoring for this test is 5 times the number correct plus the number omitted.
3. TURN OFF ALL CELL PHONES.
4. No calculators may be used on this test.
5. Any inappropriate behavior or any form of cheating will lead to a ban of the student and/or school from future National Conventions, disqualification of the student and/or school from this Convention, at the discretion of the Mu Alpha Theta Governing Council.
6. If a student believes a test item is defective, select “E) NOTA” and file a dispute explaining why.
7. If an answer choice is incomplete, it is considered incorrect. For example, if an equation has three solutions, an answer choice containing only two of those solutions is incorrect.
8. If a problem has wording like “which of the following could be” or “what is one solution of”, an answer choice providing one of the possibilities is considered to be correct. Do not select “E) NOTA” in that instance.
9. If a problem has multiple equivalent answers, any of those answers will be counted as correct, even if one answer choice is in a simpler format than another. Do not select “E) NOTA” in that instance.
10. Unless a question asks for an approximation or a rounded answer, give the exact answer.

For all questions, answer E) NOTA means none of the above answers is correct. Any question mentioning “die/dice”, “cards”, or “coins” assumes that those objects are fair and standard, unless otherwise stated. Good luck and have fun!

- Let N be the number of 7-digit phone numbers that are possible – 0 and 1 cannot be used as the first digit and the first 3 digits cannot be 800, 555, or 911. What is the sum of the digits of N ?
A. 23 B. 37 C. 43 D. 59 E. NOTA
- How many integers between 250 and 500 can be formed using only the digits 1,2,3,4,5, and 6.
A. 84 B. 90 C. 96 D. 104 E. NOTA
- In how many ways can 7 students (Snow and Wiggie among them) be seated in a row if Snow and Wiggie must be seated next to each other?
A. 120 B. 240 C. 720 D. 1440 E. NOTA
- How many distinguishable circular permutations of the letters in the name DEGREE can be formed?
A. 20 B. 24 C. 60 D. 120 E. NOTA
- How many diagonals are there in a dodecagon?
A. 35 B. 45 C. 54 D. 66 E. NOTA
- How many committees of 5 can be chosen from the FAMAT board of 11 people if President Snow and VP Santos are not to serve on the same committee?
A. 252 B. 378 C. 504 D. 712 E. NOTA

7. The FAMAT board had a X-mas party with every board member and their significant other present. (No significant other is also on the board.) Everyone shook hands(pre-virus) with every other person exactly once except members did not shake hands with their partners. There was a total of 220 handshakes. How many board members were present?
- A. 11 B. 12 C. 22 D. 24 E. NOTA
8. How many 5-letter sequences can be formed using the letters in the word matrices if each arrangement has 2 vowels and 3 consonants?
- A. 336 B. 720 C. 1440 D. 3600 E. NOTA
9. What is the coefficient of the fourth term in the expansion of $(L - 2U)^9$?
- A. -672 B. -84 C. 84 D. 672 E. NOTA
10. Right triangle ZLU is to be constructed in the xy-plane so that the right angle is at Z and ZU is parallel to the x-axis. The x and y coordinates of Z, L, and U are to be integers that satisfy $-2 \leq x \leq 7$ and $2 \leq y \leq 12$. How many different triangles with these properties could be constructed?
- A. 110 B. 1100 C. 9900 D. 10000 E. NOTA
11. If the coefficient of the 4th and 10th terms in the expansion of $(M - U)^n$ are equal, find the coefficient of the 8th term.
- A. -3003 B. -792 C. -495 D. -330 E. NOTA
12. If $(M + U)^6$ is expanded and all 64 terms are displayed, and one term is selected at random. What is the probability that the exponent of M is greater than 3?
- A. $\frac{27}{64}$ B. $\frac{15}{64}$ C. $\frac{5}{32}$ D. $\frac{11}{32}$ E. NOTA

13. How many subsets are there for the set $\{R, S, N, O, W, M, A\}$
A. 127 B. 128 C. 720 D. 5040 E. NOTA
14. Wiggie rolled two standard fair-die and calculated the product of the two numbers. The probability that the number has an odd number of positive integral factors is $\frac{J}{36}$. What is J?
A. 6 B. 7 C. 8 D. 9 E. NOTA
15. Three fair standard die are tossed. The probability that exactly 2 of the dice show the same number is $\frac{J}{216}$. What is J?
A. 60 B. 72 C. 90 D. 96 E. NOTA
16. Two marbles are drawn from a bag containing 2 red, 3 white, and 6 blue. The probability one marble is blue and the other is non-white is $\frac{L}{110}$. Find the value of L .
A. 12 B. 42 C. 54 D. 84 E. NOTA
17. The Snowman has 100 coins in his piggybank, all nickels, dimes, and quarters, whose total value is \$8.35. His piggybank does not necessarily contain coins of all three types. What is the difference between the largest and smallest number of dimes that could be in the Snowman's piggybank?
A. 13 B. 59 C. 64 D. 83 E. NOTA
18. How many ordered pairs (M, U) of positive integers satisfy $20M + 21U = 2021$ and satisfy $M > U$?
A. 3 B. 4 C. 5 D. 10 E. NOTA

19. Snow's life savings consists of 4 coins: a penny, a nickel, a dime, and a quarter. The probability that at least 15 cents worth of coins come up heads if he were to flip all of them is $\frac{S}{16}$. What is S?
- A. 4 B. 9 C. 10 D. 11 E. NOTA
20. How many ordered pairs of digits (J,W) is the integer 5JW4 a multiple of 9?
- A. 9 B. 10 C. 11 D. 12 E. NOTA
21. A parking lot has 12 spaces in a row. Eight cars arrive, each of which requires one parking space, and their drivers chose spaces at random from among the available spaces. Sohan then arrives in his SUV, which requires 2 adjacent spaces. What is the probability that he is able to park?
- A. $\frac{1}{3}$ B. $\frac{2}{3}$ C. $\frac{41}{55}$ D. $\frac{56}{165}$ E. NOTA
22. Given that L and U are natural numbers such that $\frac{3}{5} < \frac{L}{U} < \frac{5}{8}$, what is the smallest possible value for U?
- A. 11 B. 13 C. 40 D. 80 E. NOTA
23. M is the smallest composite number that has no prime factors less than 10. What is the sum of the digits of M?
- A. 4 B. 8 C. 9 D. 11 E. NOTA
24. A positive integral factor of 20! is selected uniformly and at random. What is the probability that this factor will be divisible by 20?
- A. $\frac{35}{54}$ B. $\frac{40}{57}$ C. $\frac{68}{95}$ D. $\frac{18}{25}$ E. NOTA

25. The sum of all the positive integers less than 1000 that have exactly five positive integer divisors is X . What is the sum of the digits of X ?
- A. 11 B. 16 C. 20 D. 24 E. NOTA
26. What is the largest integer value of n such that 8^n divides $100!$ evenly?
- A. 13 B. 27 C. 31 D. 32 E. NOTA
27. Find the number of values of n such that a regular n -gon has the property that the degree measure of its interior angles are integers.
- A. 18 B. 22 C. 23 D. 24Find E. NOTA
28. Find the 2021st digit past the decimal point expansion of $\frac{5}{14}$
- A. 1 B. 2 C. 7 D. 8 E. NOTA
29. How many numbers are on the list $-2021, -1978, -1935, \dots, 1935, 1978, 2021$?
- A. 92 B. 93 C. 94 D. 95 E. NOTA
30. How many multiples of 3 between 100 and 1000 can be formed using only the digits 1,4,5,6, and 8?
- A. 20 B. 24 C. 36 D. 41 E. NOTA