

# Interschool Test Solutions

Mu Alpha Theta National Convention  
July 10-15, 2022

## 1 Trivia

### 1.1 Two Lies and a Truth

1. c
2. b
3. c
4. c
5. a
6. c
7. b
8. a
9. a

### 1.2 Spoiler Warning!

1. Brad Pitt
2. Alphonse Elric (also acceptable: Alphonse, Al)
3. Norma Bates (also acceptable: Mrs. Bates, his mother)
4. *Stranger Things*
5. *Iron Man 3*
6. The Red Wedding
7. Ryuk

8. *Mission: Impossible III* (also acceptable: 3, third)

9. spoiler

### 1.3 Bash

1. 3,646,259,816,065,458,050

2.  $(x - 71)(x - 4)(x - 2)(x + 1)(x + 6)(x + 36)(x + 55)(24x + 1)$ . By the Rational Root Theorem, the numerator of any rational root will be a factor of 68,792,932 and the denominator will be a factor of 24. Using synthetic division is one method of determining if such a fraction is a root and what the quotient polynomial is.

3. 390,543. The determinant can be found by multiplying the cofactor of each element in the same row or column by the determinant of its minor and summing. For example, using the first row, the sum would be

$$8 \begin{vmatrix} 3 & 1 & 3 & 83 \\ 5 & 2 & 3 & 7 \\ 0 & 5 & 20 & 8 \\ 3 & 5 & 31 & 8 \end{vmatrix} + (-11) \begin{vmatrix} 7 & 1 & 3 & 83 \\ 9 & 2 & 3 & 7 \\ 5 & 5 & 20 & 8 \\ 1 & 5 & 31 & 8 \end{vmatrix} + 4 \begin{vmatrix} 7 & 3 & 3 & 83 \\ 9 & 5 & 3 & 7 \\ 5 & 0 & 20 & 8 \\ 1 & 3 & 31 & 8 \end{vmatrix} + 0 \begin{vmatrix} 7 & 3 & 1 & 83 \\ 9 & 5 & 2 & 7 \\ 5 & 0 & 5 & 8 \\ 1 & 3 & 5 & 8 \end{vmatrix} + 8 \begin{vmatrix} 7 & 3 & 1 & 3 \\ 9 & 5 & 2 & 3 \\ 5 & 0 & 5 & 20 \\ 1 & 3 & 5 & 31 \end{vmatrix}$$

The determinants of each 4x4 matrix can be found with the same method.

4. 30,280,085,544. The area between two consecutive zeros will always be either positive or negative, so this integral is equivalent to summing the absolute values of the integrals between every zero in the range.

$$\left| \int_{-31}^{-9} (12x^4 + 528x^3 + 3348x^2 - 3888x) dx \right| + \left| \int_{-9}^0 (12x^4 + 528x^3 + 3348x^2 - 3888x) dx \right| \\ + \left| \int_0^1 (12x^4 + 528x^3 + 3348x^2 - 3888x) dx \right| + \left| \int_1^{95} (12x^4 + 528x^3 + 3348x^2 - 3888x) dx \right|$$

5. 489

## 2 Math

\*Click the test name for the solution link\*

### 2.1 AMC

1. **Thursday** ; AMC 10A 2014 #2

2. **48** ; AMC 10B 2012 #7

3.  $\frac{7}{27}$  ; AMC 10B 2017 #9

4.  $\frac{12}{25}$  ; AMC 10A 2015 #11
5. **8078** ; AMC 10A 2019 #15
6. **16089** ; AMC 12B 2011 #17
7. **7** ; AMC 10B 2012 #20
8. **151** ; AMC 12A 2016 #19
9. **-7007** ; AMC 12A 2017 #23
10. **2024** ; AMC 12B 2015 #25

