For this test, E) NOTA means "None Of These Answers". Define $i = \sqrt{-1}$. Good luck!

1. Let's start with an easy question: What is the sum of the first 2021 positive **odd** integers?

A) 2,041,210 B) 4,082,420 C) 4,084,341 D) 4,084,441 E) NOTA

2. To go along with question 1, what is the sum of the first 2021 positive **even** numbers?

A) 2,043,241 B) 4,084,441 C) 4,086,362 D) 4,086,462 E) NOTA

3. Andrew starts doing his 10 question discrete math homework at 8 pm. Every question takes him 3 minutes to do. However, after doing each question, he gets distracted and plays BrawlStars for 2 min. What time does Andrew finish his homework?

A) 8:45 pm B) 8:50 pm C) 8:55 pm D) 9 pm E) NOTA

4. Evaluate:

$$\sum_{a=1}^{50}\left(\sum_{b=1}^{50}(a+b)\right)$$

A) 127500 B) 227500 C) 250000 D) 255000 E) NOTA

5.Evaluate:

$$\sum_{n=1}^{2021} i^n$$

A) 0 B) $i-1$ C) 1 D) $1-i$ E) NOTA

6. There is an arithmetic sequence where the 3^{rd} term is $2-\log 5$ and the 6^{th} term is $1+4\log 2$. What is the first term of this sequence?

A) $\log 5 - \log 2$ B) $\log 2$ C) $\log 5$ D) 1 E) NOTA

7.What is the sum of the first 12 Fibonacci numbers, beginning with $F_1 = 1$, $F_2 = 1$ and $F_n = F_{n-2} + F_{n-1}$?

A) 369 B) 375 C) 376 D) 384 E) NOTA

8. After combining like terms, how many terms are in the expansion of $(3a+2b+3c+4d)^{15}$

A) 716 B) 816 C) 3060 D) 3876 E) NOTA

9. Eric is very bad at finding palindromes and needs your help. What is the 150^{th} smallest positive palindrome?

A) 4444 B) 4774 C) 4994 D) 5115 E) NOTA

10. There is a geometric sequence such that the third term is $e^{\frac{6\pi}{7}i}$ and the sixth term is $e^{\frac{12\pi}{7}i}$. What is the sum of the first six terms?

A) -1 B) 0 C) $\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$ D) $\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$ E) NOTA

11. Let $a_1 = 12$ and $a_2 = 6$, and for $n > 2$, $a_n = \dfrac{n}{a_{n-1} \cdot a_{n-2}}$. Find $a_1 \cdot a_2 \cdot a_3, \dots \cdot a_{12}$

A) 1296 B) 1944 C) 2048 D) 3072 E) NOTA

12. Triangle ΔABC has angles in an arithmetic sequence with $m\angle A = 45°$ and the side opposite $\angle A$ has length 4. What is the area of ΔABC?

A) $6+2\sqrt{3}$ B) $2\sqrt{6}+6\sqrt{2}$ C) $12+\sqrt{3}$ D) $12+4\sqrt{3}$ E) NOTA

13. Let $a_1, a_2, a_3 ...$ be a sequence of numbers such that $a_n = a_{n-1} + a_{n-2}$ for every integer n where $n > 2$. Given that $a_7 = 17$, what is the sum of the first 10 terms of this sequence?

A) 187 B) 204 C) 224 D)248 E) NOTA

14. There is a sequence of numbers, $\frac{1}{6}, \frac{1}{20}, \frac{1}{42}, \frac{1}{72}, \frac{1}{110}, \dots$. What is the 10^{th} term in this sequence?

A) $\frac{1}{300}$ B) $\frac{1}{396}$ C) $\frac{1}{420}$ D) $\frac{1}{450}$ E) NOTA

15. There is a sequence of numbers only containing digits of 0,1,2 and 3 in increasing order. It's 0,1,2,3,10,11,12,13... What is the 2021^{st} term in this sequence?

A) 130432 B) 133210 C) 133211 D) 210321 E) NOTA

16. There is a sequence of numbers, a_n, where $a_{k+1} = \frac{2}{3}a_k + 2$, and $a_1 = 2021$. Evaluate:

$$\lim_{k\to\infty} a_k$$

A) 0 B) 3 C) 6 D) 8 E) NOTA

Use the following information for questions 17-18

A continued fraction is a representation of a number as $a_0+\cfrac{1}{a_1+\cfrac{1}{a_2+\ddots\frac{1}{a_n}}}$ where every number besides a_0 is positive and every a_n is an integer. This is also denoted as $[a_0; a_1, a_2, ..., a_n]$, referred to as bracket notation.

If we find the continued fraction for $\frac{17}{12}$. We would first write it out as $1+\frac{5}{12}$, then as $1+\frac{1}{\frac{12}{5}}$, then as $1+\frac{1}{2+\frac{2}{5}}$, then as $1+\frac{1}{2+\frac{1}{\frac{5}{2}}}$, and finally as $1+\frac{1}{2+\frac{1}{2+\frac{1}{2}}}$. This would be notated as $[1;2,2,2]$. Note: $[3;1,2,3,1,2,3...] = [3;\overline{1,2,3}]$

17. What is the sum of the numbers in the bracket, when the continued fraction for $\frac{43}{12}$ is written in bracket notation with the fewest number of entries?

