Choice E is NOTA, meaning 'none of the above answers are correct' and D.N.E means that you think a limit or integral does not exist based on the information in the question. Be sure to read the questions carefully and have fun!

1) Find the total area bounded by the curve $k(x) = x^2 - 4$ and the *x*-axis.

A. $-\frac{32}{3}$ B. $-\frac{8}{3}$ C. $\frac{16}{3}$ D. $\frac{32}{3}$ E. NOTA

2) Let *R* be the region bounded by the curves $f(x) = 2x^3 + 5x^2 - x + 7$ and $g(x) = 2x^3 + 4x^2 + 13$ in the Cartesian plane . Find the area of *R*.

A. $-\frac{32}{3}$ B. $\frac{21}{6}$ C. $\frac{95}{6}$ D. $\frac{125}{6}$ E. NOTA

3) Referring to the previous question, which of the following integrals gives the volume obtained when *R* is revolved around the line $x = -3$?

A. $\pi\int_{-2}^{3}\left(\left(2x^3+5x^2-x+8\right)^2-\left(2x^3+4x^2+14\right)^2\right)dx$ C. $2\pi\int_{-2}^{3}\left(x^3-4x^2-3x+18\right)dx$

B. $2\pi\int_{-2}^{3}\left(-x^3-2x^2+9x+18\right)dx$ D. $2\pi\int_{-2}^{3}\left(-x^3+7x+6\right)dx$. E. NOTA

4) In March of 2017, Kim decided to build a rectangular enclosure for her dogs. One side of which is her house. She had enough money to purchase approximately 100 meters of fencing and successfully erected the fencing so that the dogs have maximum grassy area to run around. Two years later, she moved out, and the new homeowners want to put a semicircular pool into the fenced area. What is the largest possible area of the pool, in square meters?

A. 250 B. 1250π C. $\frac{625}{2}\pi$ D. 250π E. NOTA

5) Rob is riding is snowmobile in a straight line southward in an Antarctic snowstorm. His velocity, in meters per second, can be represented by $v(t) = e^{\sqrt[3]{t}}$ for $t \geq 0$ and *t* is measured in seconds. What is the total distance, in meters, he traveled in the first 27 seconds of his journey?

A. $15e^3 - 6$ B. $21e^3$ C. $27e^3 - 18$ D. $27e^3 - 15e^2 + 5$ E. NOTA

6) Jamie asks Steven to find the area bound by the *x*-axis and $f'(x)$ on the interval $[-e, e]$, where $f'(x) = \frac{1}{x}$.

What is the **most plausible** reason why Steven refused to answer the question?

A. Within the interval given, the value of *c* that satisfies Rolle's Theorem on $f(x)$ is not the same one that satisfies the Mean Value Theorem for Derivatives
B. The area is divergent because he did not apply the absolute value property to $f(x)$.
C. The area is divergent because $f'(x)$ exhibits discontinuity or is not defined completely on the specified interval.
D. He should have answered the question, as the area is 0.
E. NOTA

7) Jennifer works at a construction company. She props up two ladders, each 6 feet tall, on the floor and leans them against one another at the top forming an isosceles triangle with the ground. She begins to pull the bases of the ladders apart such that the top of the triangle is coming closer to the ground. When the ladders form an equilateral triangle, she notices the angle between them is increasing at $\frac{1}{4}$ radians per second. At what rate is the area of the triangle increasing in that exact moment? Assume appropriate units.

A. $6\sqrt{3}$ B. $6\sqrt{2}$ C. $\frac{9\sqrt{3}}{2}$ D. $\frac{9}{2}$ E. NOTA

8) Find the area of the region that is above the x −axis, below the curve $y = \frac{2x^4}{x^5-3}$, and to the right of $x = 2$. Note that this region is unbounded on the right.

A. -1 B. 0 C. $\frac{1}{2}$ D. 1 E. NOTA

9) Let R be the Quadrant I region bound by the coordinate axes and the line $y = -3x + 6$. If R is the base of a solid in which cross sections of R perpendicular to the x −axis are squares, then what is the volume of the solid?

A. 24 B. 48 C. 72 D. 96 E. NOTA

10) Help Darsha evaluate the following, because she seems to have forgotten her double angle identities:

$$\int_0^{\frac{\pi}{4}} \frac{1}{1+\cos(2x)}\,dx$$

A. $\frac{1}{2}$ B. $\frac{\sqrt{2}}{4}$ C. $\frac{\sqrt{3}}{3}$ D. $\frac{\sqrt{3}}{6}$ E. NOTA

11) Tanya is just beginning grade 8, and she is currently in her math class. Today, they are learning about polygons and the geometric relationships between a circle and the polygons with sides less than 8. Her teacher shows her a regular hexagon and draws an incircle with circumference $2\sqrt{3}\pi$ inches. What is the area of the regular hexagon, in squared inches?

