ANSWERS

1) B
2) D
3) A
4) D
5) C
6) B
7) E (0)
8) D
9) B
10) A
11) A
12) C
13) B
14) A
15) D
16) B
17) D
18) C
19) A
20) C
21) B
22) B
23) D
24) D
25) C
26) A
27) C
28) A
29) A
30) B
SOLUTIONS

(1)

(a) \( y = 44x - 24 \)
(b) \( y = 440x - 420 \)
(c) \( y = 440x - 440 \)
(d) \( y = 440x - 460 \)
(e) NOTA

SOLUTION

\[
\frac{d}{dx}(20x^{22}) \bigg|_{x=1} = 440(1)^{21} = 440 \rightarrow y - 20 = 440(x - 1) \rightarrow y = 440x - 420. \]

(2)

(a) 10
(b) \( \frac{21}{2} \)
(c) 11
(d) \( \frac{23}{2} \)
(e) NOTA

SOLUTION

\[ x^{23} = Ax^2 - A + 1 \rightarrow x^{23} - 1 = A(x^2 - 1) \rightarrow x = 1. \] Further, \( 23x^{22} = 2Ax \) at \( x = 1 \rightarrow A = \frac{23}{2}. \]

(3)

(a) \( 2a^3 - 2a^7 + C \)
(b) \( 2a^7 - 2a^3 + C \)
(c) \( a^3 - a^7 + C \)
(d) \( a^7 - a^3 + C \)
(e) NOTA

SOLUTION

\[
dM = -\frac{2}{a^3} da \rightarrow \int a^5(7a^4 - 3)\left(-\frac{2}{a^3}\right) da = \int -14a^6 + 6a^2 \, da = 2a^3 - 2a^7 + C. \]

(4)

(a) \( \frac{\pi}{2} \)
(b) \( \pi \)
(c) \( \frac{3\pi}{2} \)
(d) \( 2\pi \)
(e) NOTA

SOLUTION


When $x \geq 2$, $y = \sqrt{1 - (x-3)^2}$. When $0 \leq x < 2$, $y = \sqrt{1 - (x-1)^2}$. When $-2 \leq x < 0$, $y = \sqrt{1 - (x+1)^2}$. When $x < -2$, $y = \sqrt{1 - (x+3)^2}$. These are four semicircles of radius one, so the total area is $2\pi$. [D]

(5)

(a) $-360$  
(b) $360$

(c) $-\frac{1}{360}$  
(d) $\frac{1}{360}$

(e) NOTA

SOLUTION

$$\frac{d}{dx}(20e^{2x})\bigg|_{x=\ln(3)} = 40e^{2\ln(3)} = 40e^9 = 40(9) = 360.$$ So the normal slope is $-\frac{1}{360}$. [C]

(6)

(a) $\frac{3 \times 10^{13}}{(10^8 - 9^8)^2}$  
(b) $\frac{3 \times 10^{12}}{(10^8 - 9^8)^2}$

(c) $\frac{4 \times 10^{13}}{(10^8 - 9^8)^2}$  
(d) $\frac{4 \times 10^{12}}{(10^8 - 9^8)^2}$

(e) NOTA

SOLUTION

$$\left(\frac{9}{10}\right)^3 \left(\frac{1}{10}\right) + \left(\frac{9}{10}\right)^{11} \left(\frac{1}{10}\right) + \left(\frac{9}{10}\right)^{19} \left(\frac{1}{10}\right) + \cdots = \left(\frac{9^{-5}}{10^{-4}}\right) \sum_{n=1}^{\infty} n \left(\frac{9}{10}\right)^{8n} = \left(\frac{10^4}{9^5}\right) \frac{\frac{9^8}{10^8}}{(1-\frac{9^8}{10^8})} = \left(\frac{10^{16}}{9^{10}}\right) \frac{\frac{9^8}{10^8}}{(1-\frac{9^8}{10^8})}.$$ [B]

(7)

(a) $-2022$  
(b) $-1011$

(c) $1011$  
(d) $2022$

(e) NOTA

SOLUTION

$$\int_0^\infty \frac{\ln(x)}{2022x^2 + x + 2022} \, dx.$$ Let $x = \frac{1}{u}$, then $dx = -\frac{du}{u^2}$.

