2022 Mu Alpha Theta National Convention **Washington DC**

Alpha Analytic Geometry Answers and Solutions

ANSWERS :

1. B	11. D	21. C
2. A	12. D	22. B
3. E	13. A	23. E
4. C	14. C	24. C
5. E	15. B	25. B
6. E	16. B	26. A
7. D	17. B	27. C
8. B	18. A	28. B
9. C	19. D	29. D
10. C	20. C	30. B

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SOLUTIONS :

1. $y=-\frac{3}{2}x+12 \to 3x+2y=24.$ Any non-zero multiple of this equation will result in a line parallel or coincident to the given line. **B**
2. First, find the displacement vectors from (6, 1, 2): $\langle 4,-3,0\rangle$ and $\langle 0,3,1\rangle$. Then find the cross product of these two vectors. This gives $\langle -3,-4,12\rangle$. Using the first point, we can find the equation of the plane: $-3(x-6)-4(y-1)+12(z-2)=0 \to -3x-4y+12z=2.$ Finally, use the point-to-plane formula: $\frac{|-3(1)-4(1)+12(1)-2|}{\sqrt{3^2+4^2+12^2}}=\frac{3}{13}$, **A.**
3. If $\theta=-140°$, the value of r must be -3. **A**
4. $\sqrt{21}=\sqrt{r^2+5^2-2(5)r\cos 60°}=\sqrt{r^2-5r+25} \to r^2-5r+25=21 \to (r-4)(r-1)=0 \to r=1,4.$ **C**
5. The side lengths are $2\sqrt{10}$, $4\sqrt{10}$, and $6\sqrt{10}$, which cannot form a triangle. **E**
6. $y=\frac{x^4+x^3-9x^2-3x+18}{x^3+3x^2-4x-12}=\frac{(x+3)(x-2)(x^2-3)}{(x+3)(x-2)(x+2)} \to \frac{x^2-3}{x+2}$. Cross-multiplying, we get $xy+2y-x^2+3=0 \to x^2-xy-2y-3=0 \to x=\frac{-y\pm\sqrt{y^2-4(1)(-2y-3)}}{2}=\frac{-y\pm\sqrt{y^2+8y+12}}{2}=$ $\frac{-y\pm\sqrt{y^2+8y+12}}{2}=\frac{-y\pm\sqrt{(y+6)(y+2)}}{2}$. From here, the domain of the radical expression should give the range of the original function: $(-\infty,-6]\cup[-2,\infty)$; however, there is a removable discontinuity at $(-3,-6)$, so the range is actually $(-\infty,-6)\cup[-2,\infty)$, **E.**
7. $y=\frac{x^2-3}{x+2}=x-2+\frac{1}{x+2}$, so the slant asymptote is $y=x-2$. The vertical asymptote is $x=-2$. $ac+b=(1)(-2)-2=-4$, **D.**
8. The two line intersect at $y=2.5$ and have opposite slopes, so $y=2.5$ is the angle bisector. **B**
9. $\cos 2\theta-2\sin 2\theta=0 \to \cos^2\theta-\sin^2\theta-2\sin\theta\cos\theta=0 \to r^2\cos^2\theta-r^2\sin^2\theta-r^2\sin\theta\cos\theta=0 \to$ $x^2-y^2-4xy=0$, **C.**
10. The plane will intersect both cones, making a hyperbola C.
11. The displacement vector from A to B is $\left\langle 4,\frac{3}{2}\right\rangle$. If AB:PB = 4, then P is located three-fourths of the distance from A to B. $\left(\frac{3}{4}\right)\left\langle 4,\frac{3}{2}\right\rangle=\left\langle 3,\frac{9}{8}\right\rangle$, so we need to add these values to the coordinates of A. $\left(-1+3,\frac{5}{2}+\frac{9}{8}\right)=\left(2,\frac{29}{8}\right)$, **D.**

