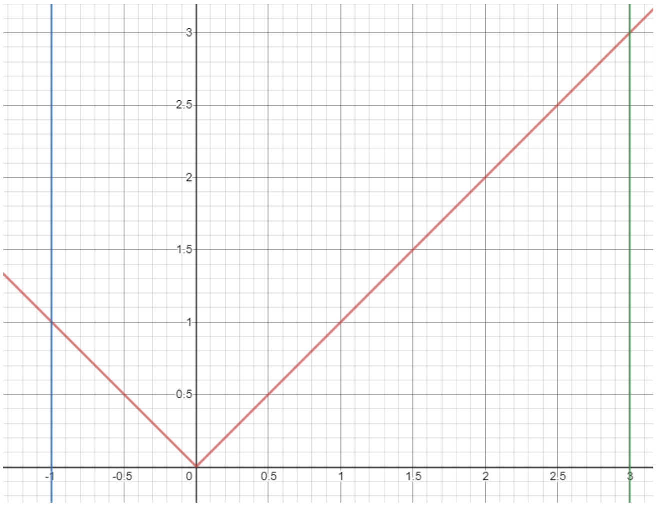
2022 Mu Alpha Theta National Convention
Calculus Hustle Answers and Solutions

ANSWERS :

1. 11	2. none	3. $\ln\lvert x+6\rvert + C$	4. 4	5. 5
6. $\theta^2\left(\frac{5\pi}{12}\right)$	7. EVT	8. -8	9. 6.4	10. -2.8
11. $\frac{3}{4}\sqrt[3]{16}-\frac{5}{4}$ OR $\frac{3}{2}\sqrt[3]{2}-\frac{5}{4}$	12. $\frac{1372}{3}$	13. $\frac{4\pi}{3}$	14. I OR Increasing	15. $y = 45x + \frac{331}{3}$
16. $-\frac{1}{\pi}$	17. 0	18. $\frac{2}{\pi}$	19. $1296\ln 6 + 864$	20. -69.5 OR $-\frac{139}{2}$
21. $\left(0,\frac{3}{8}\right)\cup\left(\frac{9}{16},\infty\right)$	22. $-\frac{\mathbf{14}}{\mathbf{3}}$	23. C	24. C, D	25. $\frac{1}{6}$

SOLUTIONS :

1. The graph will not be differentiable at any "sharp turns" such as x = 2, 4 and 5. The sum is 11.
2. We are looking for a place on the derivative graph where the y-values change from negative to positive. There is no such place on this graph, so the answer is none.
3. To calculate the value of the indefinite integral, I would first factor the denominator: $\int \frac{x+2}{x^2+8x+12} dx = \int \frac{(x+2)}{(x+2)(x+6)} dx = \int \frac{1}{(x+6)} dx =$
   $ln|x+6| + C$
4. $\lim_{x \to 2} g(x) = 1 \quad + \lim_{x \to -2^+} g(x) = \quad 1 \quad + \lim_{x \to -2^-} g(x) = 2$
   The values can be found by substituting into the piecewise function.
   Because 2 is a part of two pieces of the function, you must substitute it into both to ensure continuity. (1/2 (2^2)-1)=(-2+3) = 1
   For the one-sided limits, you only substitute into the appropriate piece. From the right, (1/2 (-2)^2 – 1) = 1. From the left, -(-2) = 2.
   1+1+2 = 4
5. The integral in this case can be considered equal to the area under the curve for the graph of absolute value of x. The two triangles have an area sum of (1/2)(1)(1) + (1/2)(3)(3) = 5



6. Note that "x" is the variable of interest here, NOT theta.

$$\frac{1}{3}\theta^2 x\Big]_{\frac{\pi}{4}}^{\frac{3\pi}{2}} = \frac{1}{3}\theta^2\left(\frac{3\pi}{2}-\frac{\pi}{4}\right) = \frac{1}{3}\theta^2\left(\frac{6\pi}{4}-\frac{\pi}{4}\right) = \frac{1}{3}\theta^2\left(\frac{5\pi}{4}\right) = \theta^2\left(\frac{5\pi}{12}\right)$$

7. If you want to learn more about the theorem: https://www.khanacademy.org/math/ap-calculus-ab/ab-diff-analytical-applications-new/ab-5-2/v/extreme-value-theorem#:~:text=The%20Extreme%20value%20theorem%20states,a%20minimum%20on%20the%20interval. Extreme Value Theorem **EVT**