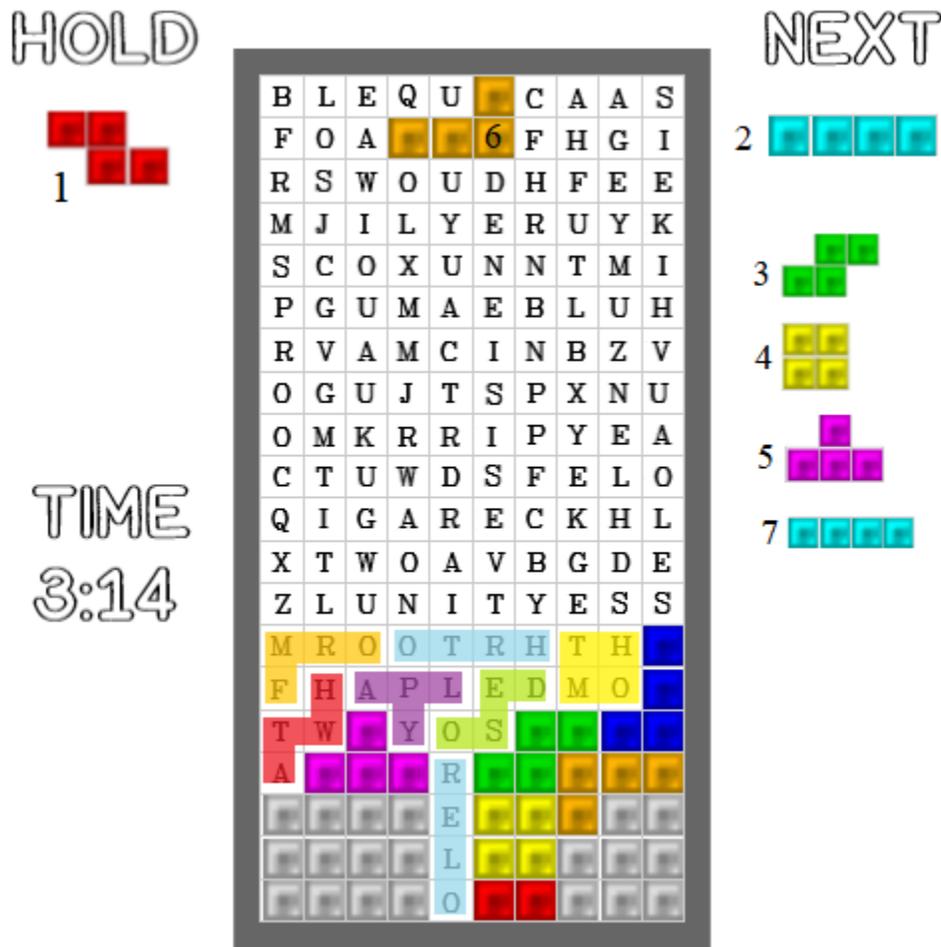
## 2.2 AIME

1. **85** ; AIME I 2011 #1
2. **216** ; AIME I 2012 #3
3. **180** ; AIME II 2016 #4
4. **58** ; AIME I 2015 #6
5. **147** ; AIME I 2018 #8
6. **13** ; AIME II 2017 #9
7. **163** ; AIME II 2010 #10
8. **958** ; AIME II 2012 #12
9. **10** ; AIME II 2013 #13
10. **65** ; AIME I 2019 #15

### 3 Puzzles

#### 3.1 Perfect Clear

Note the title of this puzzle. In Tetris, a player can achieve a “Perfect Clear” by clearing all tetrominoes from the field without any extra pieces left. Thus, in layman’s terms, all of the tetrominoes shown in the image must fit perfectly in one 10-block wide rectangle. There is only one configuration of tetrominoes that results in a Perfect Clear, as shown below.



In order to decode the message in the puzzle, note the positions of each tetromino. The four letters that each tetromino takes up can be unscrambled to form a word. For example, the letters behind the red z-tetromino can be rearranged to produce the word “WHAT”. While the order in which the tetrominoes are placed can be arbitrary, the permutation shown in the solution uses the fewest holds. Repeating the same process with all of the tetrominoes in the order described results in the question “WHAT ROLE DOES TOM H PLAY FROM THOR?” The correct answer is LOKI.

## 3.2 Chocolate Confections

The “directions” the instructions refer to aren’t the recipe directions, but rather the compass directions hidden within each line of the recipe. Every odd numbered step consists of a capitalized ingredient and a number or a reference to a number. The initials of every capitalized ingredient are a certain compass direction (ie, Nutella for North, Sweetened Elderberries for Southeast, etc), and the number is the units to move that direction in. Every even step contains a certain number of degrees, which is the number of degrees counterclockwise to turn between each step.

