2022 Mu Alpha Theta National Convention Algebra II Hustle Answers and Solutions

<u>SOLUTIONS :</u>

1. f(-2) = 2^2 – 1= 3, f(3) = 3^2 – 1, and finally f(8) = $\sqrt{8+4} = \sqrt{12} = 2\sqrt{3}$

2. Sum the three equations to get 4x + 4y + 4z = 16. Dividing both sides of the equation by 4 gives the desired sum of x + y + z = 4.

3. Using Vieta's formulas, the sum of the reciprocal of the roots for a cubic function is $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$. This simplifies to $\frac{bc+ac+ab}{abc}$. Using the given coefficients of the polynomial, $\frac{bc+ac+ab}{abc}$ simplifies to $\frac{3}{2}$. Without using Vieta's formulas, $x^3 - 3x + 2$ can be factored into $(x-1)^2(x+2)$. Using the roots 1, 1, and -2 will also produced the desired result of $\frac{3}{2}$.

4. Since the seventeenth number is 29, there are 16 differences and the sum is 29-(-3)=32, so the common difference is 2. The tenth number is nine differences away from -3, so the tenth number is 9*2+(-3)=15. The average of the first ten numbers is (15-3)/2=6, so the sum of the first ten numbers is 6*10=60.

5. C = $2^{\frac{1}{2}}$ because $\sqrt[3]{\sqrt{8}} = (2^{\frac{3}{2}})^{1/3}$. Therefore, $\log_2 C^{\frac{4}{3}} = \log_2 2^{\frac{1}{2}\cdot\frac{4}{3}}$ = 2/3 $\log_2 2$ = 2/3

6. The given equations are in Ax + By = C form. The slope of the line is -A/B so the slope of the first equation is 6. The perpendicular slope is -1/6. $-\frac{k}{(-15)} = -\frac{1}{6}$, so 6k = -15 and k = $-\frac{5}{2}$

7. The number of terms in an expansion of x terms to the power of y is $\binom{x+y-1}{x-1}$ This gives us $\binom{11}{4}$ = 330.

8. $H(x) = ax^4 + bx^3 + cx^2 + dx + 24$. Use the given roots to find the product of the roots, which is -48. Therefore, with Vieta's formula for product of roots, we have $\frac{24}{a} = -48$ and a = $-\frac{1}{2}$. H(x) = $-\frac{1}{2}$(x – 2)(x – 3)(x – 4)(x + 2). To find the sum of the coefficients of H(x), we find H(1) = $-\frac{1}{2}$(1 - 2)(1 – 3)(1 – 4)(1 + 2) = 9.

9. If both sides are multiplied by x^2- 5x - 6, 3=a(x-2)+b(x-3). Substituting 2 for x and solving we get b = -3. Substitute 3 for x and solving we get a = 3. The sum of a and b is 0.

10. Canceling the denominator of previous term, the numerator of the succeeding term leaves just the expression $\log_{10}\frac{1}{100}$ which gives us an answer of -2.

11. The summation of cubes is given by $\left(\frac{n(n+1)}{2}\right)^2 = \left(\frac{10(11)}{2}\right)^2$ = 3025

12. The given equation simplifies to $4 \le n/3 < 5$. From here we arrive at $12 \le n < 15$. Therefore, the integer values that satisfy this inequality are 12, 13, and 14. Their sum is 39.

13. $f(x) = \frac{(x-2)(3x-1)}{x+1} = \frac{3x^2-7x+2}{x+1} = 3x - 10 + \frac{12}{x+1}$.
The slant asymptote is therefore $y = 3x - 10$

14. $\log_4(256^{2020}) = \log_4(4^{4*2020}) = 4 * 2020 = 8080$

15. Squaring both sides of the equation gives us $x^2 = 2x + 35$. Rearranging and factoring gives $(x - 7)(x + 5) = 0$, which has solutions $x = 7$ and $x = -5$. Because we squared both sides of the equation, we must check that solutions are not extraneous. The original equation is not satisfied by $x = -5$, so the sum of the solutions is 7.

