For all questions below, the answer E. NOTA means “None of these answers”.

1. Dan is trying to decide between two phone plans. Plan A costs 2 cents per minute with a monthly fee of $12. Plan B costs 3 cents per minute with a monthly fee of $10. For what number of minutes will both plans have the same monthly cost?

A. 20       B. 100       C. 200       D. 300       E. NOTA

2. A coat is discounted by \( p \% \) then increased by \( p \% \). For what value of \( p \) will the overall decrease of the original coat price be marked down by 36%?

A. 6       B. 36       C. 40       D. 60       E. NOTA

3. A car gets \( G \) miles per gallon and gas costs \( C \) dollars per gallon. What expression represents the total cost in dollars to travel \( D \) miles?

A. \( \frac{DC}{g} \)       B. \( \frac{D}{CG} \)       C. \( \frac{G}{DC} \)       D. \( DCG \)       E. NOTA

4. The equation \( h(t) = -16t^2 + 48t + 4 \) models the height \( h \) of a baseball off the ground in feet at time \( t \) in seconds. What is the maximum height of the ball off the ground?

A. 16 feet       B. 32 feet       C. 40 feet       D. 48 feet       E. NOTA

5. How many liters of pure water should be added to 4 liters of 25% salt solution to make a solution that is 5% salt?

A. 20 liters       B. 18 liters       C. 17 liters       D. 16 liters       E. NOTA

6. The equation \( \ell = \ell_0\sqrt{1 - \frac{v^2}{c^2}} \) represents the relativistic length \( \ell \) of an object of original length \( \ell_0 \) when the object moves at velocity \( v \) where the speed of light is \( c \). At what fraction of the speed of light must an object travel to appear 96% its original length?

A. 0.04c       B. 0.28c       C. 0.32c       D. 0.64c       E. NOTA

7. The time it takes for the crack of thunder to reach someone is proportional to the distance the storm is away from an observer. If a thunder crack takes 2.5 seconds to travel 850 meters, how far is a storm, in meters, if a thunder crack takes 1.5 seconds to reach the observer?

A. 340       B. 510       C. 1410       D. 3185       E. NOTA

8. A set of 16 numbers has a mean of 20. If the maximum value was to be removed, the mean would go down by 2. Instead, if the minimum value was to be removed, the mean would go up by 1. What is the range of the original set?

A. 15       B. 30       C. 35       D. 45       E. NOTA
9. Consider the figure shown, assuming each angle is right. If the perimeter of the shape is 120, find the value of $x$.

\[
\begin{align*}
2x + 10 & \\
4x - 10 & \\
\end{align*}
\]

A. 4 \hspace{1cm} B. 5 \hspace{1cm} C. 10 \\
D. 15 \hspace{1cm} E. NOTA

10. At a hardware store, 3 hammers, 1 screwdriver, and 1 wrench cost $25. 1 hammer, 3 screwdrivers and 1 wrench cost $20. And 1 hammer, 1 screwdriver, and 3 wrenches cost $17.50. How much does 1 hammer, 1 screwdriver, and 1 wrench cost?

A. $10.50 \hspace{1cm} B. $12.50 \hspace{1cm} C. $15.00 \hspace{1cm} D. $17.00 \hspace{1cm} E. NOTA

11. Parallelogram $ALPH$ has $AL = 2\sqrt{3}, LP = 3\sqrt{3}, m\angle ALP = 150^\circ$. Find the exact area interior to $ALPH$, in square units.

A. $\frac{9}{2}$ \hspace{1cm} B. $3\sqrt{3}$ \hspace{1cm} C. 9 \hspace{1cm} D. $9\sqrt{3}$ \hspace{1cm} E. NOTA

12. A calculator only has two buttons: subtract 1 or divide by 5. After a button is pressed, the output is shown. Starting with the number 2020, what is the least number of button presses needed to display an output of 1 exactly?

A. 9 \hspace{1cm} B. 10 \hspace{1cm} C. 11 \hspace{1cm} D. 12 \hspace{1cm} E. NOTA

13. A semicircle has the same perimeter as a square. Which of the following is the ratio of the interior area of the semicircle to that of the square?

A. $\frac{16\pi}{(2+\pi)^2}$ \hspace{1cm} B. $\frac{4}{2+\pi}$ \hspace{1cm} C. $\frac{8\pi}{(2+\pi)^2}$ \hspace{1cm} D. $\frac{4\pi}{(1+\pi)^2}$ \hspace{1cm} E. NOTA

14. If the digits 1 to 9 can each be used at most once, what is the sum of the digits of the largest possible three-digit prime number that can be made?

A. 24 \hspace{1cm} B. 22 \hspace{1cm} C. 20 \hspace{1cm} D. 18 \hspace{1cm} E. NOTA

15. A ball is dropped from a height of 20 cm above the floor. Each time it bounces off the floor, it bounces up $\frac{3}{4}$ of the previous height. What is the total vertical distance (both up and down) in cm that the ball will travel before it comes to rest?