8. $u = x^2\,;u' = 2x\,;v = \cos(x)\,;v' = -\sin(x)$

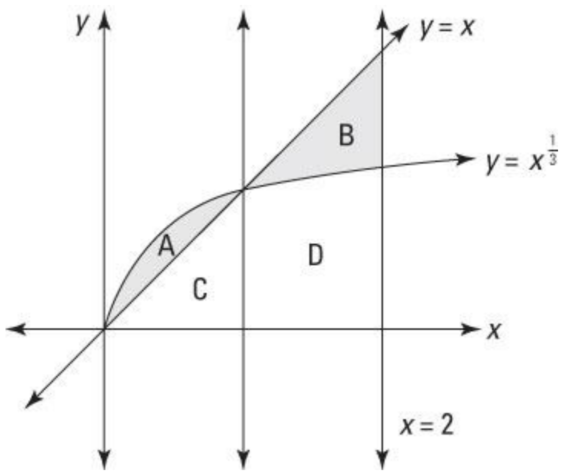
$$u'v + v'u = (2x)(\cos(x)) + (-\sin(x))(x^2)\,;2\frac{\pi}{4}\cos\frac{\pi}{4} - \left(\frac{\pi}{4}\right)^2 sin\frac{\pi}{4}\,;\frac{\pi}{4}(2)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\pi}{4}\right)^2\left(\frac{\sqrt{2}}{2}\right) = \left(\frac{\sqrt{2}}{4}\right)\left(\pi - \frac{\pi^2}{8}\right)$$

9. $V = \pi r^2 d\,; V = \pi(4^2)d\,; \frac{dV}{dt} = 16\pi \frac{dd}{dt} = 16\pi\,(-0.4) = -6.4\pi$

$$6.4\pi\ feet\ per\ second$$

10. Average Value: $\frac{1}{2-0}\int_0^2 f(x)dx = 1.4\,;\ \int_2^0 f(x)dx$ = -2(1.4)= -2.8

11.



Find the difference between Area D and Area C:

Area D: $\int_1^2 x^{\frac{1}{3}}dx - \int_0^1 xdx = \left[\frac{3}{4}x^{\frac{4}{3}}\right]_1^2 - \left[\frac{1}{2}x^2\right]_0^1 = \frac{3}{4}\left(\sqrt[3]{16} - 1\right) - \left(\frac{1}{2}\right) = \frac{3}{4}\sqrt[3]{16} - \frac{5}{4}$

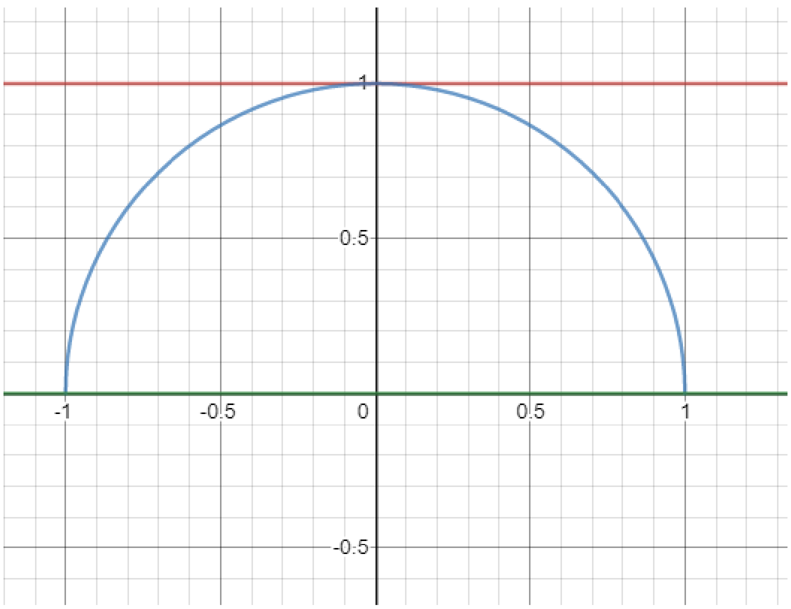
12. First we will need to find the points of intersection: $x^2 + 2x = 48; x^2 + 2x - 48 = 0; (x+8)(x-6) = 0; x = 6, -8$

$$\int_{-8}^{6} 48 - (x^2 + 2x)dx\ ;\ \left[48x - \frac{1}{3}x^3 - x^2\right]_{-8}^{6}$$

$$= \left[(48)(6) - \frac{1}{3}6^3 - 6^2\right] - \left[(48)(-8) - \frac{1}{3}(-8^3) - (-8)^2\right]$$

$$= 288 - \frac{216}{3} - 36 - 384 + \frac{512}{3} + 64 = \frac{1372}{3}$$

13. Here is the initial graph:



About the x-axis: We can use the disk method –

$$V = \pi \int \left(\sqrt{1 - x^2}\right)^2 dx = \pi \int 1 - x^2 dx = 2\pi \left[x - \frac{1}{3}x^3\right]_0^1 = \frac{2\pi 2}{3}$$