For example, following the first few steps gives: Move north 1 unit, turn 45 degrees, move northwest 1.5 units, turn 180 degrees (flip over), move southeast 1.5 units or as far as you moved northwest, turn 90 degrees, move 1.5 units northeast. This traces out the letter Y. Completing parts 2 and 3 in the same manner trace out the letters U and M, respectively. The answer is YUM.

## 3.3 A Contemporary Piece

While it might seem like advanced music theory would be necessary for this puzzle, it can be solved with only basic understanding of rhythm and note lengths. Observe that the title of the piece, “Comer’s Ode” is simply an anagram of “Morse Code”; in similar fashion, the name of the composer is also an anagram for Samuel F.B. Morse. These two clues trivially point out that the message hidden in the puzzle is encrypted with aural Morse Code. In regards to the system of dots and dashes, an eighth note corresponds to a dot and a quarter note corresponds to a dash. Each letter is comprised of these combinations of notes. The eighth rests separate individual letters, while the quarter rests indicate the end of an entire word.

Working out the entire piece, the encrypted message is: “GREEN DAY HIT INSPIRED BY THE CATCHER IN THE RYE”. The correct answer is WHO WROTE HOLDEN CAULFIELD.

## 3.4 Engineers Are Not Handymen

This puzzle introduces the concept of logic gates and circuits, which implement Boolean functions often employed in computer and electrical engineering. Each of the two circuits are designed in a way so that they accept a five digit binary input, denoted by the numbered nodes, in which the leftmost bit is the most significant. As explained by the supposed terrorist, only some combinations of ones and zeroes will result in a HIGH value (or synonymously turn the light bulb on). A logic expression can be written for each circuit using boolean algebra, but for those less familiar with this topic, the circuits can be evaluated on a case-by-case basis with the given information about each gate.

Using whichever method, we can see that only five unique bitwise combinations of switches can turn the light bulb on in both circuits, and they must be in increasing order, as described. Converting each of these binary numbers into their decimal equivalents, we are left with five numbers for each circuit that are each less than or equal to 26. This is a glaring clue that the numbers should be converted into letters based on their position in the alphabet. Doing so results in the words AEGIS and DEITY. Some knowledge of Greek

mythology is required here, as well as some word association. In regards to deities, the aegis is most closely connected to Zeus and Athena, but the terrorist mentioned a four-letter code. The correct answer is ZEUS.

### 3.5 Mastermind

From a cursory look, it is apparent that the question marks following each group of words are letters that each make up a word and the objective of the puzzle is to fill in the blanks with the numbered letters. However, what do the dots symbolize? This puzzle is based off a well-known game known as “Mastermind” in which a player must crack the code by attempting guesses to reveal the positions of certain letters in the code. As such, an empty dot signifies that one of the letters in the guess appears in the code, but is located in the wrong position. A filled black dot indicates that one of the letters in the guess appears in the exact same position as it does in the code. Therefore, each group of words can be considered as an individual game in which the code can be deduced from the given information.

The intermediary solutions are, in order, MATH, BLITZ, CAROL, PEANUT, ROBINS, SARDINE, and YOUNGER. After filling in the proper letters, the resulting question is: “COLORED PART IN EYE?” The correct answer is IRIS.

### 3.6 A Strange Digital Device

The one sentence description contains a couple grammar errors. “A some machine” doesn’t read right and there are four buttons, not “for.” Whoever discovered this machine evidently mixed up their homophones, and the sentence should be written “I found a sum machine with four buttons and a small display.”

There are 9 symbols. Since the machine is outputting a sum, each symbol must stand for a number (the equals sign on the answer sheet as opposed to a colon also hints at this), and the blank display when nothing is pressed yet represents zero. Though it may be logical that the symbols stand for the numbers 1-9, a small hint to this is the fact that it’s a “digital” device, not only because of the display but also because it deals entirely with digits. It’s also guessable that the four buttons given are 1-4, but this is further supported by the fact that all four buttons pushed output a blank display. 1-4 sum to 10, and the machine can’t output more than one digit at a time (note that the display is small and only has room for one digit).