A. 160 \hspace{1cm} B. 140 \hspace{1cm} C. 120 \hspace{1cm} D. 80 \hspace{1cm} E. NOTA

16. Jack and Jill are captains for their hockey team, along with four other starting players. If all six players line up in a row for a photo, what is the probability that Jack and Jill are at the two ends?

A. $\frac{1}{360}$ \hspace{1cm} B. $\frac{1}{30}$ \hspace{1cm} C. $\frac{1}{20}$ \hspace{1cm} D. $\frac{1}{15}$ \hspace{1cm} E. NOTA
17. The first three figures in a pattern are shown consisting of 3, 10, and 21 unit squares. How many unit squares are in the 100th figure?

A. 19701  B. 19900  C. 20100  D. 20301  E. NOTA

18. There are 101 pennies in row. Starting at the beginning of the row each time, the following procedure is done, in this order:
- Coins 2, 4, 6, …, and so on, are replaced by a nickel
- Coins 3, 6, 9, and so on, are replaced with a dime
- Coins 4, 8, 12, and so on, are replaced with a quarter.

What is the total value now of the 101 coins?


19. Consider the pyramid with square $ABCD$ as its base. Let $M, N, O,$ and $P$ be the midpoints of each of the four lateral edges. What fraction of volume of the largest pyramid is left once the small pyramid with base $MNOP$ is removed?

A. $\frac{1}{2}$  B. $\frac{3}{4}$  C. $\frac{4}{5}$  D. $\frac{7}{8}$  E. NOTA

20. You open a long novel and realize that the page numbers of the two pages facing you multiply to 1980. What is the sum of these two page numbers?

A. 85  B. 87  C. 89  D. 91  E. NOTA

21. An auditorium's first row has 20 seats and each successive row has two more seats than the previous row. If the auditorium has 840 total seats, how many total rows are there?

A. 18  B. 19  C. 20  D. 22  E. NOTA
22. Let $-2 \leq x \leq 2$. Let points $A$ and $B$ lie on $f(x) = 4 - x^2$ and $C$ and $D$ lie on $y = -f(x)$ with $AB$ and $CD$ parallel to the $x$-axis. If $m\angle AOD = 60^\circ$, find the exact area of rectangle $ABCD$.

A. $2\sqrt{3}$  
B. $4\sqrt{3}$  
C. 6  
D. $8\sqrt{3}$  
E. NOTA

23. Relative to point $O$, Ship A is 20 miles due north and Ship B is 50 miles due east. Ship A travels at a speed of 10 mph due south while Ship B travels 4 mph due east. After $t$ hours, which expression correctly expresses the straight-line distance between Ships A and B?

A. $2\sqrt{725 + 29t^2}$  
B. $2\sqrt{725 - 29t^2}$  
C. $\sqrt{725 + 200t + 29t^2}$  
D. $\sqrt{725 + 29t^2}$  
E. NOTA

24. In a 2 by 2 matrix, each of the four values is randomly selected to be a 0 or 1. What is the probability that the matrix is singular?

A. $\frac{1}{4}$  
B. $\frac{3}{8}$  
C. $\frac{5}{8}$  
D. $\frac{3}{4}$  
E. NOTA

25. Radleigh's blood pressure, $p(t)$ in millimeters of mercury, can be modeled by the function $p(t) = 120 + 15\sin(150\pi t)$, at time $t$ in minutes. Let $M$ be Radleigh's maximum blood pressure and $N$ be Radleigh's minimum blood pressure. Find $M - N$.

A. 150  
B. 135  
C. 30  
D. 15  
E. NOTA

26. A population of deer is expected to grow exponentially by 20% over 10 years. If this trend holds, by what percentage will the population increase over 20 years?

A. 20%  
B. 40%  
C. 44%  
D. 60%  
E. NOTA

27. A 3 by 3 cube is made up of 27 cubes of edge length 1. At each corner of the large cube, a unit cube is removed. What is the new total surface area?

A. 19  
B. 30  
C. 38  
D. 54  
E. NOTA
28. All the letters in the word COMBINATIONS are scrambled at random. The probability that all the vowels are together can be written in simplified form as \( \frac{m}{n} \). What is \( m + n \)?

A. 67  
B. 100  
C. 103  
D. 793  
E. NOTA

29. A bike rental currently rents an average of 36 bicycles a day for $10 each. A consultant tells the company that for each fifty-cent decrease in charge for a bike rental, they will rent 3 more bikes per day. What is the optimum price that the bike company should charge per bike rental to maximize their daily revenue?

A. $6.50  
B. $8.00  
C. $10.00  
D. $12.00  
E. NOTA

30. For a box in the shape of a right rectangular prism, the distinct areas of three non-congruent faces are 40, 60, and 96 square units, respectively. In cubic units, what is the volume of the prism?

A. 240  
B. 320  
C. 480  
D. 640  
E. NOTA