2022 MU ALPHA THETA NATIONAL CONVENTION
MU BOWL - CONDENSED VERSION

Question 0

The function $f(x) = 20x^{2021} + 2x + 1$ has 2021 roots $r_1, \dots, r_{2021}$. Compute the value of $\sum_{i=1}^{2021} r_i$.

Question 1

Define the following functions over real values of x:

- $f_1(x) = \frac{x^5+1}{x^4}$
- $f_2(x) = x^2 \, ln^2 \, x$
- $f_3(x) = arcsin\, x \;\, arccos^2\, x + arcsin^2\, x \; arccos\, x$
- $f_4(x) = \frac{1}{x}\int_0^x (t \cos t) dt$

Evaluate the following at the given points:

A. $f_1{}'(2)$
B. $f_2{}'(e)$
C. $f_3{}'(\frac{\sqrt{3}}{2})$
D. $f_4{}'(\frac{\pi}{2})$

Compute $\frac{6ABCD}{e}$.

Question 2

Compute the values of the following integrals:

A. $\int_0^1 ln^2\, x\, dx$
B. $\int_0^{\frac{\pi}{4}} cos\, 2x\, (sin\, x + cos\, x)^4\, dx$
C. $\int_0^2 x^2(2-x)^4\, dx$
D. $\int_{-1}^{1} \frac{x^2}{e^x+1} dx$

Compute $ABCD$.

Question 3

Compute the values of the following limits (if the limit does not exist, let the answer to that part be 12):

A. $\lim\limits_{x \to 0} \frac{1 - cos x}{x sin x}$
B. $\lim\limits_{x \to \infty} \sqrt{x^2 + 4x} - \sqrt{x^2 - 4x}$
C. $\lim\limits_{n \to \infty} \binom{n}{2} \left(\frac{1}{n}\right)^2 \left(1 - \frac{1}{n}\right)^{n-2}$
D. Let $f(x)$be equal to 0 if $x \leq 0$ and equal to x otherwise. Compute the value of $\lim\limits_{x \to 0^+} \frac{f(arctan(x))}{x}$

Compute $ABCD$.

Question 4

Let $f(x) = x^4 + 4x - 1$ for all $x \neq -1$and $f(x) = \pi$ if $x = -1$. If any of the following do not exist or are infinite, let the answer to that part be 12.

A. $\lim\limits_{x \to -1} f(x)$
B. $\int_{-\pi}^{\pi} f(x) \sin x \, dx$
C. Compute the global minimum value of $f(x)$.
D. For how many positive real values of x does $f(x) = 0$?

Compute $ABCD$.

Question 5

Let R be the region bounded by the curves $f(x) = x^2 + 4$ and $f(x) = 8$.

A. Compute the area of R.
B. Compute the volume when R is revolved around the x axis.
C. Compute the volume when R is revolved around the y axis.
D. The y coordinate of the centroid of R.

Compute $\frac{BD}{AC}$.

Question 6

Let $f(x) = x^3 + 4x + 2$.

A. The value of the only real root of $f(x)$ is contained in $(n, n+1)$ for an integer n. What is the value of n?
B. The Mean Value Theorem for Derivatives guarantees a value c when applied to f on the interval $(0,1)$. What is c?
C. The normal line to $f(x)$ at $x = 0$ makes an angle θ with respect to the positive y axis. What is $sin\theta$?
D. The tangent line to $f(x)$ at $x = 0$ intersects $f(x)$ at how many places?

Compute $A^2B^2C^2D^2$.

Question 7

Let a curve C be defined by the parametric equations:

- $x(t) = 4t$
- $y(t) = \frac{1}{1+t^2}$

A. Compute $\frac{d^2y}{dx^2}$ at time $t = 1$.
B. Compute the area of the region below C and above the x axis.

Compute AB.

Question 8

Let $r = 3 + 3\ cos\ \theta$ and $r = 3 + 3\ sin\ \theta$ be curves in the polar plane.

A bug begins at the polar coordinate $(6,0)$ on the curve $r = 3 + 3\ cos\ \theta$ and begins moving in the following fashion:

- It moves along a curve until it reaches an intersection point between the two curves. Whenever this happens, it adjusts itself and begins moving along the curve it intersected with. Note that if there are 12 intersection points from $[0, 2\pi)$, the bug will change direction 12 times when moving according to this.
- The bug always will move counterclockwise (always on the "outside" curve)
- It stops moving when it returns to its starting point.

Given these conditions, let the total distance the bug travels be d.

Compute the value of $d \cdot \sqrt{2 - \sqrt{2}}$

Question 9

If $\{x\}$is the fractional part and $[x]$ is the integer part of the real number x:

A. Let $f(n) = \sum_{i=0}^{n} \frac{1}{i+1}$ and $g(n) = \int_0^n \frac{\{x\}}{1+[x]}\, dx$. Compute the value of $\lim\limits_{n\to\infty} \frac{f(n)}{g(n)}$.

B. $\int_0^1 (-1)^{\left[\frac{1}{x}\right]}\, dx$

Compute e^{2A-B}.

Question 10

Compute the global maximum for each of the following functions (if you believe the function has no global maximum value, use 12 as your answer for that part)

Let A be the x value for which $f(x) = \frac{2x}{x^2+1}$ reaches a global maximum value.

Let (B, C)be the input $(x, y)$for which$g(x, y) = 4xy - 2x^2 - 5y^2 + 8x + 4y + 6$ reaches a global minimum value.

Let D be the x value for which $h(x) = x^{\frac{1}{x}}$reaches a global maximum value.

Compute $ABCD$.

Question 11

How many of the following limits are finite and not equal to 0?

- $\lim\limits_{n\to\infty} (1+\frac{1}{n})^n$
- $\lim\limits_{n\to\infty} (1+\frac{1}{n})^{n^2}$
- $\lim\limits_{x\to 0} \frac{1+x^4-\cos x}{x^2}$
- $\lim\limits_{x\to 0} \frac{\arctan x-\tan x}{\sin x - x}$
- $\lim\limits_{x\to 2} \frac{x^2-5x-6}{x^2-4}$
- $\lim\limits_{x\to 3} \frac{x^2-8x+15}{x^2-6x+9}$

Question 12

Compute the following series sums:

A. $\sum_{n=0}^{\infty} \frac{2^n}{n!}$

B. $\sum_{n=0}^{\infty} \frac{n^2}{n!}$

C. $\sum_{n=0}^{\infty} [arctan\, n - arctan\, (n+1)]$

D. $\sum_{n=0}^{2021} \frac{e^x}{e^{2021-x} + e^x}$

Compute $ABCD$.

Question 13

If any part is unable to be evaluated, the value for that part is 0.

A. If $f(x) = (x-1)^6(2x+1)^4$, find $f'(-1)$.
B. $\frac{d}{dx}\left(ln(3-x)\right)$ at $x = 7$.
C. If $f(x) = x^x e^{cos(x)}$, the value of $\frac{df(\pi)}{dx}$.

Compute $A + B + C$

Question 14

Let $f(x) = 1 + x + x^2 + x^3 + \ldots + x^{2020}$

Compute the value of

$$log_2\left(\frac{f(2) + f'(2)}{2021}\right)$$