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12. We first need the constant to be 1, so $\frac{7}{16}x^2-\frac{3}{8}\sqrt{3}xy+\frac{13}{16}y^2=1$. Using the formula $\frac{2\pi}{\sqrt{4ac-b^2}}$:

$$\frac{2\pi}{\sqrt{4\left(\frac{7}{16}\right)\left(\frac{13}{16}\right)-\left(\frac{3}{8}\sqrt{3}\right)^2}}=\frac{2\pi}{\sqrt{\frac{91}{64}-\frac{27}{64}}}=2\pi,\ \textbf{D}.$$

13. $\tan 2\theta=\frac{-6\sqrt{3}}{7-13}=\sqrt{3}\to\theta=30°$, $x=\frac{\sqrt{3}}{2}x'-\frac{1}{2}y'$, $y=\frac{1}{2}x'+\frac{\sqrt{3}}{2}y'$. Substituting these values into the original equation gives $4(x')^2+16(y')^2-16=0\to\frac{x^2}{4}+y^2=1$. Minor axis length is 2, **A**.

14. $25x^2+16y^2+150x-128y-1119=0\to25\left(x^2+6x+9\right)+16\left(y^2-8y+16\right)=1119+225+256\to$ $25(x+3)^2+16(y-4)^2=1600\to\frac{(x+3)^2}{64}+\frac{(y-4)^2}{100}=1$. The directrices will be located at $y=4\pm\frac{a^2}{c}=4\pm\frac{100}{6}$, the positive value being $\frac{62}{3}$, **C**.

15. It is a property of odd functions that the product of two odd functions is even. **B**

16. The given vectors are the displacement vectors used in problem 2. The magnitude of their cross-product is 13, so the area is 13, **B**.

17. From the given information, we can see that the hyperbola is vertical and $a=3$, giving us $\frac{(y-2)^2}{9}-\frac{(x-5)^2}{b^2}=1$. The slope of the asymptote is $\frac{a}{b}$, so $\frac{2}{1}=\frac{3}{b}\to b=\frac{3}{2}$. $a^2+b^2=c^2=\frac{45}{4}$, so $c=\frac{3}{2}\sqrt{5}$, **B**.

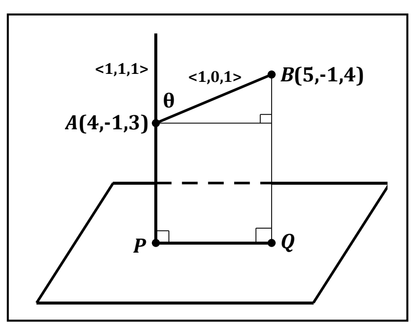
18. The denominator must contain the coordinates of a displacement vector or a non-zero multiple of those coordinates. Choice A does not have that. All the numerators in the choices are $x-/y-/z-$ a point that the line passes through. In the case of D, the midpoint between the given points. **A**

19. $(mx+k)^2=4px\to m^2x^2+(2km-4p)x+k^2=0$. We need to find where the discriminant is 0. $\left(2km-4p^2\right)^2-4\left(m^2\right)\left(k^2\right)=0\to-16kmp+16p^2=0\to k=\frac{p}{m}$, **D**.

20. The first equation generates a circle. The other three generate 8-petal roses. **C**

21. The intercepts are $\left(-\frac{k}{2},0\right)$ and $\left(0,-\frac{k}{3}\right)$. $27=\frac{1}{2}\left(-\frac{k}{2}\right)\left(-\frac{k}{3}\right)\to k^2=324\to|k|=18$, **C**.

22. Let $A(4,-1,3)$ and $B(5,-1,4)$, and let $\overline{PQ}$ be the projection on the plane. Then, $\overrightarrow{AB}=\langle1,0,1\rangle$. Let θ be the angle between the $\langle1,1,1\rangle$ normal vector and $\overrightarrow{AB}$. $\cos\theta=\frac{\langle1,1,1\rangle\cdot\langle1,0,1\rangle}{\sqrt{1^2+1^2+1^2}\sqrt{1^2+0^2+1^2}}=\frac{\sqrt{6}}{3}$, so $\sin\theta=\frac{\sqrt{3}}{3}$. $\sin\theta=\frac{PQ}{AB}\to\frac{\sqrt{3}}{3}=\frac{PQ}{\sqrt{2}}\to PQ=\frac{\sqrt{6}}{3}$, **B**.



