

“NOTA” stands for none of the above answers is correct. Good luck and have fun!

You may assume that all fractions, radicals, and logarithms in answers are completely simplified.

All trigonometric arcfunctions have their traditional domains and ranges, unless otherwise noted.

1.) Consider the parabolic function $P(x) = 3x^2 - 4kx + 7$. $P(x)$ has only one real root. The positive value of k can be written in the form $\frac{\sqrt{m}}{n}$. Find $m + n$.

A) 23

B) 25

C) 89

D) 101

E) NOTA

2.) Convert the following rectangular equation to polar form: $x^2 + y^2 = 2y$.

A) $r = 2 \cos \theta$

B) $r = -2 \cos \theta$

C) $r = 2 \sin \theta$

D) $r = -2 \sin \theta$ E) NOTA

3.) The vertex of the graph of the following parametric equations can be written in the form (A, B) . Find the product AB .

$$x = \frac{t}{2} + 1$$

$$y = t^2 - 1$$

A) -5

B) -1

C) 4

D) 6

E) NOTA

4.) How many integral solutions exist for the following inequality?

$$x + 1 \leq 2x + 3 \leq -3x + 8$$

A) 4

B) 5

C) 6

D) Infinitely Many E) NOTA

5.) The distance from the point (a, b) to the origin is 8. The arithmetic mean of a and b is 5. Find ab .

A) $-\frac{39}{2}$

B) $\frac{17}{2}$

C) 18

D) 46

E) NOTA

6.) The x solution for the following equation is also the area of circle P. Find the radius of circle P.

$$\sqrt{x+1} = \sqrt{2x-5} + 1$$

- A) $\frac{\sqrt{3\pi}}{\pi}$ B) $\frac{\sqrt{15\pi}}{\pi}$ C) $\frac{3\sqrt{\pi}}{\pi}$ D) $\frac{15\sqrt{\pi}}{\pi}$ E) NOTA

7.) Find the value of the ordinate value of the vertex of $Q(x)$ given that $Q(x) = 3x^2 - 5x + 11$.

- A) 11 B) $\frac{5}{3}$ C) $\frac{5}{6}$ D) $\frac{107}{12}$ E) NOTA

8.) How many integral solutions exist for the following inequality? $|2x - 3| < 5$

- A) 4 B) 6 C) 8 D) 21 E) NOTA

9.) Find the sum of the solutions within the domain $0 \leq x < 2\pi$: $\sec^4 x - 6 \tan^2 x + 2 = 0$.

- A) $4\pi + 4\sqrt{\tan^{-1} 3}$ B) $4\pi + 8\sqrt{\tan^{-1} 3}$ C) 6π D) 8π E) NOTA

10.) Which of the following is the acute angle between the vectors $\langle -3, 5, 2 \rangle$ and $\langle 1, 2, 4 \rangle$?

- A) $\cos^{-1} \frac{15\sqrt{5}}{59}$ B) $\cos^{-1} \frac{5\sqrt{798}}{266}$ C) $\cos^{-1} \frac{13\sqrt{133}}{266}$ D) $\cos^{-1} \frac{\sqrt{798}}{798}$ E) NOTA

11.) Given that $f(x) = x^2 - 5x + 1$ and $g(x) = \frac{1}{x-5}$ find $f(g(g(4)))$.

- A) $-\frac{1}{6}$ B) $-\frac{3}{16}$ C) $\frac{1}{2}$ D) $\frac{67}{36}$ E) NOTA

12.) Consider the polynomial $P(x) = \frac{1}{4}x^2 + 1$. A triangle is constructed such that one of its vertices is the focus of the graph of P ; call this point A. Another one of its vertices is located at one of the points where the polynomial and its latus rectum intersects; call this point B. The last point of the triangle is located on the directrix of the polynomial such that the distance from it and point B is minimized; call this point C. Find the area of $\triangle ABC$.

- A) $\frac{1}{8}$ B) $\frac{1}{2}$ C) 1 D) 2 E) NOTA

13.) Find the sum of the squares of all possible real values of x that can make the system of equations $|y| - \frac{2x}{|x|} = -1$ and $x|x| + y|y| = 24$ true.

- A) 44 B) 46 C) 48 D) 50 E) NOTA

14.) Simplify: $\frac{\frac{x-1}{x+1} \div \frac{x-1}{x}}{\frac{x+1}{x+1} - \frac{x}{x-1}}$ where defined.

- A) $\frac{x}{x-1}$ B) $\frac{(x-1)^2}{x}$ C) $\frac{x(x-1)}{x^2-2x-1}$ D) $\frac{x^2-2x-1}{x-1}$ E) NOTA

15.) Solve: $2x^2 - 11x - 40 \geq 0$.

- A) $(-\infty, \frac{-5}{2}] \cup [8, \infty)$ B) $(-\infty, -8] \cup [\frac{5}{2}, \infty)$ C) $[\frac{-5}{2}, 8]$
 D) $[-8, \frac{5}{2}]$ E) NOTA

16.) Solve: $\sin 2\theta = \cos \theta$.