$$\int_0^\infty \frac{\ln\left(\frac{1}{u}\right)}{2022\left(\frac{1}{u}\right)^2 + \frac{1}{u} + 2022} \left(-\frac{du}{u^2}\right) = -\int_0^\infty \frac{\ln(u)}{2022 + u + 2022u^2} \, du.$$ If the integral is the negative of itself, it must be zero. [E]
(8)  
(a) \[ \int_{0}^{2\pi} \sqrt{\theta^2 + 901} \, d\theta \]  
(b) \[ \int_{0}^{2\pi} \sqrt{\theta + 30} \, d\theta \]  
(c) \[ \int_{0}^{2\pi} \sqrt{60\theta + 2} \, d\theta \]  
(d) \[ \int_{0}^{2\pi} \sqrt{\theta^2 + 60\theta + 901} \, d\theta \]  
(e) NOTA

**SOLUTION**

\[ r = 30 \text{ when } \theta = 0 \text{ and } r \to r + 2\pi \text{ as } \theta \to \theta + 2\pi. \] Therefore \( r(\theta) = \theta + 30. \) Therefore the arc length is \[ \int_{0}^{2\pi} \sqrt{r^2 + r'^2} \, d\theta = \int_{0}^{2\pi} \sqrt{\theta^2 + 60\theta + 901} \, d\theta. \] [D]

(9)  
(a) 125  
(b) 250  
(c) 500  
(d) 1000  
(e) NOTA

**SOLUTION**

Let \((x_0, 0)\) be the point the Flash is at. Then \(D^2 = (x - x_0)^2 + (\sqrt{x})^2 = (x - x_0)^2 + x \to 2DD' = 0 = 2(x - x_0) + 1 \to x = x_0 - \frac{1}{2} \to D = \sqrt{\frac{1}{4} + x_0 - \frac{1}{2}} = \sqrt{x_0 - \frac{1}{4}} \text{ will be the distance from the curve to the Flash's location. Therefore } \frac{dD}{dt} = \frac{1}{2} \sqrt{x_0 - \frac{1}{4}} = \frac{1000}{2\sqrt{4}} = 250. \] [B]

(10)  
(a) \[ \frac{3\ln(20)}{\ln(6) - \ln(5)} \]  
(b) \[ \frac{1}{3} \ln \left( \frac{5}{2} \right) \]  
(c) \[ \frac{3\ln(20)}{\ln(5) - \ln(2)} \]  
(d) \[ \frac{1}{3} \ln \left( \frac{6}{5} \right) \]  
(e) NOTA

**SOLUTION**

\[ \frac{dT}{dt} = k(T - T_0) \to \ln(T - T_0) = kt + C \to T(t) = Ce^{kt} + T_0. \] The ambient room temperature \( T_0 = 40 \) so \( T(t) = Ce^{kt} + 40. \) Cap starts at -20 so \( T(0) = -20 = C + 40 \to C = -60. \) Finally after three hours Cap has warmed 10 to -10. That means \( -10 = 40 - 60e^{3k} \to k = \frac{1}{3} \ln \left( \frac{5}{6} \right). \) Therefore Cap will reach 37 when \( 37 = 40 - 60e^{kt} \to kt = \ln \left( \frac{1}{20} \right) \to t = \frac{\ln \left( \frac{1}{20} \right)}{\frac{1}{3} \ln \left( \frac{5}{6} \right)} = \frac{3\ln(20)}{\ln(6) - \ln(5)}. \] [A]
(11)

(a) \((e^{2022} - 1)\ln(2022)\)  
(b) 2022

(c) \(\ln(2022)\)  
(d) \(e^{2022} - 1\)  
(e) NOTA

SOLUTION

Let \(f(x) = \left(1 + \frac{2022}{x}\right)^x\). Therefore \(f(2022x) = \left(1 + \frac{1}{x}\right)^{2022x}\). Now, in general \(\frac{d}{da db} f(ab) = \frac{d}{db} f(ab) = \frac{1}{x} f(2022x) - f(x)\) \(\frac{dx}{x} f(xy)dydx = \int_0^\infty \int_0^{2022} \frac{1}{y} dydx f(xy)dydx = \int_0^\infty \int_0^{2022} \frac{1}{y} dydx f(xy)dydx = (e^{2022} - 1)\ln(2022)\). \(\boxed{A}\)