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23. $t=\dfrac{y-1}{2}\to x=3\left(\dfrac{y-1}{2}\right)^2=\dfrac{3}{4}\left(y^2-2y+1\right)\to 3y^2-4x-6y+3=0$, **E.**

24. $\text{comp}_{\mathbf{b}}\mathbf{a}=\dfrac{a\bullet b}{|b|}=\dfrac{8+6}{\sqrt{4+9}}=\dfrac{14\sqrt{13}}{13}$, **C.**

25. The midpoint is (–2, 4) and the slope between the given points is –1/3, so the slope of the perpendicular bisector is 3. $y-4=3(x+2)\to 3x-y+10=0$, **B.**

26. $\vec{a}\bullet\vec{b}=0$, so $\vec{a}$ and $\vec{b}$ are perpendicular. Since they are also unit vectors, $|\vec{a}|=|\vec{b}|=1$. Since they are perpendicular, the angle α between them is $90°$. We also know that, by definition, $|\vec{a}\times\vec{b}|=|\vec{a}||\vec{b}|\sin\alpha$; since $\alpha=90°$, we have $|\vec{a}\times\vec{b}|=|\vec{a}||\vec{b}|=1$. We also know that the dot product of a vector with itself is 1, and that the cross product of two vectors gives a vector perpendicular to those two vectors. Therefore, the dot product of either vector with its cross product will be 0.
$\vec{c}\bullet\vec{c}=2^2=4=\left(x\vec{a}+y\vec{b}+\left(\vec{a}\times\vec{b}\right)\right)\left(x\vec{a}+y\vec{b}+\left(\vec{a}\times\vec{b}\right)\right)=\left(x^2+0+0\right)+\left(0+y^2+0\right)+\left(0+0+|a\times b|\right)=$
$x^2+y^2+1\to x^2+y^2=3$. $\vec{a}\bullet\vec{c}=|\vec{a}||\vec{c}|\cos\theta=\vec{a}\bullet\left(x\vec{a}+y\vec{b}+\left(\vec{a}\times\vec{b}\right)\right)=(1)(2)\cos\theta=x+0+0\to$
$x=2\cos\theta$. $\vec{b}\bullet\vec{c}=|\vec{b}||\vec{c}|\cos\theta=\vec{b}\bullet\left(x\vec{a}+y\vec{b}+\left(\vec{a}\times\vec{b}\right)\right)=(1)(2)\cos\theta=0+y+0\to y=2\cos\theta$.
$x^2+y^2=3\to 4\cos^2\theta+4\cos^2\theta=3=8\cos^2\theta$, **A.**

27. The given vector has length $\sqrt{4+36+9}=7$, so multiply each coordinate by –5/7. **C**

28. The centroid divides the distance from the orthocenter to the circumcenter in a ratio of 2:1. The *x*-coordinates of the orthocenter and centroid increase by 6, so the distance from centroid to circumcenter increases by 3; likewise, the *y*-coordinates decrease by 2, so the distance from centroid to circumcenter decreases by 1. This lands the circumcenter at $C(6, 2)$.
$AC=\sqrt{90}=3\sqrt{10}$, so the radius is $\dfrac{3}{2}\sqrt{10}$, **B.**

29. $r=\dfrac{15}{4+\cos\theta}=\dfrac{\frac{15}{4}}{1+\frac{1}{4}\cos\theta}$, so we have a horizontal ellipse. The vertices will be at $(3, 0)$ and $(5, \pi)$, which are 8 units apart. **D**

30. $\overline{AB}$ is the chord of contact. If we plug in (0, 3) we will get the equation of the chord and the *y*-coordinates of the endpoints of the chord of contact: $4(0)(x)-(3)(y)=36\to y=-12$.
$4x^2-(-12)^2=36\to x^2=45\to x=\pm 3\sqrt{5}$. Now we have area $=\dfrac{1}{2}\left(6\sqrt{5}\right)(15)=45\sqrt{5}$, **B.**