

**2022 MAO NATIONAL CONVENTION
MENTAL MATH TEST**

22) 89

ANSWERS

1) 22020

23) 4:3 or $\frac{4}{3}$

2) 75

24) $\frac{8}{81}$

3) 13216

25) 4

4) 14

26) singular or cannot be determined

5) $\frac{5}{4}$

27) 460

6) 5

28) 122

7) $\frac{7}{297}$

29) circle

8) 6

30) 3,628,800

9) 300π

31) 2

10) 58

32) 41

11) 67

33) 26,400

12) -3

34) 500,500

13) 20

35) $\frac{11}{8}$

14) 5

36) 11

15) 50

37) 30,240

16) alphabetically

38) 316

17) 25

39) December 31

18) 2, 5, 11

40) 1

19) 7560

20) 91

21) 2450

SOLUTIONS

1) The 2nd & 3rd and 4th & 5th numbers each add to 9999, or 20,000 - 2. Added to 2022, you get **22020**.

2) $\sqrt{5 \cdot 5 \cdot 5 \cdot 5 \cdot 3 \cdot 3} = 5 \cdot 5 \cdot 3 = \mathbf{75}$

3) $11 \cdot 12 - 132$ and $8 \cdot 2 = 16 \rightarrow \mathbf{13216}$

4) $111_3 = 9 + 3 + 1 = 13_{10} = \mathbf{14}_9$

5) $4^{2x+3} = 4^{6x-2}$, set the exponents equal to each other.

$$2x + 3 = 6x - 2 \rightarrow 4x = 5 \rightarrow x = \frac{5}{4}$$

6) $\log_2 5x = \log_2(x + 20) \rightarrow 5x = x + 20 \rightarrow 4x = 20 \rightarrow x = \mathbf{5}$

7) $\frac{7}{99} \cdot \frac{1}{3} = \frac{7}{297}$

8) A hexecontagon has 60 sides. $\frac{360}{60} = \mathbf{6}$

9) $SA - 3\pi r^2 = 3\pi(10)^2 = \mathbf{300\pi}$

10) $36 \div (2)(3) + 4 = (18)(3) + 4 = 54 + 4 = \mathbf{58}$

11) **67**

12) This is the difference of cubes. $5 - 8 = -3$

13) $100 - 16(5) = 100 - 80 = \mathbf{20}$

14) 9 evenly divides the first 11 digits. So, the remainder is **5**.

15) The watermelon had one pound of "meat" originally and after evaporation. $x = \text{new weight of the watermelon}$. $0.02x = 1 \rightarrow x = \mathbf{50}$.

16) alphabetically.

17) $\sqrt{(7-14)^2 + (11+13)^2} = \sqrt{49 + 576} = \sqrt{625} = \mathbf{25}$

18) 2 & 5 are obvious. The sum of the digits is 58 \rightarrow not 3. For 11, take the alternating sum of the digits which is 0. So, 11 also works. **2, 5, 11**

19) $12^3 + 18^3 = (12 + 18)(144 - 216 + 324) = (30)(252) = \mathbf{7560}$

20) We need multiples of 7 that fit the other descriptions $\rightarrow \mathbf{91}$

21) 70% of 700 = 490. $\frac{x}{5} = 490 \rightarrow x = 5 \cdot 490 = \mathbf{2450}$

22) $a + b = 45$, $b + c = 56$, $a + c = 77 \rightarrow$ By elimination $a = 33$, $b = 12$, and $c = 44$. $33 + 12 + 44 = \mathbf{89}$

23) $V = \frac{1}{3}\pi r^2 h$. Divide each volume by 5. C1 : $16\pi/3$ $\rightarrow r = 4$. C2 : $3\pi \rightarrow r = 3$ (other 3's cancel).

Ratio is **4 : 3** or $\frac{4}{3}$

24) $a_n = a_1 \cdot r^{n-1} \rightarrow a_7 = 72 \cdot (\frac{1}{3})^{7-1} = 72 \cdot \frac{1}{729} = \frac{72}{729} = \frac{8}{81}$

25) This is a convergent geometric series. $S = \frac{a_1}{1-r} \rightarrow$

$$S = \frac{2}{1-0.5} = \mathbf{4}$$

26) determinant = $20 - 20 = 0$. **Singular** matrix.

27) $200 + 200 + 200 - 40 - 60 - 50 + 10 = \mathbf{460}$

28) 11-60-61 is a Pythagorean Triple $\rightarrow \mathbf{122}$ inches

29) **circle** (see $5x^2 + 5y^2$)

30) **3,628,800**

31) First two terms are reciprocals $\rightarrow = 1$.

$$\log_4 625 = \log_2 25 = 2\log_2 5. \frac{2\log_2 5}{\log_2 5} = \mathbf{2}$$

32) $80 - 54 = 26$ which is the age difference. So, mom was 39 when Nancy was 13. $80 - 39 = \mathbf{41}$ years ago.

33) 1 mile = 5280 feet, 10 miles = 52800 feet, so 5 miles = **26,400** feet.

34) $\frac{(1000)(1001)}{2} = (500)(101) = \mathbf{500,500}$

35) $\frac{12+8+6+4+2+1}{24} = \frac{33}{24} = \frac{\mathbf{11}}{8}$

36) $\frac{18}{5} \cdot \frac{55}{18} = \frac{55}{5} = \mathbf{11}$

$$37) \frac{10!}{(10-5)!} = \frac{10!}{5!} = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = \mathbf{30,240}$$

$$38) (80 + 78)(80 - 78) = (158)(2) = \mathbf{316}$$

39) **December 31**

40) **1**