16. 9^{x-1} can be written as 3^{2x-2} using properties of exponents. $3^{2x-2} = \frac{3^{2x}}{3^2} = \frac{(3^x)^2}{9} = \frac{36}{9} = 4$

17. $\frac{5+12i}{2-3i} * \frac{2+3i}{2+3i} = \frac{10+15i+24i-36}{4+6i-6i+9} = \frac{-26+39i}{13} = -2 + 3i$

18. If $x - 1$ is a factor of the given polynomial, then $f(1) = 0$. $f(1) = 1^3 + 3k + k^2 + k - 1 = k^2 + 4k$. $k^2 + 4k = 0$, then $k(k + 4) = 0$ and $k = 0, -4$

19. We start by completing the square. $4x^2 - 8x + 9y^2 + 90y = -193$. $4(x^2 - 2x) + 9(y^2 + 10y) = -193$. $4(x^2 - 2x + 1) + 9(y^2 + 10y + 25) = -193 + 4 + 225$. This simplifies $4(x - 1)^2 + 9(y + 5)^2 = 36$. Dividing both sides of the equation by 36, we arrive at $\frac{(x-1)^2}{9} + \frac{(y+5)^2}{4} = 1$, which is the equation of an ellipse. The a term is $\sqrt{9} = 3$ and the b term is $\sqrt{4} = 2$. The area $= \pi ab = 6\pi$

20. If $f(x)$ is divided by $(x - a)$, the remainder is $f(a)$. Since we are dividing by $7x - 14$, we have the form $7(x - 2)$ and our remainder is $f(2)$. $2^{10} - 2*2^6 + 4 = 900$

21. Arrange the terms on one side and we get $x^2 + y^2 - 6x + 2y = 10$. Completing the square, we obtain $(x - 3)^2 + (y + 1)^2 = 20$. The radius of the circle is $2\sqrt{5}$ and thus the diameter is equal to $4\sqrt{5}$, the side length of the square. Area of the square is $(4\sqrt{5})^2 = 80$

22. $|(2 + 2i)^6| = |2 + 2i|^6 = (\sqrt{2^2 + 2^2})^6 = \sqrt{8}^6 = 8^3 = 512$

23. There are 6! ways to arrange the six students in a line. We view the two students who refuse to stand beside each other as a single block and therefore there are 5!*2 ways to arrange the six students with the two who refuse next to each other (multiplied by two because they can be arranged two ways). Therefore, the number of ways to arrange the six students without having the two students standing next to each other is 6! – 5!*2 = 720 – 120*2 = 480

24. This series can be written as $\sum_{\text{n}=1}^{\infty}\frac{n}{2(3^{n-1})}$, which can be written as $\sum_{n=1}^{\infty}\frac{3n}{2(3^{n})}$.
    The $\frac{3}{2}$ can be taken out of the summation to become $\frac{3}{2}\sum_{n=1}^{\infty}\frac{n}{(3^{n})}$. Now, this series can be written out as $\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots=\frac{\frac{1}{3}}{1-\frac{1}{3}}=\frac{1}{3}\times\frac{3}{2}=\frac{1}{2}$

    $$+\frac{1}{9}+\frac{1}{27}+\ldots=\frac{1}{2}-\frac{1}{3}=\frac{1}{6}$$

    $$+\frac{1}{27}+\ldots=\frac{1}{6}-\frac{1}{9}=\frac{1}{18}$$

    $\frac{1}{2}+\frac{1}{6}+\frac{1}{18}+\ldots$can be seen as an infinite geometric series with $\frac{1}{2}$ as the first term and $\frac{1}{3}$ as the common ratio. This series is equal to $\frac{\frac{1}{2}}{1-\frac{1}{3}}=\frac{1}{2}\times\frac{3}{2}=\frac{3}{4}$. So $\frac{3}{2}\times\frac{3}{4}=\boldsymbol{\frac{9}{8}}$**.**

25. The formula for the eccentricity of the hyperbola is $\frac{\sqrt{a^2+b^2}}{a}$. *a* is smaller than *b*, so we know that *a* is the square root of the denominator (25) and *b* is the square root of the other denominator (144), so a = 5 and b = 12. We can substitute now. $\frac{\sqrt{25+144}}{5}=\frac{\sqrt{169}}{5}=\boldsymbol{\frac{13}{5}}$**.**

ANSWERS :

1. $2\sqrt{3}$
2. 4
3. $\frac{-3}{2}$.
4. 60
5. 2/3
6. -$\frac{5}{2}$
7. 330
8. 9
9. 0
10. -2
11. 3025
12. 39
13. y = 3x - 10
14. 8080
15. 7
16. 4
17. $-2 + 3i$
18. $0, -4$
19. 6π
20. 900
21. 80
22. 512
23. 480
24. $\mathbf{\frac{9}{8}}$
25. $\mathbf{\frac{13}{5}}$