Once it’s established that the buttons are the digits 1-4, figuring out the other digits is a simple matter of logic, or writing out all the combinations. For example,  $\underline{\quad}$  is repeated when pairing up the buttons, and the only number repeated in the pairwise sums is 5.  $\ominus$  pairs with two other numbers to make one of the numbers 1-4, so  $\ominus$  must be 1. Now that 1 and 5 are known,  $\approx$  and  $\ominus$  summing to  $\underline{\quad}$  means that  $\approx$  must be 4. Continuing in the same manner gives the final solution:

$$\text{H} = 3 \quad \approx = 4 \quad \ominus = 1 \quad \mathcal{V} = 2 \quad \mathcal{U} = 7 \quad \underline{\quad} = 5 \quad \mathcal{B} = 6 \quad \mathcal{A} = 8 \quad \mathcal{Q} = 9$$

### 3.7 Only 375 Possibilities

The table of numbers may be daunting, but following the riddle line by line may prove to be a fruitful task. The first line of the poem hints that seven numbers must be chosen from the table. The phrase “of first importance” can be said to be the definition for the word “prime”, and there are “infinitely many” prime numbers, all of which have no other factors except 1 and themselves. Continuing to follow the riddle step by step, the first two lines of the second stanza indicates that lines must be drawn between each of the seven numbers, resulting in 21 lines (not necessarily distinct). After drawing these lines, there are three specific points of interest in which three lines all cross at the same point, hence the allusion to Oedipus, where he slain his father at a place where “three roads meet”. These three points are all distinct from the initial seven. Finally, note that these three points form a triangle. The last stanza reveals that the three digit password can be found at the triangle’s centroid. Consult the picture below for a diagrammed solution.

330	999	678	567	246	590	121	144	046	771	998	812	432	434	500	
422	686	105	387	185	205	969	089	297	975	051	574	236	177	888	
693	329	784	266	768	707	098	551	413	696	024	366	913	212	077	
226	327	123	374	754	959	281	668	381	361	582	675	800	040	282	
315	388	289	063	963	897	482	600	176	847	294	282	964	925	094	
034	886	758	299	595	452	687	621	305	906	054	650	926	053	004	
749	208	203	535	298	924	966	192	579	275	912	112	857	069	130	
380	950	654	597	632	025	813	572	102	273	965	685	195	274	981	
934	753	865	725	922	958	188	923	995	241	332	949	111	091	586	
200	006	012	635	663	576	369	361	848	447	755	254	038	171	553	
549	398	708	209	72	565	849	841	721	948	864	286	190	045	867	
845	290	873	472	91	583	723	8	7	171	092	657	689	250	198	679
573	795	221	226	532	759	676	936	799	584	220	513	852	128	076	
065	67	958	756	921	475	442	121	968	837	101	666	552	860		
788	481	606	009	375	883	909	738	056	153	248	256	542	992	386	
698	640	546	77	750	081	512	729	210	869	036	555	291	143	049	
395	961	831	584	939	222	703	408	152	514	575	062	752	166	952	
245	114	008	760	107	44	649	507	075	254	764	945	204	855	645	
168	032	455	681	899	656	890	887	651	730	519	742	096	176	020	
609	016	933	078	596	368	902	002	671	496	674	530	363	581	772	
278	682	217	392	662	261	430	973	538	356	412	237	796	904	334	
885	54	580	489	700	262	667	050	216	240	365	033	129	115	646	
623	706	183	798	680	194	228	672	901	957	027	589	448	979	119	
352	834	807	288	106	780	100	486	598	989	055	213	348	866	253	
789	495	322	343	633	868	195	175	993	044	861	169	670	497	382	

The correct answer is 723.