$$= \frac{4\pi}{3}$$

14. $x^2 y = 4\ ; y = \frac{4}{x^2}\ ; y = 4x^{-2}; y' = -8x^{-3}; y'(-1) = 8$

15. $y'(-4) = 2(-4)^2 - 3(-4) + 1 = 45 = slope$

$$y(-4) = \frac{2}{3}(-4)^3 - \frac{3}{2}(-4)^2 - 4 + 1 = -\frac{209}{3}$$

$$y + \frac{209}{3} = 45(x+4); \boldsymbol{y = 45x + \frac{331}{3}}$$

16. $\int_{-1}^{\frac{3}{2}} \cos(\pi y)\, dy\ ;\ \left[\frac{1}{\pi}\sin(\pi y)\right]_{-1}^{\frac{3}{2}} = \frac{1}{\pi}(-1) - 0 = -\frac{1}{\pi}$

17. When you originally substitute in infinity, the result is indeterminate. Therefore we will use L'Hopital's Rule. $\frac{d\left(x^6-7x\right)^{\frac{1}{5}}}{d(e^{6x})} = \frac{\frac{1}{5}\left(x^6-7x\right)^{-\frac{4}{5}}(6x^5-7)}{6e^{6x}} = \frac{\frac{1}{5}(6x^5-7)}{6e^{6x}(x^6-7x)^{\frac{4}{5}}}$; ... The denominator is growing at a faster pace, which makes the limit = 0.

18. $\frac{1}{\pi}[\sec(\pi x)]_{-1}^{0} = \frac{1}{\pi}[1+1] = \frac{2}{\pi}$

19. $u = 6^{3x^2}\ ; u' = 6^{3x^2} ln6\ (6x)\ ; v = x^4; v' = 4x^3$

I would substitute in x =1 now instead of using the algebraic expressions to simplify to make it easier on myself:

$$u = 6^3 = 216; u' = (216)ln6(6); v = 1^4 = 1\ ; v' = 4(1)^3 = 4$$

$$u'v + v'u = (6)(216)ln6 + (4)(216) = 1296ln6 + 864$$

20. $y = -2x^3 - \frac{7}{2}x^2 + 5x + 1; y' = -6x^2 - 7x + 5; 0 = -6x^2 - 7x + 5\ ; 0 = 6x^2 + 7x - 5\ ; 0 = (2x - 1)(3x + 5); x = \frac{1}{2}, -\frac{5}{3}$

Check the direction of the slope:

x	$-\frac{5}{3}$	-1	$\frac{1}{2}$	3
Y'(x)	Out of domain	6	0	-70

Because of the direction change from positive to negative of the graph, we can know that there is relative maximum at the point when x = ½. Because we are on a fixed domain, I will additionally check the endpoints for minimum values:

x	-1	$\frac{1}{2}$	3
y	-5.5	19/8	-69.5

21. $s(t) = -6t^3 + 8t^4 + 5; s'(t) = v(t) - 18t^2 + 32t^3; s''(t) = a(t) = -36t + 96t^2$

For the speed to increase, we are looking for the velocity and the acceleration to be working together (both positive or both negative). First, I will look for the zeroes.

$$s'(t) = v(t) - 18t^2 + 32t^3 = 0 = 2t^2(16t - 9); t = 0, \frac{9}{16}$$

t	0	¼	$\frac{9}{16}$	1
V(t)	0	-5/8	0	+14

$$s''(t) = a(t) = -36t + 96t^2 = 0 = 6t(16t - 6); t = 0, \frac{3}{8}$$

t	0	¼	$\frac{3}{8}$	1
a(t)	0	-3	0	+60

They are both negative on the interval (0,3/8) and both positive on (9/16,infinity)

$$\left(0, \frac{3}{8}\right) \cup \left(\frac{9}{16}, \infty\right)$$

22. $\frac{1}{3-(-1)}\int_{-1}^{3} -2x^3 + 4x^2 - 5x + 1dx$; $\frac{1}{4}[-\frac{1}{2}x^4 + \frac{4}{3}x^3 - \frac{5}{2}x^2 + x]_{-1}^{3}$; $-\mathbf{\frac{14}{3}}$

23. You use the slope formula:

A: $\frac{-5-(-1)}{0-(-2)} = -2$

B: $\frac{0-(4)}{0-(-2)} = -2$

C: $\frac{0-(6)}{0-(-2)} = -3$

D: $\frac{4-(0)}{0-(-2)} = 2$

C is the least

24. Points of inflection are where the second derivative changes direction. On the derivative graph, this can be found when it has a relative min or max. Letters C and D correspond to relative max and minimums, which would be POI on the original graph.

C and D

25. $A = \int_0^1 x - x^2 dx; \frac{1}{2}x^2 - \frac{1}{3}x^3; \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$