(Note: n represents an arbitrary integer)

- A) $\theta = 2\pi n + \frac{\pi}{6}, 2\pi n + \frac{5\pi}{6}, \pi n + \frac{\pi}{2}$ B) $\theta = 2\pi n + \frac{\pi}{6}, 2\pi n + \frac{5\pi}{6}, \pi n$
 C) $\theta = 2\pi n + \frac{\pi}{3}, 2\pi n + \frac{2\pi}{3}, \pi n + \frac{\pi}{2}$ D) $\theta = 2\pi n + \frac{\pi}{3}, 2\pi n + \frac{5\pi}{3}, \pi n + \frac{\pi}{2}$ E) NOTA

17.) Evaluate: $\lim_{x \rightarrow \infty} \frac{x^4 - 6x^3 + x^2 - x^7 + x - 1}{2x^4 + x^3 - x^2 - 6x + 5}$.

- A) -1 B) $-\frac{1}{2}$ C) $\frac{1}{2}$ D) 1 E) NOTA

18.) The eccentricity of $2x^2 + y^2 + 12x + 4y - 22 = 0$ can be written in the form $\frac{\sqrt{m}}{n}$ when fully reduced. Find the units digit of $m + n$.

- A) 0 B) 4 C) 7 D) 8 E) NOTA

19.) A circle whose center is $(-4,9)$ with radius 4 can be written in the form

$Ax^2 + Cy^2 + Dx + Ey + F = 0$, where all variables are integers, A is positive, and D and F are relatively prime. Find the units digit of $A + C + D + E + F$.

- A) 3 B) 4 C) 7 D) 9 E) NOTA

20.) Simplify the expression: $\frac{1 + \cos x}{\sin x} \cdot \frac{1 - \cos x}{\cot x}$ where defined.

- A) $-\sin x$ B) $\cos x$ C) $\frac{\sin^2 x}{\cos x}$ D) $\tan x$ E) NOTA

21.) Evaluate: $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ if $f(x) = x^2 + 6x + 9$.

- A) $2x + 6$ B) $2x - 6$ C) $x + 3$ D) $x - 3$ E) NOTA

22.) Find $f^{-1}(x)$ given that $f(x) = \frac{x+1}{x-1}$.

- A) $\frac{1-x}{x+1}$ B) $\frac{x+1}{1-x}$ C) $\frac{x+1}{x-1}$ D) $\frac{x-1}{x+1}$ E) NOTA

23.) Find the area of the region bounded by the graph: $|2x| + |y - 1| = 8$

- A) 32 B) 48 C) 64 D) 128 E) NOTA

24.) Solve, where $0 \leq x \leq \frac{\pi}{2}$ is an acute angle: $\sin(\cos^{-1}(\tan(\cos^{-1}(\tan x)))) = 1$.

- A) 0 B) $\frac{\pi}{4}$ C) $\frac{\pi}{3}$ D) $\frac{\pi}{2}$ E) NOTA

25.) Let $\lfloor x \rfloor$ represent the greatest integer less than or equal to x (the floor function).

Find the sum of all possible values of b , where (a, b, c) is an ordered triple of positive real numbers that satisfies the following system of equations.

$$\lfloor a \rfloor \cdot b \cdot c = 3$$

$$a \cdot \lfloor b \rfloor \cdot c = 4$$

$$a \cdot b \cdot \lfloor c \rfloor = 5$$

- A) $\frac{17\sqrt{30}}{12}$ B) $\frac{2\sqrt{15}}{3}$ C) $\frac{\sqrt{30}}{2}$ D) $\frac{3\sqrt{30}}{4}$ E) NOTA

26.) Which of the following is/are true?

I. If two lines in the xy -plane are perpendicular, then the product of their slopes is -1

II. If a line is tangent to a circle, then the intersection of the line and a radius of the circle forms right angles.

III. The graph of the function $f(x) = \frac{x^2}{x}$ has an asymptote at $x = 0$

- A) I only B) II only C) I and III D) II and III E) NOTA

27.) If $g(x) = \frac{x-2}{2x+1}$ find $g'(2)$.

- A) $\frac{1}{25}$ B) $\frac{1}{20}$ C) $\frac{1}{10}$ D) $\frac{1}{5}$ E) NOTA

28.) Which of the following is the value of $\tan\left(\frac{x}{2}\right)$ given that $\cos x = \frac{a}{b}$ where both a and b are positive and that $\frac{3\pi}{2} \leq x \leq 2\pi$.

A) $\frac{\sqrt{b^2-a^2}}{a-b}$

B) $\frac{\sqrt{b^2-a^2}}{b-a}$

C) $\frac{-\sqrt{b^2-a^2}}{b+a}$

D) $\frac{\sqrt{b^2-a^2}}{b+a}$

E) NOTA

For questions 29 and 30 note that $\cosh x = \frac{e^x+e^{-x}}{2}$ and $\sinh x = \frac{e^x-e^{-x}}{2}$.

29.) If $\sinh x = \frac{2}{3}$, then x can be written in the form $\ln\left(\frac{a+\sqrt{b}}{c}\right)$. Find $a + b + c$.

A) 16

B) 18

C) 20

D) 24

E) NOTA

30.) The solution for the equation $2 \cosh 2x + 6 \sinh 2x = 12$ can be written in the form $\left(\frac{1}{a}\right) \ln\left(\frac{b+\sqrt{c}}{d}\right)$ (which is fully in simplest form). Find $a + b + c + d$.

A) 16

B) 18

C) 20

D) 24

E) NOTA