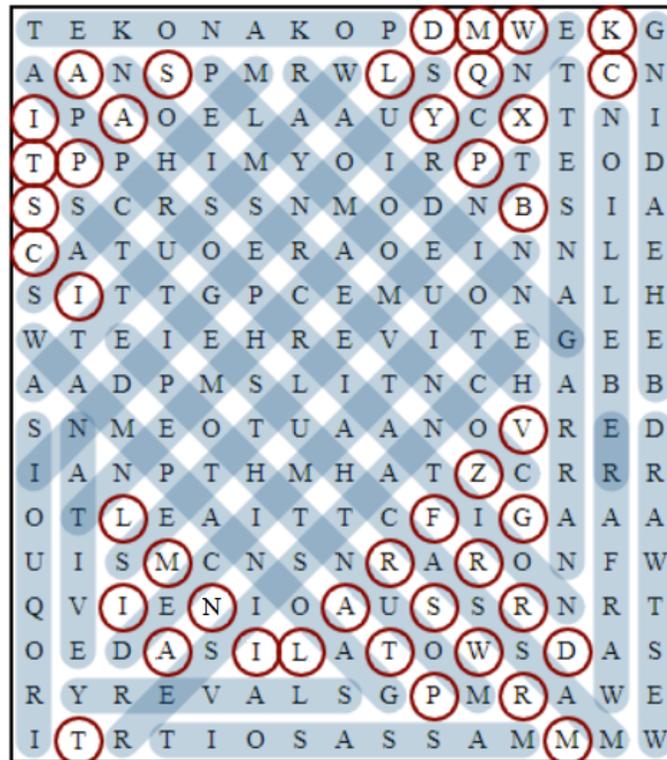
### 3.8 A Coordinate Space

The title “A Coordinate Space” isn’t referring to graphing in 3D, but coordinates in outer space. This combined with the arrangement of circles should bring to mind the solar system. Therefore, the letters A-K stand for Neptune, Asteroid Belt, Mars, Venus, Saturn, Sun, Mercury, Jupiter, Earth, Moon, and Uranus, respectively. Note that the “y” coordinates are the numbers 1-15. The first number in the coordinate gives the position of a certain letter in the name, and the second coordinate gives the position of that letter in the final word. For example, (1,13) next to Sun means the first letter of Sun is the 13th letter of the answer. Continuing in the same manner for all the coordinates gives the answer philanthropists.

### 3.9 History Teacher’s Substitute Lesson Plans

At first, the puzzle seems very straightforward, presenting itself as an ordinary word search. After finding and circling all of the words in the bank, consider the remaining letters that have not been utilized yet, as shown below in the image of the solution.

## Metacomet's Revenge



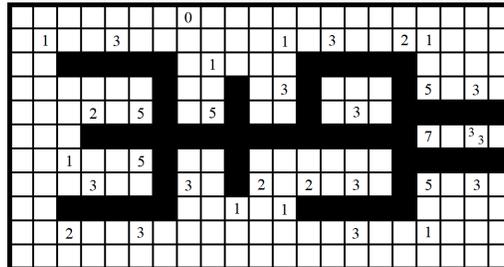
Reading from top to bottom, left to right, reveals the phrase “ITS CAPITAL IS MANILA” followed by filler letters. The correct answer is PHILIPPINES.

### 3.10 “Serious” Bibliophilia

Note the word “Serious” is in quotation marks. Looking at its word parts instead of the word’s actual meaning, it can be taken to mean “Having to do with series.” This, combined with the narrator mentioning that the books have an order, should clue in to looking at the books which are part of a series. Of the books on the bookshelf, these are the following: Mockingjay, Artemis Fowl: The Time Paradox, The Battle of the Labyrinth, A Song of Ice and Fire: A Dance with Dragons, The Lion the Witch and the Wardrobe, Harry Potter and the Deathly Hallows, and The Fellowship of the Rings. The sentence description especially emphasizes the order of the books, so looking at the order each book appears in within its series gives the following: Mockingjay: 3, The Time Paradox: 6, The Battle of the Labyrinth: 4, A Dance with Dragons: 5, The Lion the Witch and the Wardrobe: 2, Harry Potter and the Deathly Hallows: 7, and The Fellowship of the Rings: 1. Finally, the sentence says we are not choosing books but authors. Replacing each book title with the letter of the author who wrote it (at least according to these covers) gives A:3, B:6, C:4, E:5, L:2, O:7, P:1. Ordering the letters accordingly reveals the answer to be PLACEBO.

### 3.11 Memory Lane

Completing the tapa puzzle according to the rules gives:



$$3 + 9 = \boxed{12}$$