Extra Instruction: The last 10 questions on this test each have a single answer but two steps/parts to get to it. These questions are meant to encourage collaboration with your partner, but can obviously be done however you chose. They are scored no differently that regular questions. E) NOTA means "None Of These Answers".

1) Find $\frac{d}{dx}\left[\lim_{n\to\infty}\left(1-\frac{3}{n}\right)^{9n}\right]$.

A) e^3 **B)** $3e^{3x}$ **C)** e^{-27} **D)** $-27e^{-27x}$ **E)** $NOTA$

2) Let region R be a bounded second-quadrant region area equal to a constant a. When R is rotated fully about the x axis, it creates a solid of volume 4. When R is rotated fully about the y axis, it creates a solid of volume 8. Let the centroid of R be the point (p, q). Whats is $\frac{p}{q}$?

A) -2 **B)** $-\frac{1}{2}$ **C)** $\frac{1}{2}$ **D)** 2 **E)** $NOTA$

3) Two concentric circles have radii 5 and 8. Andy the Ant lies on the circle of radius 5 and Buffy the Baboon lies on the circle of radius 8 such that Andy and Buffy are initially 3 units apart. Andy starts running counterclockwise along his circle and Buffy starts running clockwise around his circle. Andy and Buffy run at different but constant rates and both complete exactly one lap on their respective circles in 1 minute. In units per minute, how fast is the distance between Andy and Buffy increasing the first time they are 7 units apart?

A) $\frac{40\pi\sqrt{3}}{7}$ **B)** $\frac{80\pi\sqrt{3}}{7}$ **C)** $\frac{120\pi\sqrt{3}}{7}$ **D)** $\frac{160\pi\sqrt{3}}{7}$ **E)** $NOTA$

4) How many of the following functions are continuous at $x = 0$?

I. $f(x) = \frac{\sin(x)}{x}$ for all $x \neq 0$ and $f(0) = 1$

II. $g(x) = \frac{|x|}{x}$ for all $x \neq 0$ and $g(0) = 1$

III. $h(x) = \{x\}$, where $\{x\}$ denotes the unique number in $[0, 1)$ such that $x - \{x\}$ is an integer

A) 0 **B) 1** **C) 2** **D) 3** **E)** $NOTA$

5) Let $f(x) = x^3 + x$ and $g(x)$ be the inverse of $f(x)$. Let $h(x) = f(g(x))$. Let $j(x) = \frac{h(x)}{g(x)}$. Find the slope of the tangent line to $j(x)$ at $x = 10$.

A) -32 **B)** -6 **C)** $\frac{4}{13}$ **D)** $\frac{36}{13}$ **E)** $NOTA$

6) Eric is trying to find the area between two polar curves, c_1 and c_2 on the interval $(\frac{\pi}{4}, \frac{5\pi}{4})$. The curves the following property: when they are intersected by a line going through the pole, the square of the distance d_1 from the point of intersection on c_1 to the pole minus the square of the distance d_2 from the point of intersection on c_2 to the pole is always equal to π. Find the area.

A) $\boldsymbol{\pi^2}$ **B)** $\boldsymbol{\pi}$ **C)** $\boldsymbol{\frac{\pi\sqrt{2}}{2}}$ **D)** $\boldsymbol{\frac{\pi^2}{2}}$ **E)** $\boldsymbol{NOTA}$

7) Find the amount of area enclosed by the curves $y = 2x^3 - 4x^2 - x + 5$ and $y = x^3 + 2x^2 - 6x - 7$.

A) $\mathbf{30.5}$ **B)** $\mathbf{31.25}$ **C)** $\mathbf{31.5}$ **D)** $\mathbf{32.75}$ **E)** $\boldsymbol{NOTA}$

The following information is provided as background for Questions 8-10:

Helena is making rubber band art. She hammers nails into a board and then connects various nail pairs with rubber bands, which do not stretch after they are placed. Both the nails and the bands are infinitely thin (they are effectively dots and line segments).