(12)

(a) \(-\frac{3\sqrt{5}}{5}\)  
(b) \(-\frac{3\sqrt{25}}{25}\)

(c) \(-\frac{3\sqrt{5}}{25}\)  
(d) \(-\frac{3\sqrt{25}}{5}\)  
(e) NOTA

SOLUTION

By similar triangles we know that \(\frac{h}{r} = \frac{200}{50} = 4\) \(\Rightarrow h = 4r\) \(\Rightarrow \frac{dh}{dt} = 4 \frac{dr}{dt}\) \& \(V = \frac{1}{3} \pi r^2 h = \frac{4}{3} \pi r^3\).

Therefore \(\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}\). The initial volume is \(V = \frac{1}{3} \pi (50^2)(200) = \frac{500000\pi}{3}\) so half that will be \(\frac{250000\pi}{3} = \frac{4}{3} \pi r^3 \Rightarrow r = 5\sqrt{3} \Rightarrow \frac{dV}{dt} = -\pi = 20\pi \sqrt{25} \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = -\frac{1}{20\sqrt{25}} \Rightarrow \frac{dh}{dt} = -\frac{1}{3\sqrt{25}} = -\frac{3\sqrt{5}}{25}\). \(\boxed{C}\)

(13)

(a) \(\frac{\pi}{6}\)  
(b) \(\frac{\pi}{4}\)

(c) \(\frac{\pi}{3}\)  
(d) \(\frac{\pi}{2}\)  
(e) NOTA

SOLUTION

\(\lim_{n \to \infty} \sum_{k=1}^n \frac{\pi}{k^2 + n^2} = \lim_{n \to \infty} \sum_{k=1}^n \frac{1}{n} \frac{1}{(\frac{k}{n})^2 + 1} = \int_0^1 \frac{dx}{x^2 + 1} = [\arctan (x)]_0^1 = \frac{\pi}{4}\). \(\boxed{B}\)
(14)

(a) $\frac{1}{e^2}$  
(b) $\frac{1}{e}$  
(c) $e$  
(d) $e^2$  
(e) NOTA

**SOLUTION**

The area of $R$ is given by $\int_0^{\infty} \frac{dx}{(x+1)^2} = \left[-\frac{1}{x+1}\right]_0^{\infty} = 1$. Therefore we need $\frac{1}{2} = \int_0^{\infty} b^{-x} dx = \left[-\frac{1}{\ln(b)} b^{-x}\right]_0^{\infty} = -\frac{1}{\ln(b)} \ln(b) = -2 \rightarrow b = \frac{1}{e^2}$. [A]

(15)

(a) $\frac{\pi}{2}$  
(b) $\frac{\pi}{6}$  
(c) $\frac{\pi^2}{2}$  
(d) $\frac{\pi^2}{6}$  
(e) NOTA

**SOLUTION**

$$\int_0^{\infty} \frac{x}{e^{x-1}} dx = \int_0^{\infty} \frac{xe^{-x}}{1-e^{-x}} dx = \sum_{n=1}^{\infty} x e^{-nx} dx = \sum_{n=1}^{\infty} \int_0^{\infty} xe^{-nx} dx = \sum_{n=1}^{\infty} \left[-\frac{1}{n} xe^{-nx} - \frac{1}{n^2} e^{-nx}\right]_0^{\infty} = \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} [D]$$

(16)

(a) $\frac{1}{69}$  
(b) $\frac{1}{61}$  
(c) 69  
(d) 61  
(e) NOTA

**SOLUTION**

$$\frac{d}{dx} \left[ f^{-1}(x) \right]_{x=22} = \frac{1}{f'(f^{-1}(22))}$$

Clearly $f(1) = 22$ so $\frac{1}{f'(f^{-1}(22))} = \frac{1}{f'(1)} = \frac{1}{60(1^2)-9(1^2)+10(1)} = \frac{1}{61}$ [B]

(17)

(a) $3x^2 \sin(x^{66}) - 2x \sin(x^{44})$  
(b) $\sin(x^{44}) - \sin(x^{66})$  
(c) $\sin(x^{66}) - \sin(x^{44})$  
(d) $2x \sin(x^{44}) - 3x^2 \sin(x^{66})$  
(e) NOTA
SOLUTION

\[ \frac{d}{dt} \int_{x^3}^{x^2} \sin(t^{22}) \, dt = 2x \sin(x^{44}) - 3x^2 \sin(x^{66}). \]