A. $3\sqrt{3}$ B. $6\sqrt{3}$ C. $2\sqrt{6}$ D. 2 E. NOTA

12) Given that the area of the region bounded by the x −axis, the line $x = \frac{\pi}{4}$, and the curve $y = \sin^5(x)$ is equal to K, then compute, in terms of K:

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^5(x)\,dx$$

A. $-K$ B. 0 C. K D. $2K$ E. NOTA

13) Ryan is beginning calculus level 3-D coordinate geometry. His final exam is coming up. Help him find, to the nearest square unit, the area of triangle *ABC* determined by the coordinates:

$A(0,1,7)$ $B(3,1,1)$ $C(4,-2,-2)$

A. 5 B. 10 C. 15 D. 20 E. NOTA

14) What is the surface area of a rectangular prism with dimensions $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$?

A. $\frac{1}{24}$ B. $\frac{3}{8}$ C. $\frac{3}{4}$ D. $\frac{13}{12}$ E. NOTA

15) Anthony, Miles, and Danielle are competing at a Mu Alpha Theta convention. They stumble upon an area question that has them evaluate:

$$\int_0^{\ln(2)} \left(\frac{3}{e^x - 3} \right) dx$$

They each selected the same correct answer but solved it different ways. Talking about it afterwards, Danielle and Anthony used partial fraction decomposition to help, but Miles said he did not need to do so and showed them an arguably equally as interesting way to solve it. What was their answer (i.e., what is the value of the integral)?

A. $-3\ln(2)$ B. $-\ln(4)$ C. $-\ln(2)$ D. $\ln(4)$ E. NOTA

16) In a manner of speaking, the Mean Value Theorem for Integrals guarantees that for a definite integral, a rectangle with the same area and width exists. What value of *c* satisfies the Mean Value Theorem for Integrals on the interval $[0, \ln(4)]$ for the function:

$$g(x) = \frac{3}{e^x + 3}?$$

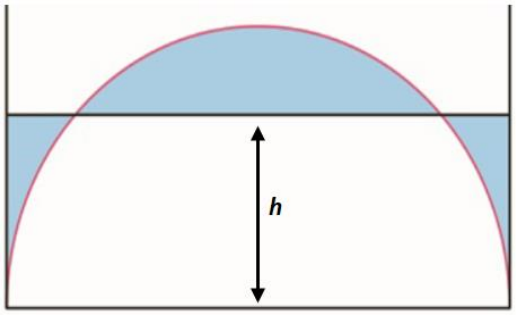
A. 0 B. $\ln(2)$ C. $\ln(3)$ D. $\ln(4)$ E. NOTA

17) The region bounded by the graphs $f(x) = \frac{1}{2}x^3$ and $g(x) = \sqrt{8x}$ in Quadrant I of the Cartesian plane is split into two regions, *S* and *K*, of unequal area by the graph of $h(x) = 2x$. What is the positive difference between the areas of *S* and *K*?

A. $\frac{2}{3}$ B. 1 C. $\frac{6}{5}$ D. $\frac{4}{3}$ E. NOTA

18) Consider semicircle *S* of radius 1 accompanied by two vertical lines tangent to *S* at the diameter, as shown. At height *h* above the diameter of *S* there is a horizontal line such that the combined area of the shaded regions shown is minimized. What is the value of *h*? *This figure is not necessarily drawn to scale.*

Note: the horizontal line is such that it intersects the semicircle exactly twice. It is not a tangent line at the top of *S* nor is it the diameter of *S*.



A. $\frac{3}{5}$ B. $\frac{\sqrt{2}}{2}$ C. $\frac{\sqrt{3}}{3}$ D. $\frac{\sqrt{3}}{2}$ E. NOTA

19) Maria is standing on her back porch $(x=0)$ and is flying a kite with a very long string. The wind is blowing Maria's kite due west. The wind is strong, and the height of the kite as it is flying is represented by $h(x)=\frac{x^3}{6}+\frac{1}{2x}$ where x is the straight horizontal distance, in kilometers, from Maria. When the kite is located at $x=\frac{1}{2}$, a gust of wind comes and blows the kite to $x=1$, at which point Maria was finally able to lock the kite into place. What is the total distance, in kilometers, the kite has flown thanks to this unexpected gust of wind?

