1. D
2. B
3. C
4. D
5. E
6. A
7. B
8. B
9. E
10. A
11. A
12. D
13. E
14. A
15. A
16. D
17. C
18. E
19. C
20. C
21. C
22. D
23. A
24. C
25. B
26. A
27. E
28. A
29. B
30. A

1.	D	There are $\binom{5}{4} = 5$ ways to choose exactly 4 of the 5 coins to land as tails. Since there are a total of $2^5 = 32$ ways to flip a coin, the probability is $\boxed{\frac{5}{32}}$
2.	B	We count by compliment. The number of ways to roll a sum of 11 is 2 and the number of ways to roll a sum of 12 is 1. Thus, our answer is $1 - \frac{3}{36} = \boxed{\frac{11}{12}}$
3.	C	There are $\binom{9}{3}$ ways to select the first group, $\binom{6}{3}$ ways to select the second group, and the last group is set. However, since the groups are not distinguishable, we divide by the ways to number the groups $3! = 6$. Then our total is $\frac{\binom{9}{3}\binom{6}{3}}{6} = \boxed{280}$
4.	D	The number of ways to select a black marble, a pink marble, then a blue marble is $4 \cdot 3 \cdot 3 = 36$. The three colors may be selected in any order so the number of ways to select three different colors is $36 \cdot 6 = 216$. The total number of ways to select 3 marbles without replacement is $10 \cdot 9 \cdot 8 = 720$. So, $\frac{216}{720} = \boxed{\frac{3}{10}}$
5.	E	Let a, b, c, d, e represent the number of seats to the left of the first student, the number of seats between the first and second student, and so on. Then $a + b + c + d + e = 6$ where $b, c, d > 0$. By stars and bars, the number of solutions to this equation is $\binom{(6-3)+5-1}{5-1} = 35$. The number of ways to seat the four students is $4! = 24$. In total, $35 \cdot 24 = \boxed{840}$
6.	A	$\frac{1}{4}\left(4 \cdot 0 + 2 \cdot (0 + 12) + \frac{4}{3} \cdot \left(0 + \frac{16}{3} + \frac{64}{3}\right) + 1 \cdot (0 + 3 + 12 + 27)\right) = \boxed{\frac{457}{18}}$
7.	B	We must choose one suit, and five cards within that suit. $4 \cdot \binom{13}{5} = \boxed{5148}$
8.	B	$f'(x) = -\frac{x}{\sqrt{2-x^2}}$ so that $-\frac{x}{\sqrt{2-x^2}} > \sqrt{2-x^2} \Rightarrow x^2 - x - 2 > 0 \Rightarrow x < -1, x > 2$. The domain of f is $[-\sqrt{2}, \sqrt{2}]$, thus the probability $f'(k) > f(k)$ is $\frac{\sqrt{2}-1}{2\sqrt{2}} = \boxed{\frac{2-\sqrt{2}}{4}}$
9.	E	By the Principle of Inclusion-Exclusion, $$49 + 25 + 36 - (10 + 6 + 15) + 3 = 82$$
10.	A	There are $3 \cdot 3 \cdot 3 \cdot 3 = 81$ ways to roll exactly two even numbers and one odd number and $3 \cdot 3 \cdot 3 = 27$ ways to roll three even numbers. To get a sum less than 7, the possible numbers showing are $(1,2,2), (1,2,4), (3,2,2), (2,2,2)$ for a total of $3 + 6 + 3 + 1 = 13$ possibilities. So, our probability is $\boxed{\frac{13}{108}}$
11.	A	Set the top row to be [1,2,3]. Then the second row can be [2,3,1] or [3,1,2], with the third row determined. There are 6 possible ways to permute the first row. So the total number of possibilities is $2 \cdot 6 = \boxed{12}$
12.	D	$3^4 = \boxed{81}$
13.	E	Arbitrarily color one vertex any of the 3 possible colors. The two vertices adjacent to this vertex can either be the same color or different colors. If they are the same color there are 2 possible ways to color them, and there is 2 possible ways to color the final vertex. If they are different colors, then they must be the two colors not taken