(18)

(a) 20  
(b) 22  
(c) 24  
(d) 26  
(e) NOTA

SOLUTION

\[ D'(1) + M'(2) = L'(R(1))R'(1) + L'(R(2)) + L(2)R'(2) = L'(3) * 4 + (-1)(2) + (1)(6) = (5)(4) + 4 = 24. \]

(19)

(a) 6  
(b) 7  
(c) 8  
(d) 9  
(e) NOTA

SOLUTION

\[ \int_{6}^{12} f(x) \, dx = 6 \rightarrow \int_{0}^{12} f(x) \, dx = -6 \text{ since } f(x) \text{ is odd. Now } \int_{6}^{22} f(x) \, dx = 22 = \int_{-6}^{6} f(x) \, dx + \int_{6}^{22} f(x) \, dx = \int_{6}^{22} f(x) \, dx. \text{ Finally } \int_{12}^{6} f(x) \, dx = 10 = \int_{12}^{6} f(x) \, dx + \int_{6}^{6} f(x) \, dx = \int_{-12}^{6} f(x) \, dx. \]

So \( \int_{6}^{12} f(x) \, dx = -10. \text{ Thus } \int_{0}^{22} f(x) \, dx = \int_{6}^{22} f(x) \, dx + \int_{0}^{12} f(x) \, dx - \int_{12}^{6} f(x) \, dx = 22 - 6 - 10 = 6. \]

(20)

(a) 4140  
(b) 2025  
(c) 2205  
(d) 270  
(e) NOTA

SOLUTION

Let \( x \) be the height of the mouse from the ground. Then, to move a small height \( dx \) will result in an amount of work \( dW = 4dx + 2x \, dx \rightarrow W = \int_{0}^{45} (4 + 2x) \, dx = [4x + x^2]_{45}^{15} = 180 + 2025 = 2205. \)
(21)

(a) \(\log_2(11)\)  
(b) \(\log_{22}(2)\)  
(c) \(\log_2(22)\)  
(d) \(\log_{11}(2)\)  
(e) NOTA  

**SOLUTION**

\[ A = 22e^{kt} \rightarrow 1 = 22e^k \rightarrow k = -\ln(22) \rightarrow \frac{1}{2} = e^{\frac{-\ln(22)t_1}{2}} \rightarrow -\ln(2) = -\ln(22) \rightarrow t_\frac{1}{2} = \frac{\ln(2)}{\ln(22)} = \log_{22}(2). \quad [B] \]

(22)

(a) 0  
(b) \(e^{23}\)  
(c) \(e^{46}\)  
(d) 23  
(e) NOTA  

**SOLUTION**

\[ \lim_{x \to 23} \frac{e^x - e^{23}}{x - 23} = \frac{d}{dx} [e^x]_{x=23} = e^{23}. \quad [B] \]

(23)

(a) \(\frac{1}{2022!}\)  
(b) \(\frac{1}{2018!}\)  
(c) \(\frac{1}{1011!}\)  
(d) \(\frac{1}{1009!}\)  
(e) NOTA  

**SOLUTION**

\[ x^4 e^{x^2} = x^4 \sum_{n=0}^{\infty} \frac{(x^2)^n}{n!} = \sum_{n=0}^{\infty} \frac{x^{2n+4}}{n!}. \quad \text{The } 2022^{\text{nd}} \text{ derivative at } x=0 \text{ will be the coefficient of } x^{2022} \rightarrow 2022 = 2n + 4 \rightarrow n = 1009 \rightarrow \text{The answer is } \frac{1}{1009!}. \quad [D] \]

(24)

(a) \(\frac{33}{49}\)  
(b) \(\frac{33}{94}\)  
(c) \(-\frac{33}{49}\)  
(d) \(-\frac{33}{94}\)  
(e) NOTA  

**SOLUTION**
\( y^4x^3 + x^2 - 14y = 22 \rightarrow 4y^3x^3y' + 3y^4x^2 + 2x - 14y' = 0 \) → \( y' = \frac{-3y^4x^2 - 2x}{4y^3x^3 - 14} \) → \( y'(3,1) = \)