A. $\frac{1}{2}$ B. $\frac{5}{8}$ C. $\frac{17}{48}$ D. $\frac{31}{48}$ E. NOTA

20) A target for a darts game is made by drawing a circle inscribed inside a square with side length 1. The area inside the square is considered the target area. To the nearest tenth, what is the probability that a randomly-thrown dart that hits the target area lands outside of the square?

A. 0.1 B. 0.2 C. 0.3 D. 0.4 E. NOTA

21) Find the orthogonal trajectory for the family of curves given by $y=kx^2$, where k is an arbitrary and real constant. In the answer choices, let C represent the usual constant of integration.

A. $y=Ce^{Cx}$ B. $x^4-y^3=C$ C. $x^2+2y^2=C$ D. $y^2=-Cx$ E. NOTA

22) What is the area inside one petal of the rose with polar equation $r=6\cos(2\theta)$?

A. $\frac{9\pi}{4}$ B. $\frac{9\pi}{2}$ C. $\frac{27\pi}{4}$ D. 9π E. NOTA

23) Fahan is pouring apple juice into his glass that is in the shape of a right truncated cone at a constant rate of 2 cubic inches per second. The lower base is 4 inches in diameter and the upper (open) base has a diameter of 6 inches. The glass has a total height of 4 inches. At what rate, in inches per second, is the height of liquid in the glass increasing when the glass is 25% full?

Note: This truncated cone can also be considered a frustum formed from a right circular cone, whose smaller base is the bottom of the glass.

A. $\frac{8}{25\pi}$ B. $\frac{32}{81\pi}$ C. $\frac{2}{9\pi}$ D. $\frac{16}{65\pi}$ E. NOTA

24) In reference to the previous question, which of the following is true? Assume appropriate units.

A. The rate at which the juice level is rising in the glass is greater when the glass is 25% full in comparison to when it is 50% full.
B. If the glass was not truncated, and was shaped like a right circular cone, it would have a height of 10 inches.
C. As the juice is being poured into the glass, the rate at which the radius of the surface of the liquid is increasing is half the rate at which its height is rising.
D. Because the juice is being poured at a constant rate, the height of the juice is rising at a constant rate.
E. NOTA

25) Divya and Kyle are practicing for the upcoming Mu Alpha Theta competition. Divya was tasked with the question:

$$\int_{-2}^{2} \sqrt{4-x^2}\,dx$$

She tells Kyle that she successfully performed the substitution $x = 2\sin(u)$ to help and she obtained the correct answer. What did she select? Hint: was her method the most efficient?

A. π B. $\frac{\pi}{2} - 2$ C. 2π D. 4π E. NOTA

26) What is the area bounded by the graph of $r = 2\cos(\theta) + \sin(2\theta)$ on the interval $\theta = 0$ to $\theta = \frac{\pi}{2}$?

A. $\frac{3}{2}$ B. $\frac{5\pi}{2} + \frac{3}{2}$ C. $\frac{7\pi}{2} - \frac{5}{2}$ D. $\frac{5\pi}{8} + \frac{4}{3}$ E. NOTA

27) What is the ratio of the volume of an inscribed sphere to the volume of a cube that circumscribes it?

A. $\frac{3}{5}$ B. $\frac{\pi}{4}$ C. $\frac{3\pi}{20}$ D. $\frac{\pi}{6}$ E. NOTA

28) An ellipse with equation $\frac{x^2}{36} + \frac{y^2}{9} = 1$ is revolved about its major axis, forming an ellipsoid. What is the volume of the ellipsoid?

A. 9π B. 18π C. 72π D. 144π E. NOTA

29) Let $m(x) = -4x + 12$. What is the largest possible area of a rectangle that is bound by the *x*-axis, *y*-axis, and $m(x)$ in quadrant I, given that two sides of the rectangle are along the axes?

A. 9 B. 10 C. 18 D. 20 E. NOTA

30) Let $p(x) = (x-3)^2$ and let $L(x)$ be the tangent line to $p(x)$ at point (a,b), where $0 < a < 3$. Point (a,b) is such that it maximizes the area of a triangle bounded by the *x*-axis, the *y*-axis, and $L(x)$ in the first quadrant. What is the area of the maximized triangle, given that two sides of the triangle are along the axes?

A. $\frac{15}{2}$ B. 8 C. 10 D. $\frac{23}{2}$ E. NOTA