

1) Let's start this test off with a very easy question. How many factors does 2520 have?

- A) 28            B) 32            C) 36            D) 48            E) NOTA

2) Here's a slightly harder question. What is the sum of the factors of 2520?

- A) 4536            B) 8064            C) 9360            D) 10480            E) NOTA

3) How many zeroes does 2520! end in?

- A) 627            B) 628            C) 629            D) 630            E) NOTA

4) What is the remainder when the following is divided by  $x - 1$ ?

$$\sum_{n=1}^{2520} x^n$$

- A) 1260            B) 1850            C) 2021            D) 2520            E) NOTA

5) What is the sum of the two smallest roots of  $y = 2x^4 + 6x^3 - 22x^2 - 6x + 20$ .

- A) -6            B) -4            C) 0            D) 3            E) NOTA

6) Simplify:

$$\log_8 343 \cdot \log_{49} 729 \cdot \log_9 256$$

- A)  $\frac{9}{2}$             B) 8            C) 12            D)  $\frac{27}{2}$             E) NOTA

7) Zach's favorite number is 7. What is the **tens** digit of  $7^{2021}$ ?

- A) 0            B) 1            C) 3            D) 4            E) NOTA

8) Which of the following vectors are orthogonal to  $\langle 2, -1, 3 \rangle$ ?

- A)  $\langle 5, 6, 2 \rangle$       B)  $\langle -2, -3, 5 \rangle$       C)  $\langle 8, -4, 12 \rangle$       D)  $\langle 7, 10, 2 \rangle$       E) NOTA

9) There is a triangle  $\triangle ABC$  such that  $AB = 6$ ,  $AC = 10$ , and  $BC = 14$ . The angle bisector from vertex  $A$  intersects  $BC$  at  $D$  and the median drawn from vertex  $C$  intersects  $AB$  at  $E$ .  $AD$  and  $CE$  intersect each other at point  $O$ . What is  $\frac{AO}{OD}$ ?

- A)  $\frac{8}{5}$       B) 2      C)  $\frac{8}{3}$       D) 3      E) NOTA

10) The function  $f(x) = x^4 - 12x^3 + 51x^2 - 92x + 60$  can be written in factored form as  $f(x) = (x - r_1)(x - r_2)(x - r_3)^2$ . Find  $r_1^2 + r_2^2 + r_3^2$ .

- A) 10      B) 20      C) 38      D) 42      E) NOTA

11) What type of conic formed by the equation  $r = \frac{8}{5 - 8 \cos(\theta)}$

- A) Circle      B) Ellipse      C) Hyperbola      D) Parabola      E) NOTA

12) How many squares are on a standard 8x8 chess board?

- A) 196      B) 204      C) 212      D) 220      E) NOTA

13) What is the sum of the solutions of  $\cot^2 \theta \cos^2 \theta = \csc^2 \theta - \frac{3}{2}$  where  $0 \leq \theta < 2\pi$ ?

- A)  $\frac{3\pi}{2}$       B)  $2\pi$       C)  $\frac{8\pi}{3}$       D)  $4\pi$       E) NOTA

14) There is a rectangle  $ABCD$  with point  $P$  inside of it. If  $AP^2 + CP^2 = 128$ , what is  $BP^2 + DP^2$ ?

- A) 96      B) 128      C) 144      D) Not enough information      E) NOTA

15) What is  $\cos(36^\circ)\cos(72^\circ)$ ?

- A)  $\frac{1}{8}$       B)  $\frac{1}{4}$       C)  $\frac{\sqrt{2}}{4}$       D)  $\frac{1}{2}$       E) NOTA

16) What is the determinant of  $\begin{bmatrix} 2 & 6 & 4 & 10 \\ 1 & 5 & 3 & 8 \\ 2 & 4 & 3 & 7 \\ 6 & 1 & 10 & 2 \end{bmatrix}$ ?

- A)  $-300$       B)  $60$       C)  $210$       D)  $720$       E) NOTA

17) Amy and Alice are playing a number guessing game. Alice is thinking of a number and gives Amy four clues. The number leaves a remainder of 5 when divided by 13, a remainder of 3 when divided by 6, a remainder of 4 when divided by 11, and is smaller than 250. What is the sum of the digits of the number Alice is thinking of?