A) 6 B) 9 C) 10 D) 13 E) NOTA

18. This one is a little harder. What is the continued fraction for $\sqrt{2}$?

A) $[1;\overline{1,2}]$ B) $[1;\overline{2}]$ C) $[1;\overline{2,1}]$ D) $[1;\overline{2,1,1}]$ E) NOTA

19. Alex and Zach are sworn enemies. After Zach flexes his wealth to Alex, Alex challenges Zach to a game which Zach promptly accepts. The game is to take turns throwing pies at each other until one of them hits the other. However, there is another condition. Amy is a spectator and watching this spectacle. If either Zach or Alex hits Amy with a pie, she destroys the person that hit her and that person loses the game. Alex has a 30% chance of hitting Zach and a 10% chance of hitting Amy with a pie while Zach has a 20% chance of hitting Alex and a 20% chance of hitting Amy. Given that Alex gets to go first, what is the probability that Alex wins the game?

A) $\dfrac{15}{32}$ B) $\dfrac{9}{16}$ C) $\dfrac{5}{8}$ D) $\dfrac{21}{32}$ E) NOTA

20. Given the definitions below, compute $\dfrac{A}{B}$:

$$A=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} \qquad B=\sum_{n=1}^{\infty}\frac{1}{(2n)^3}$$

A) 5 B) 6 C) 7 D) 8 E) NOTA

21. Kira and Jack are playing a game. They take turns giving each other math problems and trying to solve them until one person has solved the math problem given to them. For example, if Kira went first, Jack would give her a math problem, and she would have two minutes to solve it. If she doesn't solve it, she would then give Jack a math problem which he attempts to solve in two minutes. They each get a new problem on each of their turns. Now, Kira is a high IQ individual while Jack is a low IQ individual, so Kira has a $\frac{3}{4}$ chance of solving her math problem on her turn while Jack only has a $\frac{1}{4}$ chance of solving his math problem on his turn. Given that Kira gets to go first, what is the probability that Kira wins the game?

A) $\frac{12}{13}$ B) $\frac{13}{14}$ C) $\frac{14}{15}$ D) $\frac{15}{16}$ E) NOTA

22. There is a number, 1.1491625364964... where after the decimal point is the sequence of perfect squares, starting from 1. What is the 288th digit after the decimal point?

A) 2 B) 4 C) 7 D) 9 E) NOTA

23. A geometric sequence exists such that the first term is $a_1 = \cos c$ and the second term is $a_2 = \sin c$. Find the sum of the values of $c \in [0, 2\pi)$ such that

$$\sum_{n=1}^{\infty} a_n = \cos c + \sin c$$

A) $\frac{\pi}{2}$ B) $\frac{3\pi}{4}$ C) π D) 2π E) NOTA

24. A 9 term geometric sequence of real numbers has product 512. What is the middle term?

A) $\frac{1}{2}$ B) $\sqrt{2}$ C) 2 D) $2\sqrt{2}$ E) NOTA

25. An infinite geometric sequence has total sum j, where $j > 0$ and j is finite. What is the maximum possible sum of every third term in the series, starting with the first term? This means $a + ar^4 + ar^7$... where a is the starting term and r is the common ratio.

A) $\frac{2}{3}j$ B)$\frac{4}{3}j$ C) $2j$ D) $3j$ E) NOTA

26. Amy's favorite number is 26. Kira's favorite number is 1113. Which of the following is equivalent to

$$\sum_{n=26}^{1113} \sin(n^\circ)$$

A) 0 B) $\sum_{n=1105}^{1113} \sin(n^\circ)$ C) $\sum_{n=1}^{8} \sin\left[(n+1104)^\circ\right]$ D) $\sum_{n=1106}^{1113} \sin(n^\circ)$ E) NOTA

27. Evaluate:

$$\sum_{n=0}^{\infty} \frac{(n+1)^2}{2^n}$$

A) 6 B) 11 C) 12 D) 16 E) NOTA

28. Pokemon was created on Tuesday, February 27th, 1996. What day of the week does February 27th, 2396 fall on?

A) Monday B) Tuesday C) Wednesday D) Thursday E) NOTA

29. The squares of a standard 8x8 chessboard are numbered left to right, and top to bottom such that the top left corner is numbered #1, the top right corner is numbered #8, and the bottom right corner is numbered #64. There is 1 grain of wheat placed on square #1, 3 on square #2, 9 on square #3, 27 on square #4, and so on. What fraction of wheat is placed on the column that contains the square marked #6?

A) $\frac{1}{18}$ B) $\frac{27}{2551}$ C) $\frac{243}{3280}$ D) $\frac{1}{3}$ E) NOTA

30. Congratulations for making it to the end! What is the sum of an infinite geometric series with first term 1 and common ratio $\frac{1}{2}$?

A) $\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) 2 E) NOTA