

2022 MAΘ NATIONAL CONVENTION
MU CIPHERING
CONDENSED VERSION

0) Find the equation of the tangent line to the graph $y = \sin x + x$ at the point $(0, 0)$.

1) Find the length of the arc from $x = 1$ to $x = 3$ if $y = \frac{x^3}{12} + \frac{1}{x}$

2) Find the area inside $r = 3\sin \theta$ and outside $r = 2 - \sin \theta$

3) Evaluate $\int_{\sqrt{3}}^2 \frac{\sqrt{x^2 - 3}}{x} dx$

4) A snowman has a sphere of diameter 6. He wishes to inscribe a cylinder with the largest possible volume. What would be the height of the cylinder?

5) Iron-Wiggie must cross a river 4 miles wide to have dinner with Captain Lu 4 miles down stream using any combination of swimming across the river and flying down the shore. If Wiggie can fly 10 mph and swim 6mph, how long is the total route in miles in order to get to dinner in the least amount of time?

6) The tangent line to a point on the curve $y = 6 - x^2$ in the first quadrant intersects the axes at two points. The goal is to minimize the area enclosed by the triangle whose vertices are these two points and the origin, If the tangent point is (L, U) , what does $LU = ?$

7) $\lim_{x \rightarrow \infty} \sum_{i=1}^n \frac{i}{in + n^2} = L + \ln U$. What does $L + U = ?$

8) Given: $\frac{dy}{dx} = x^2 y - 2 - 2x^2 + y$ and $y(0) = 5$, what does $y(3) = ?$

9) $\int_0^{\frac{\pi}{3}} \frac{\sin^3 x}{\cos^4 x} dx = ?$

10) Evaluate: $\int_0^6 \frac{x}{1 + \frac{x}{1 + \dots}} dx$

11) A right circular cone is shrinking. When the radius is 3m and the height is 4m, the volume is decreasing at $4 \text{ m}^3 / \text{min}$. At that time, what is the rate at which the surface area of the cone is decreasing, in m^2 / min .

12) If $g(x) = \left(-3x + \frac{2}{x}\right)^5$, what is the value of the constant term of $g'(x)$?