2021 Log1 Contest Answers: ALPHA

1. **B** The polar coordinate $(-1,\frac{\pi}{4})$ is $\left(-\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$ in rectangular coordinates, which implies that there is an axis of symmetry about the y-axis.

2. **C** Adding 2 to each side of the equation yields expressions that can be factored as $(x+1)^2+(y+1)^2=3$, which is a circle at center (-1,-1).

3. **E** By inspection, this circle has radius $\sqrt{3}$, which has an area of 3π.

4. **THROWN OUT**

5. **THROWN OUT**

6) **D**, suppose that we call the roots of $P(x)$ x, y, and z. The product of the roots taken two at a time is equal to $(xy)(yz)(xz) = (xyz)^2$. Thus, we are simply trying to find the product of the roots, squared. This can be found by Vieta's formula. The product of the roots is $= \frac{-constant}{a}$ if the degree of the highest term is odd and $\frac{constant}{a}$ if the degree of the highest term is even. In our case,

Product $= \frac{-(-16)}{1} = 16$

$16^2 = 256$

7) **C and E are acceptable.**

I is true because if we have 2 corresponding angles then the third must also be a corresponding angle. With all three congruent angles we can determine that one triangle is simply a dilation of another as the sides are constructed from the angles.

II is false because if we have 2 corresponding sides we must have the included angle (the angle that lies at the vertex of the two sides) as opposed to any two angles. The included angle with the two sides proves congruency

III is true because if we have 3 corresponding and proportional sides between the triangles we know (like stated in I) that the triangles are simply a dilation of each other.

Thus, only I and III are true. **= Answer C**

Statement I says "If two angles of a triangle are congruent". It should say "If two corresponding angles of two triangles are congruent". Thus Statement I is false and only Statement III is true. **= Answer E**

8) . **D** Observe that because $\frac{\pi}{12}=\frac{\pi}{3}-\frac{\pi}{4}$, we can use the sine addition rule:

$\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)=\sin\frac{\pi}{3}\cos\frac{\pi}{4}-\cos\frac{\pi}{3}\sin\frac{\pi}{4}=\frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2}-\frac{1}{2}\frac{\sqrt{2}}{2}=\frac{\sqrt{6}-\sqrt{2}}{4}$. Because cosecant is the reciprocal of sine, we simplify $\frac{4}{\sqrt{6}-\sqrt{2}}\cdot\frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}=\frac{4(\sqrt{6}+\sqrt{2})}{4}=\sqrt{6}+\sqrt{2}$

9) **B** All of these are valid parametrizations of the segment shown except for B. While they take the same basic structure of applying the linear property to a linear, sinusoidal, quadratic, and exponential function respectively, because the range of sin(t) is only from -1 to 1, it cannot draw the entire segment.

10) **C** Converting from polar to rectangular coordinates, we observe:

$$3\sqrt{x^2+y^2}+2y=6$$

$$9x^2+9y^2=(6-2y)^2$$

$$9x^2+9y^2=36-24y+4y^2$$

$$9x^2+5y^2+24y=36$$

$$9x^2+5\left(y^2+\frac{24}{5}y+\frac{144}{25}\right)=36+\frac{144}{5}$$

$$9x^2+5\left(y+\frac{12}{5}\right)^2=\frac{324}{5}$$

This is an ellipse with center (0,-12/5).

11) **A** Factoring this expression yields $\left(\tan(x)+\frac{\sqrt{3}}{3}\right)\left(\tan x+\sqrt{3}\right)$, which has zeroes where tangent is the opposite of the square root of three, or the opposite of its reciprocal. The sum of these angles on the requested interval are

$$\frac{-\pi}{6}+\frac{-\pi}{3}+\frac{2\pi}{3}+\frac{5\pi}{6}=\pi$$

12) **E** Draw a picture to see that this is computing the area of a circle sector with an internal angle of 120 degrees, a third of a circle of radius 5.

13) **B** This infinite sum is geometric. Observe that by change of base formula, we can phrase each term in log base 3, because $\log_9(x)=\frac{\log_3(x)}{\log_3(9)}$ and $\log_{81}(x)=\frac{\log_3(x)}{\log_3(81)}$, so there is a common ratio of ½. By infinite sum of a geometric series, the sum is $2\log_3(x)$, which has a solution when $x=3^9$.

14) This is a Caesar cipher with a shift of 13, also known as ROT13.

Never gonna give you up, never gonna let you down, never gonna run around and desert you.

15) This is a keyword cipher using the keyword ''PRASEODYMIUM.'' A keyword cipher works similarly to a cryptogram except that a keyword appears at the leftmost part of the decryption table, as follows.

Ciphertext: ABCDEFGHIJKLMNOPQRSTUVWXYZ

Plaintext: PRASEODYMIUBCFGHJKLNQTVWXZ

I am serious, and don't call me Shirley.