- A) 2      B) 4      C) 6      D) 8      E) NOTA

18) What is the area of a triangle with side lengths:  $A = 4\sqrt{2}$ ,  $B = 3\sqrt{17}$ , and  $C = \sqrt{113}$ ?

- A) 28      B)  $8\sqrt{14}$       C) 30      D) 32      E) NOTA

19) What is  $(\sqrt{5} + 2\sqrt{2})^6$  rounded to the nearest integer.

- A) 16873      B) 16874      C) 16875      D) 16876      E) NOTA

20) What is the area of the shape formed by the roots of  $x^6 = 1$  in the Argand plane?

- A)  $\frac{3\sqrt{3}}{2}$       B) 3      C)  $3\sqrt{3}$       D) 6      E) NOTA

21) Quadrilateral  $ABCD$ , has side lengths  $AB = 16$ ,  $BC = 13$ ,  $CD = 8$ , and  $AD = 11$ . Diagonals  $AC$  and  $BD$  are drawn, which intersect at point  $E$ .  $\angle DAE$  and  $\angle CBD$  are congruent and  $\angle CAB$  and  $\angle CDB$  are congruent. What is  $AC \cdot BD$ ?

- A) 176                  B) 208                  C) 271                  D) 308                  E) NOTA

22) What is the minimum positive value of  $\frac{x^2 + 4x + 8}{3x + 6}$ ?

- A)  $\frac{1}{2}$                   B) 1                  C)  $\frac{3}{2}$                   D) 2                  E) NOTA

23) Jeffrey is trying to steal the Philosopher's stone. He walks into the potion puzzle which has been changed. Now, there are 5 potions in a row numbered 1-5 from left to right. One of the potions is the one he needs to proceed, two of them are poison, and two of them are harmless. Then, he sees the puzzle. First, the poison will always be adjacent to a harmless potion. Second, the potions at the ends are different. Third, none of the potions in prime positions will kill you. Lastly, the 3rd potion will not help you go forward. Which position is the potion that will let Jeffrey go forward.

- A) 1                  B) 2                  C) 3                  D) 4                  E) 5

24) Sean is sending gifts from Korea to Samuel, Joanne, Anna, and Olivia. However, the packages get mixed up so no one knows who was supposed to get which package. How many ways are there for no one to receive the gift that Sean meant to send them?

- A) 6                  B) 7                  C) 8                  D) 9                  E) NOTA

25) An integer partition of a number  $n$  is a way of writing  $n$  as a sum of positive integers. For example, the partitions of 3 are  $1 + 1 + 1$ ,  $1 + 2$ , and 3. How many partitions of 7 are there?

- A) 14                  B) 15                  C) 16                  D) 17                  E) NOTA

26) What is the sum of the elements of  $A^{-1}$  given that  $A = \begin{bmatrix} 3 & 7 & 1 \\ 5 & 4 & 8 \\ 2 & 10 & 12 \end{bmatrix}$ ?

- A)  $\frac{23}{362}$       B)  $\frac{20}{181}$       C)  $\frac{23}{181}$       D)  $\frac{40}{181}$       E) NOTA

27) A perfect number is a positive integer that is equal to the sum of its proper divisors. What is the sum of the digits of the 2<sup>nd</sup> smallest perfect number?

- A) 6      B) 8      C) 10      D) 12      E) NOTA

28) What is the distance between between the point  $(3, 7, 4)$  and the plane defined by

$$2x + 5y + 3z = 15$$

- A) 5      B)  $\sqrt{38}$       C) 6      D)  $\frac{\sqrt{646}}{19}$       E) NOTA

29) Chomp is a pure strategic game with the following rules. Cookies are set out in a 6x8 rectangular grid. The bottom left corner is poisoned. Two players take turns "chomping" one of the cookies which is eating the cookie they selected and every cookie above or to the right of the cookie. The player who eats the poisoned cookie loses. Who wins?(Assume each player plays optimally)(Do not choose E)

- A) Player 1      B) Player 2  
C) There is no set winner      D) They draw  
E) NOTA

30) I'm sorry if you split up and you got sent to the back, so here's a little present. What is the area of a circle with radius  $2\sqrt{2}$ ?

- A)  $2\sqrt{2}\pi$       B)  $4\pi$       C)  $6\pi$       D)  $8\pi$       E) NOTA