1) D	7) C	13) A	19) D	25) B
2) D	8) B	14) C	20) C	26) D
3) E	9) D	15) B	21) A	27) C
4) A	10) A	16) C	22) E	28) B
5) E	11) B	17) B	23) C	28) C
6) B	12) A	18) B	24) C	30) D

Alpha Sequences and Series Solutions

1. The sum of the first n odd integers is n^2, so sum of the first 2021 odd integers is 2021^2 which is 4084441. $\boxed{\textbf{D}}$

2. Since every positive even number is 1 more than it's respective odd term, it's just 2021 more than question one, which is 4086462. $\boxed{\textbf{D}}$

3. Andrew has to do 10 questions, so that's 30 minutes, however he only takes 9 breaks because after he finishes he's done, so he's done $9*2+3*10$, 48 minutes after 8. $\boxed{\textbf{E}}$

4. First, we will take the inner sum, holding a to be constant. This makes the question become $\sum_{a=1}^{50} 50a + 1275$. We can seperate these into $50\sum_{a=1}^{50} a + \sum_{a=1}^{50} 1275$. We know 1275 is the sum from 1 to 50 so this question is just 100 times the sum of integers from 1 to 50 or 127500. $\boxed{\textbf{A}}$

5. Listing out a few terms, $i - 1 - i + 1$, we see that the sum is 0 every 4 terms. Therefore, all we care about is the 2021st term which is just i. $\boxed{\textbf{E}}$

6. For this question, it will be easier to visualize if make the terms into a single log. So, the 3rd term is log20 and the 6th term is log160. So, if we call the first term a and the common difference d, $a + 2d = log20$ and $a + 5d = log160$. Subtracting them, we get $3d = log8$ so $d = log2$. Plugging this back into either of the equations, you get $a = log2$. $\boxed{\textbf{B}}$

7. So you could just sum them up, it's not that hard. However, the sum of the first n Fibonacci numbers is $F(n+2) - 1$. The 14th Fibonacci number is 377, so it's 376. It's not that hard to find the 14th, 12th is 144 and 11th is 89. $\boxed{\textbf{C}}$

8. So, this is a classic stars and bars question. We can imagine the variables as bars and the exponent as the stars since we are "giving" a certain amount of the exponent to each variable. so, there are 3 bars and 15 stars, so $18C3$ is 816. $\boxed{\textbf{B}}$

9. Trivially, there are 108 three, two, and 1 digit palindromes. There are 10 palindromes for each "thousand". So up to 4994 there are 148 palindromes, so 2 after 4994 is 5115. $\boxed{\textbf{D}}$

10. With the given information, we have $ar^2 = e^{\frac{6\pi}{7}i}$ and $ar^5 = e^{\frac{12\pi}{7}i}$. So, by dividing the 2nd equation by the 1st equation, we have $r^3 = e^{\frac{6\pi}{7}i}$ which means $r = e^{\frac{2\pi}{7}i}$. By observing the first 6 terms, we see that this is the roots of unity of $x^7 = 1$ but missing 1. So, the rest of the roots sum to -1. $\boxed{\textbf{A}}$

11. If we inspect the series, $a_{3n} = \frac{3n}{a_{3n-1}a_{3n-2}}$. This means that every 3rd term is n divided by the previous 2 terms. Since we are multiplying them, the previous 2 terms cancel so we get 3*6*9*12. $\boxed{\textbf{B}}$

12. If the angles of a triangle are in an arithmetic sequence, one of them must be $60°$. It doesn't matter which angle you call $60°$, let's say it's angle B. Using law of sins, $\frac{4}{\frac{\sqrt{2}}{2}} = \frac{B}{\frac{\sqrt{3}}{2}}$. This means $B = 2\sqrt{6}$. Now, we can find the area. We have $\frac{1}{2} * 4 * 2\sqrt{6} * \sin(75°) = 4\sqrt{6} * \frac{\sqrt{6}+\sqrt{2}}{4} = 6 + 2\sqrt{3}$. $\boxed{\textbf{A}}$