Helena begins with a circular board of radius R inches. She places n nails equally spaced around the circumference of her board and labels them $Nail$ 1 through $Nail\ n$ going clockwise. Then, she connects every generic nail $Nail\ k$ where $1 \le k \le n$ to $Nail\ ((k + f) mod\ n)$, where f is some expression or constant of her choosing. For example, if Helena uses $n = 5$ and $f = 2$, she can form a pentagon in the center of her board.

8) Helena wants to create a smooth circle of radius r inches (where $0 \le r \le R$) in the center of her board. How many nails will she need (what is n), and what is the relationship between any one rubber band and the circle?

A) $\boldsymbol{n = \infty}$ **and a rubber band is a line segment that is tangent to the circle at its midpoint**

B) $\boldsymbol{n = 2\pi r}$ **and a rubber band is a line segment that is tangent to the circle at its midpoint**

C) $\boldsymbol{n = \infty}$ **and a rubber band is the derivative of the circle at its midpoint**

D) $\boldsymbol{n = 2\pi r}$ **and a rubber band is the derivative of the circle at its midpoint**

E) $\boldsymbol{NOTA}$

9) Helena uses the correct n from Question 8. Which of the following gives a possible expression could she use as f?

A) $\boldsymbol{\frac{n\cos^{-1}\left(\frac{r}{R}\right)}{2\pi}}$ **B)** $\boldsymbol{\frac{n^2\cos^{-1}\left(\frac{r^2}{R^2}\right)}{\pi}}$ **C)** $\boldsymbol{\frac{n\cos^{-1}\left(\frac{r^2}{R^2}\right)}{2\pi}}$ **D)** $\boldsymbol{\frac{n\cos^{-1}\left(\frac{r}{R}\right)}{\pi}}$ **E)** $\boldsymbol{NOTA}$

10) Alex decides to try rubber band art, too. He starts with a square board of side length s inches. He randomly selects a point on the left edge and places a nail there. Next, he rotates that point 90° counterclockwise about the geometric center of the board and places a second nail at this transformed point. Then, he puts a rubber band between these two nails. He picks a new random spot on the left edge to start with and repeats the process indefinitely. A curve begins to appear (some might describe it as the boundary between the region where his rubber bands lie and the empty region). This curve is tangent to all of the rubber bands, continuous on $[0, s]$, and differentiable on $(0, s)$. Call this curve $f(x)$ defined for $0 \le x, f(x) \le s$. What is $f(x)\left(1 - \frac{1}{f'(x)}\right) + x(1 - f'(x))$ evaluated at $x = \frac{s}{3}$?

A) $\boldsymbol{\frac{s}{9}}$ **B)** $\boldsymbol{\frac{s}{3}}$ **C)** $\mathbf{s}$ **D)** $\boldsymbol{3s}$ **E)** $\boldsymbol{NOTA}$

11) Find the slope of the tangent line to $r = 2\cos(2\theta)$ at $\vartheta = \frac{\pi}{6}$.

A) $-2\sqrt{3}$ **B)** $-\frac{\sqrt{3}}{5}$ **C)** $\frac{\sqrt{3}}{5}$ **D)** $\frac{\sqrt{3}}{7}$ **E)** $NOTA$

12) Bailey has a right circular cone of volume 1 $unit^3$ with height to radius ratio of $\sqrt{2}$ to 1. He circumscribes a sphere about it, a a cone about the sphere, an inverted cone about the uninverted cone, a cylinder about the cone, and a sphere about the cylinder. Each time, he uses a figure of the smallest volume possible. The volume of his final sphere can be expressed as $a^b c^d unit^3$, where a and c are primes and $a < c$. Find $ad + bc$.

A) 3 **B) 5** **C) 7** **D) 9** **E)** $NOTA$

13) Kira is taking a 60 $minute$ long essay test where she spends her time either reading a passage or writing an essay about it. If she doesn't read the passage at all and writes nothing, she earns a score of 0 $point$. Her accumulated score increases at a rate of 1 $point/minute$ for time that she spends writing. Her final score is mutlitpled at a rate equivalent to being doubling for every 5 $minutes$ she spends reading the question. In order to maximize her score how much time, in $minutes$, should she spend writing?

