

**2022 MAΘ NATIONAL CONVENTION**  
**ALPHA CIPHERING**  
**CONDENSED VERSION**

- 0) What is the period of the function  $f(x) = \cos^4 x - \sin^4 x$ ?
- 1) Find the sum of the solutions to:  $k^{k\sqrt{k}} = (k\sqrt{k})^k$ .
- 2) Find all the solutions for the equation:  $\frac{\cos x - 1}{-\sin x} - 1 = 0$  over the domain:  $0 < x < 2\pi$
- 3) Mr. Lu's urn contains some black marbles and exactly 4 gold marbles. The probability of selecting a gold marble is L%. If the number of black marbles is doubled, the probability of selecting one of the 4 gold marbles drops to (L-15)%. What does L=?
- 4) Find the measure of the angle, in degrees, between the vectors  $\langle 2, 1 \rangle$  and  $\langle -2 - \sqrt{3}, -1 + 2\sqrt{3} \rangle$ .
- 5) If  $z$  is a complex number that satisfies  $2 + 8i = z + |z|$ , what does  $|z| = ?$
- 6) How many solutions does the equation:  $\cos(32x) + \cos(16x) = 0$  have for  $0 \leq x < 2\pi$ ?
- 7) L and U are two points on a circle with center K, and Z lies outside the circle, on ray  $\overline{LU}$ . Given  $LU=24$ ,  $UZ=28$ , and  $KL=15$ , find KZ.
- 8) Find the slope of the perpendicular bisector of the line segment joining the foci of the graphs of  $y^2 = 4x$  and  $x^2 - 6x - 8y - 31 = 0$ .
- 9) The fifth term of an arithmetic sequence is 4 and the  $n^{\text{th}}$  term is 104, where  $n > 5$ . How many possible values are there for  $n$  if the common difference is an integer?
- 10) If the coefficient of the  $4^{\text{th}}$  and  $10^{\text{th}}$  terms in the expansion of  $(M - U)^n$  are equal, what is the coefficient of the  $8^{\text{th}}$  term?
- 11) Find all ordered pair solutions for the following system of equations:  $h + k = 6$  and  $h^{k^2 - 7k + 12} = 1$ . What is the sum of the values of  $h$ ?

12) Evaluate the determinant:

$$\begin{vmatrix} 3 & 4 & -2 & 1 \\ 3 & 1 & 0 & -3 \\ 0 & -3 & 2 & 3 \\ 2 & -1 & 0 & -4 \end{vmatrix}$$