13. This sequence is similar to Fibonacci numbers. Since we're not given that much information, it may be useful to get every term into the same variables as less variables is easier to work with. So, we will write all the first 10 terms in terms of $a_1 and a_2$ So, we have $a_3 = a_1 + a_2, a_4 = a_1 + 2a_2, a_5 = 2a_1 + 3a_2, a_6 = 3a_1 + 5a_2, a_7 = 5a_1 + 8a_2$ and so on. (If you notice the coefficient of the a_1 term is the $n-2$ Fibonacci number and the coefficient of the a_2 term is the $n-1$ Fibonacci number). From here, if we sum all of the first 10 terms, we get $55a_1 + 88a_2$ which happens to be $11a_7$. So, our answer is 11*17=187. $\boxed{\textbf{A}}$ Alternatively, you can just set a_6 and a_5 to whatever you want as long as they sum up to 17 because the terms themselves don't actually matter. You will always get 187.

14. So, we can you flip all the fractions, get the sequence, and flip it back. We will call $f(x)$, the function defined by these points, (1,6) (2,20) (3,42) (4,72) (5,110). By taking the difference of the differences, we see that this function is a quadratic. You can either set up a 3 variable 3 equations system to solve using any 4 of the given points or, if you notice, $x = 1$ is $2 \cdot 3$, $x = 2$ is $4 \cdot 5$, $x = 3$ is $6 \cdot 7$. So, it really looks like $f(x) = (2x)(2x + 1)$. Plugging in 10 and reciprocating it, you get $\frac{1}{420}$. $\boxed{\textbf{C}}$

15. This is just listing the numbers in order in base 4. This means, the 2021st term is just 2020 in base 3 which is 133210. $\boxed{\textbf{B}}$

16. If a limit exists, which it does, then the ratio of consecutive terms approaches 1, or that the $k+1$ term equals the k term. That turns this question into $L = \frac{2}{3}L + 2$ Solving this, we get $L = 6$. $\boxed{\textbf{C}}$

17. By looking at the example, we see there is an algorithmic approach for finding the continued fraction. First, we take $\lfloor \frac{43}{12} \rfloor$ which is 3 and make this into $3 + \frac{7}{12}$ and then into $3 + \frac{1}{12/7}$. Then we take the floor of $\frac{12}{7}$ and make it into $3 + \frac{1}{1+\frac{5}{7}}$. Then, we keep doing the process of flipping the fraction, flooring it, subtracting that from the fraction until we get 1 over an integer. So, the continued fraction for $\frac{43}{12}$ is $3 + \frac{1}{1+\frac{1}{1+\frac{1}{2+\frac{1}{2}}}}$ or [3;1,1,2,2]. Summing these up, we get 9. $\boxed{\textbf{B}}$

18. Do the same process as the one used for $\frac{43}{12}$. So floor of $\sqrt{2}$ is 1, so we have $1 + \sqrt{2} - 1$, which then becomes $1 + \frac{1}{\sqrt{2}+1}$. The floor of $\sqrt{2} + 1$ is 1. So, we have $1 + \frac{1}{2+\sqrt{2}-1}$. As we see, we have $\sqrt{2} - 1$ again, so we know that the 2s repeat infinitely. So, the continued fraction is $[1; \overline{2}]$. $\boxed{\textbf{B}}$

19. We can analyze Alex winning in two way, Zach hitting Amy or Alex hitting Zach. For Zach hitting Amy, the first time would be on his first turn, which has a $\frac{3}{25}$ of happening since Alex has to miss completely and Zach has to hit Amy. For every successive Zach turn, there is a common ratio of $\frac{9}{25}$. So, for Zach hitting Amy, there is a total of $\frac{3}{16}$ chance of that happening. For Alex winning, there is a $\frac{3}{10}$ probability on the first turn. From then on, it is geometric with common ratio $\frac{9}{25}$ as well. This has a $\frac{15}{32}$ chance of happening. Adding these two up gives $\frac{21}{32}$. $\boxed{\textbf{D}}$

20. We know that $A+B = \sum_{n=1}^{\infty} \frac{1}{(n)^3}$. Simplifying B, we have B $= \frac{1}{8} \sum_{n=1}^{\infty} \frac{1}{(n)^3}$. Substituting, $B = \frac{1}{8}(A+B)$. Solving, we get $A = 7B$. So, $\frac{A}{B} = 7$. $\boxed{\textbf{C}}$