		by the first vertex. There are 2 ways to color them different colors, and there is 1 way to color the final vertex. In total, $3 \cdot (2 \cdot 2 + 2) = \boxed{18}$
14.	A	This is a geometric series with first term $\frac{1}{6}$ and ratio $\frac{25}{36}$. $\frac{\frac{1}{6}}{1-\frac{25}{36}} = \boxed{\frac{11}{25}}$
15.	A	$\binom{9}{3}\left(-\frac{1}{x^2}\right)^3 (2x)^6 = -5376$
16.	D	The total probability must be equal to 1. Therefore, $$\int_0^\infty a \cdot 3^{-x}\, dx = 1$$ $$\frac{-a \cdot 3^{-x}}{\ln 3} \Big\vert_0^\infty = 1$$ $$a = \boxed{\ln 3}$$
17.	C	$\int_1^3 \ln 3 \cdot 3^{-x}\, dx = -3^{-x} \Big\vert_1^3 = \frac{1}{3} - \frac{1}{27} = \boxed{\frac{8}{27}}$
18.	E	Using integration by parts, $$\int_0^\infty x \ln 3 \cdot 3^{-x}\, dx = -x \cdot 3^{-x} + \frac{1}{\ln\ 3} 3^{-x} \Big\vert_0^\infty = \boxed{\frac{1}{\ln 3}}$$
19.	C	$$\int_0^c \ln 3 \cdot 3^{-x}\, dx = \frac{1}{2}$$ $$-3^{-x} \Big\vert_0^c = \frac{1}{2}$$ $$-3^{-c} + 1 = \frac{1}{2}$$ $$c = \boxed{\frac{\ln 2}{\ln 3}}$$
20.	C	Note that, $$-x^4 + 5x^2 - 4 > 0 \Rightarrow -(x-1)(x+1)(x-2)(x+2) > 0$$ So that $1 < \|x\| < 2$, and $$1^2 + 2^2 = \boxed{5}$$
21.	C	Using linearity of expectation, $$\frac{1}{9} + 4 \cdot \frac{1}{10} = \boxed{\frac{23}{45}}$$
22.	D	Consider any set of 3 strangers. There is a $\frac{1}{8}$ probability that the three strangers form a friend triangle. There are $\binom{8}{3} = 56$. Using linearity of expectation, $$56 \cdot \frac{1}{8} = \boxed{7}$$
23.	A	The bounds of integration are given by $x - x^2 = kx \Rightarrow x = 0,1-k$. Then, $$\frac{\int_0^1 \int_0^{1-k} x - x^2 - kx\, dx\, dk}{1-0} = \int_0^1 \frac{(1-k)^3}{6} dk = \boxed{\frac{1}{24}}$$

24.	C	We count by compliment. There are $\frac{8!}{3!2!} = 3360$ total ways to arrange the letters. The first and last letters can be either A or S, with $\frac{6!}{2!} = 360$ and $\frac{6!}{3!} = 120$ ways, respectively. Thus, by compliment $3360 - 360 - 120 = \boxed{2880}$
25.	B	We arrange the 3 up steps and 3 right steps required to travel from $(0,0)$ to $(3,3)$. There are $\binom{6}{3}$ ways to do this. Similarly, there are $\binom{4}{2}$ ways to arrange the 2 up steps and 2 right steps to travel from $(3,3)$ to $(5,5)$. In total, $\binom{6}{3}\binom{4}{2} = \boxed{120}$
26.	A	We count by compliment. There are 2^4 ways to pick a subset of $[4]$. We want $A \cap B = \emptyset$. Thus, for each element of $[4]$ it can be in A only, it can be in B only, or it can be in neither set. Therefore, the number of ways is $2^4 \cdot 2^4 - 3^4 = \boxed{175}$
27.	E	Method 1: Given any four unlabeled points, there are 8 possible labeling which draw chords that intersect. Additionally, there are $4! = 24$ possible labelings. Therefore, the probability is $\frac{8}{24} = \boxed{\frac{1}{3}}$ Method 2: WLOG let the circumference be 1. Placement of A is arbitrary. B may be placed a distance of x counterclockwise from A. C may be placed between A and B along the length x or $1 - x$ and D must be placed opposite C, on the length $1 - x$ or x, respectively. So, the probability is given by $$\int_0^1 x(1-x) + (1-x)x\,dx = \boxed{\frac{1}{3}}$$
28.	A	We consider the possible residues of the elements in the set mod 20. For $0 \leq x \leq 9$, if x is a residue of an element in the set, then $20 - x$ cannot be a residue of an element in the set. This gives a maximum size of $\boxed{11}$. This is achieved by the set $$\{0,1,2,3,4,5,6,7,8,9,10\}$$
29.	B	We choose $1 \leq i \leq 6$, so that $f(i) = i$. The remaining elements of $[6]$ must be in a derangement. Using the recursive formula $D_n = (n-1)(D_{n-1} + D_{n-2})$, where D_n gives the number of derangements of $[n]$, and $D_1 = 0, D_2 = 1$, we find that $D_5 = 44$. Then the total number of bijections is $6 \cdot 44 = \boxed{264}$
30.	A	We wish to find probability that $(2n-1)y < x \leq 2ny$ for some $n \in \mathbb{N}$. By graphing, we find that for a given value of n, this gives a probability of $\frac{1}{2n-1} - \frac{1}{2n}$. Then, $$\frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots = \boxed{\ln 2}$$