

1. D	7. A	13. D	19. B	25. B
2. C	8. E	14. B	20. A	26. D
3. B	9. C	15. B	21. C	27. C
4. D	10. C	16. E	22. E	28. B
5. A	11. C	17. C	23. D	29. A
6. C	12. B	18. D	24. D	30. D

1. The factorization of 2520 is  $2^3 \cdot 3^2 \cdot 5 \cdot 7$ , so there are  $(1 + 3)(1 + 2)(1 + 1)(1 + 1) = 48$  factors.
2. From the factorization, the sum of the factors is  $(1 + 2 + 2^2 + 2^3)(1 + 3 + 3^2)(1 + 5)(1 + 7) = 9360$
3. To do this, we find the number of 5s in 2520! since it is clearly the limiting factor. So, we have  $\lfloor \frac{2520}{5} \rfloor + \lfloor \frac{2520}{25} \rfloor + \lfloor \frac{2520}{125} \rfloor + \lfloor \frac{2520}{625} \rfloor = 628$
4. For the remainder, we just plug in  $x = 1$  into the function, which gives 2520.
5. The equation factors into  $2(x - 1)(x + 1)(x - 2)(x + 5)$ , so the sum of the two smallest roots is  $-5$  and  $-1$  which is  $-6$ .
6. By applying change of base and changing into base 10, this is  $\frac{3 \log 7}{3 \log 2} \cdot \frac{3 \log 9}{2 \log 7} \cdot \frac{8 \log 2}{\log 9}$ . All the logs cancel, so it's just  $\frac{3}{2} \cdot 8 = 12$ .
7. Listing out the first few examples, we notice the pattern for the tens digit goes 0, 4, 4, 0. Since  $2021 \div 4$  has remainder 1, we choose 0.
8. With vectors  $\langle a, b, c \rangle$  and  $\langle d, e, f \rangle$ , they are orthogonal if  $ad + be + cf = 0$ . This is not true for any of the vectors, so it is E.
9. To do this, we just apply mass points. From the angle bisector, we set C with mass 3 and B with mass 5, making D have mass 8. A has the same mass as B since it is intersected by a median, so it has mass 3. So, the ratio of AO and OD is  $\frac{8}{3}$
10. This is.  $(x - 5)(x - 3)(x - 2)^2$  so the sum of the distinct roots is  $4 + 9 + 25 = 38$ .
11. The general form of a polar conic is  $\frac{ed}{1 \pm e \sin(\theta)}$  or  $\frac{ed}{1 \pm e \cos(\theta)}$  where e is the eccentricity. So, if we take out a 5 from the denominator, we have  $5(1 - \frac{8}{5} \sin(\theta))$ . This means the eccentricity is  $\frac{8}{5}$  so it's a hyperbola.
12. This is just the sum of the first 8 perfect squares. So, it's  $\frac{(8)(9)(17)}{6} = 204$ .
13. So, on the right side,  $\csc^2(\theta) - 1 = \cot^2(\theta)$ . So, we can change this to  $\cot^2(\theta)\cos^2(\theta) = \cot^2(\theta) - \frac{1}{2}$ . Then, moving the cotangents to one side, we have  $\cot^2(\theta)(\cos^2(\theta) - 1) = -\frac{1}{2}$ . We can simplify this to  $\cot^2(\theta)(-\sin^2(\theta)) = -\frac{1}{2}$ . So, in the 3rd, we have  $\cos^2(\theta) = \frac{1}{2}$  or  $\cos(\theta) = \pm \frac{\sqrt{2}}{2}$  So the sum of solutions is  $4\pi$
14. This is just British flag theorem so it's 128.

15. Say  $x = \cos(36^\circ) \cos(72^\circ)$ . Multiply both sides by  $\sin(36^\circ)$  to get  $\sin(36^\circ)x = \sin(36^\circ) \cos(36^\circ) \cos(72^\circ)$ . By applying the double angle identities we get  $\sin(36^\circ)x = \frac{1}{4} \sin(144^\circ)$ .  $\sin(144^\circ) = \sin(36^\circ)$  so  $x = \frac{1}{4}$
16. The second row and the third row added together is a multiple of the first row, so the determinant is zero.
17. We can apply mods. Taking the clue about 6 and 11, we clearly see that 15 works, so our answer will be  $15 \pmod{66}$  and  $5 \pmod{13}$ . You could reduce this, but since it's below 250, there's only 4 possible numbers: 15, 81, 147, and 213. The only one of those that is  $5 \pmod{13}$  is 213 so the sum of the digits is 6.
18. We can construct this triangle from a  $12 \times 7$  rectangle by taking away 3 right triangles. So, we can find the area by subtracting out the area of those 3 triangles from the rectangles area. The 3 right triangles are one with legs of 4 and 4, one with legs 7 and 12, and one with legs of 8 and 7. So, the area of this triangle is  $84 - 8 - 28 - 18 = 32$ .  
Alternatively, you can use points to construct this triangle and use shoestring to find the area.
19. To find the nearest integer we can add  $(2\sqrt{2} - \sqrt{5})^6$  to what we are trying to find. We can do this because  $2\sqrt{2} - \sqrt{5}$  is less than 1 so raising it to the 6th power will make it negligible. Doing this will make it so all the square roots cancel when adding leaving you with only integers, giving you 16874
20. The shape formed is a regular hexagon with side length 1. So, the area is  $\frac{3\sqrt{3}}{2}$
21. Basically what the information about the angles tell you is that the quadrilateral is cyclic. Therefore, we just apply Ptolemy's Theorem and so it's  $16*8 + 13*11 = 271$ .
22. We can rewrite this as  $\frac{1}{3}(\frac{(x+2)^2+4}{x+2})$ . Going further, this is  $\frac{1}{3}((x+2) + \frac{4}{x+2})$ . Since we're trying to find the least positive value, we can use AM/GM, getting  $\frac{x+2+\frac{4}{x+2}}{2} \geq \sqrt{(x+2) * \frac{4}{x+2}}$ , which means  $\frac{(x+2)^2+4}{x+2} \geq 4$  meaning the min of this function is  $\frac{4}{3}$
23. The first 3 clues basically limit down the potion that lets you go forward to positions 3 and 5. The potion forward cannot be in position 2 because then either the first clue or second clue would be broken. Then, the last game tells you it must be the 5th position.
24. This is just the number of derangements. You can use principle of inclusion and exclusion, round  $\frac{4!}{e}$ , or straight up list them out since it's small enough.
25. Just list them out since 7 is pretty small. There's 15 of them.
26. First, you find the determinant of the current matrix, which is -362. Then, you find the matrix of minors. From here, you just negate every element who's rows and columns sum to an odd number. From here, just add all the elements and divide by -362 which gives you  $\frac{40}{181}$
27. The 2nd smallest perfect number is 28, so the sum of digits is 10.
28.  $\frac{|3*2+7*5+4*3-15|}{\sqrt{2^2+3^2+5^2}} = \sqrt{38}$
29. If Player 2 has a guaranteed move to win, then if player 1 did that strategy on their turn, then they would have won, so playing optimally, player 1 has a winning strategy.
30. Area of a circle is  $\pi r^2$ , so  $8\pi$