(a) 1 \hspace{1cm} (b) \( x^{1011} \)

(c) \( \sqrt{2}x^{1011} \) \hspace{1cm} (d) \( \frac{1}{2}x^{2022} \) \hspace{1cm} (e) NOTA

\textbf{SOLUTION}

\[
\left( \frac{d}{dx} \left[ x^{2022} \right] \right) \left( \frac{d}{du} \left[ x^{2022} \right] \right) = \frac{d}{du} \left[ x^{2022} \right] = 2022x^{2021} \frac{dx}{du} = u \rightarrow 2022x^{2021} = u \frac{du}{dx} \\
\]

\( x^{2022} = \frac{1}{2}u^2 \rightarrow u = \sqrt{2}x^{1011}. \)

\[ \text{[C]} \]

(26)

(a) \( 2 + \ln \left( \frac{3}{2} \right) \) \hspace{1cm} (b) \( 6 - \ln(2) \)

(c) \( 2 + \ln \left( \frac{2}{3} \right) \) \hspace{1cm} (d) \( 4 + \ln(3) \) \hspace{1cm} (e) NOTA

\textbf{SOLUTION}

\[
\int_{3/2}^{8/22} \frac{\sqrt{1 + 22x}}{x} \, dx \rightarrow u = \sqrt{1 + 22x} \rightarrow x = \frac{u^2 - 1}{22} \rightarrow dx = \frac{u}{11} \, du \rightarrow \int_{3/2}^{8/22} \frac{\sqrt{1 + 22x}}{x} \, dx = \int_2^{\frac{2u^2}{u^2 - 1}} du = \int_2^3 2 \left( \frac{2}{u^2 - 1} \right) \, du = \int_2^3 2 + \frac{2}{u^2 - 1} \, du = \left[ 2u + \ln|u - 1| - \ln|u + 1| \right]^3_2 = 6 + \ln(2) - \ln(4) - 4 - \ln(1) + \ln(3) = 2 + \ln \left( \frac{3}{2} \right). \]

\[ \text{[A]} \]

(27)

(a) \( \frac{9}{196\pi} \) \hspace{1cm} (b) \( \frac{9}{49\pi} \)

(c) \( \frac{3}{196\pi} \) \hspace{1cm} (d) \( \frac{3}{49\pi} \) \hspace{1cm} (e) NOTA

\textbf{SOLUTION}

\[
V = \frac{4}{3} \pi r^3 \rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \rightarrow \frac{3}{4\pi(49)} = \frac{dr}{dt} = \frac{3}{196\pi}. \]

\[ \text{[C]} \]
(28)

(a) $\frac{160}{3}$  
(b) $\frac{80}{3}$  
(c) 120  
(d) 60  
(e) NOTA

**SOLUTION**

$$\int_{0}^{2} 20\sqrt{x} \, dx = 20\sqrt{2} \left[ \frac{2}{3} x^{\frac{3}{2}} \right]_{0}^{2} = 20\sqrt{2} \left( \frac{2}{3} \right) (2\sqrt{2}) = \frac{160}{3}. \quad [A]$$

(29)

(a) 4.70  
(b) 4.69  
(c) 4.65  
(d) 4.60  
(e) NOTA

**SOLUTION**

$$\sqrt{22} \approx \sqrt{25} - 3 \left( \frac{1}{2\sqrt{25}} \right) = 5 - \frac{3}{10} = 4.70. \quad [A]$$

(30)

(a) 0  
(b) 1  
(c) 2  
(d) 3  
(e) NOTA

**SOLUTION**

<table>
<thead>
<tr>
<th>n</th>
<th>Divergent since the terms do not limit to zero</th>
<th>Conditionally Convergent via alternating series test</th>
<th>Absolutely convergent (Taylor series for $e^x$)</th>
<th>Absolutely convergent via the ratio test</th>
<th>Absolutely convergent via the root test</th>
<th>Diverges via Raabe’s test</th>
<th>Convergent via the p-test</th>
<th>Diverges since there is a term dividing by zero</th>
<th>Conditionally convergent via alternating series, and the integral test diverges when negatives are removed</th>
<th>Absolutely convergent via the p-test</th>
<th>NA</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-3</td>
<td>$=1. \quad [B]$</td>
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