21. This is just a geometric sum with first term $\frac{3}{4}$ and common ratio $\frac{3}{16}$ so the sum is $\frac{12}{13}$. $\boxed{\textbf{A}}$

22. There are 3 one digit perfect square, six 2 digit, and 22 three digit ones, so that makes up a total of 81 digits. Now, we have 4 digit perfect squares and need 207 more digits. To find which square this is, we divide 207 by 4 and get 51 with remainder 3. So, the 288th digit will be the third digit of the 52nd 4 digit perfect square. This is $31 + 52$ which is 83. $83^2 = 6889$. So, 8. $\boxed{\textbf{E}}$

23. The ratio of the sequence is $\dfrac{a_2}{a_1} = \dfrac{\sin c}{\cos c} = \tan c$. The RHS of the sum cancels the first two terms of the LHS, so we have $\sin c \tan c + \sin c \tan^2 c + \cdots = 0 = \dfrac{\sin c \tan c}{1 - \tan c}$ by the geometric series sum formula. This is true when the numerator is 0, which happens precisely when $\sin c = 0$, when $c = 0$ or $c = \pi$ for a sum of $0 + \pi = \pi$. $\boxed{\textbf{C}}$

24.One might be tempted to call the first term a then each term after that $a \cdot r^n$. It's much easier instead to set b to be the middle term. The product is then

$$br^{-4} \cdot br^{-3} \cdot br^{-2} \cdot br^{-1} \cdot b \cdot br^1 \cdot br^2 \cdot br^3 \cdot br^4 = b^9 = 512 \Rightarrow b = 2$$

$\boxed{\textbf{C}}$

25. So, the sum of an infinite geometric is $\frac{a}{1-r}$ where a is the first term and r is the common ratio. If we take every third term starting from a, have the sum is $\frac{a}{1-r^3}$. Factoring the bottom, this is equal to $\frac{a}{(1-r)(1+r+r^2)}$. From the original information, we know that $\frac{a}{1-r} = j$. So, we have our new sum as $\frac{j}{1+r+r^2}$. To maximize this, since j is constant, we just need to minimize our bottom. This is where $r = \frac{-1}{2}$ Pluging this back in, we get $\frac{4}{3}j$. $\boxed{\textbf{B}}$

26. The sum of 360 consecutive sine values is 0, and since we have $1113 - 26 + 1 = 1088$ sine values, we can cancel out $3 \times 360 = 1080$ of them, leaving the last 8 terms. Option A is clearly wrong, and option B has 9 terms. Option C is indexed incorrectly (ends on $\sin(1112°)$, whereas option D has the correct setup for the last 8 terms. $\boxed{\textbf{D}}$

27. Say $x = n + 1$. Now this question is $2\sum_{x=1}^{\infty} \frac{x^2}{2^x}$.
Say $S = \frac{1^2}{2} + \frac{2^2}{2^2} + \frac{3^2}{2^3} + \frac{4^2}{2^4}$.... Multiplying both sides by 2, we get $2S = \frac{1^2}{1} + \frac{2^2}{2^1} + \frac{3^2}{2^2} + \frac{4^2}{2^3}$.... Subtracting the first equation by the second one, we get $S = 1 + \frac{3}{2} + \frac{5}{2^2} + \frac{7}{2^3}$... .Doing the same process again, we finally end up with $S = 2 + 2 + 1 + \frac{1}{2} + \frac{1}{4}$..., so we have $S = 6$. Since what we are looking for is 2 times this, the final answer is 12. $\boxed{\textbf{C}}$

28. Every 400 years, it will always be the same day. So, it's Tuesday. $\boxed{\textbf{B}}$ Alternatively, 400 is 1 mod 7 and there are 97 leap years every 400 years, so each day is 98 days ahead, which is the same day since 98 is 0mod7

29. Say the sum of grains in the first column is x. Then, the sum of the grains in the 2nd column is 3x, the third column is 9x, the 4th column is 27x, and so on. This is because each row is basically a finite geometric sequence with common ratio 3. So, the total number of grains is $x + 3x + 9x + 27x + 81x + 243x + 729x + 2187x = 3280x$. The 6th column from the left contains 243x grains, so the answer is $\frac{243}{3280}$. $\boxed{\textbf{C}}$

30. $\frac{1}{1-\frac{1}{2}} = 2$. $\boxed{\textbf{D}}$