A) 0 **B)** $\frac{5}{\ln(2)}$ **C)** $60\ln(2)$ **D)** 60 **E)** $NOTA$

14) Let $\coth(x) = \frac{e^{2x}+1}{e^{2x}-1}$. Compute $\lim_{x\to 0} \frac{x\coth(x) - 1}{x^2}$.

A) $\frac{1}{6}$ **B)** $\frac{1}{3}$ **C)** $\frac{1}{2}$ **D) 1** **E)** $NOTA$

15) Jeffrey is finding the volume of the solid formed when a region is rotated fully about a the x axis using Washer Method: $V = \pi \int_a^b [f(x)^2 - g(x)^2]dx$. Which of the following correctly describes the concepts behind each part of this formula?

A) $f(x)$ **represents the distance from the origin to the outer edge of the washer**

B) $g(x)$ **represents the larger radius of the washer**

C) π **is part of an expression to represent circumference of each washer**

D) dx **represents the represents the thickness of the washer**

E) $NOTA$

16) Jack is finding the volume of the solid formed when a region is rotated fully about the y axis using Shell Method: $V = 2\pi \int_a^b xf(x)dx$. Which of the following correctly describes the concepts behind some parts of this formula?

A) x **represents the average radius of the cylindrical shells**

B) $f(x)$ **represents the lateral surface area of each cylindrical shell**

C) 2π **is part of an expression to represent the circumferece of each spherical shell**

D) b **and** a **represent the radii of the narrowest and widest cylindrical shells, respectively**

E) $NOTA$

17) How many of the following limits are both nonzero and finite?

I. $\lim_{x\to\infty} \frac{\arctan(x)}{\ln(x)}$

II. $\lim_{x\to\infty} \frac{1}{e^{-x}+1}$

III. $\lim_{x\to\infty} x(2^{\frac{1}{x}}-1)$

IV. $\lim_{x\to\infty} \frac{\sqrt{x}}{\ln(\ln(x))}$

A) 0 **B) 1** **C) 2** **D) 3** **E) NOTA**

18) The famous Weierstrass Substitution to help with integrals in terms of x is $t=\tan\left(\frac{x}{2}\right)$. Find the sum of $\sin(x)$, $\cos(x)$, and $\frac{dx}{dt}$ in terms of t.

A) $\frac{-t^2+t+3}{1-t^2}$ **B)** $\frac{-t^2+t+3}{1+t^2}$ **C)** $\frac{-t^2+2t+1}{1+t^2}$ **D)** $\frac{-t^2+2t+3}{1+t^2}$ **E)** ***NOTA***

19) To raise a note on a violin by one octave, you must place your finger halfway between where it was before and the bridge of the violin. Alice's finger starts 10 inches away from the bridge. Therefore, $f(d)=\log_2\left(\frac{10}{d}\right)$ is an equation returning how many octaves (fractional octaves are okay) higher Alice's note will be that her original note if she places her finger a distance of d inches away from the bridge. If Alice moves her finger at a speed of $2\ inches/second$ towards the bridge, what is the instantaneous increase in the number of octaves per second by which she has raised her note after 3 *seconds*?

A) $\ln(2)$ **B)** $-\ln(2)$ **C)** $-\frac{1}{\ln(2)}$ **D)** $\frac{1}{\ln(2)}$ **E) NOTA**

20) The indefinite integral $\int \frac{1}{x+x^a}dx$, for x in a positive domain and where a is a real constant that is not equal to 1, can be written as $\frac{1}{m}\ln(x^n+p)+C$, where m, n, and p are constants which may be in terms of a. Find $m-2n-3p$.

A) a

B) $a-1$

C) $a-4$

D) $a-6$

E) ***NOTA***

Section of Two-Step Questions (see "Extra Instruction" at beginning of test):

21) In space, Anna the astronaut is tethered by a 3-foot-long rope to a plate in the shape of a filled, regular, n-sided polygon (where $n \geq 3$) of side length 10 $feet$. Assuming Anna's volume is negligible, let $V(n)$ equal the volume of the 3D region in which she can roam in terms of n.

Let $f(n) = V(n)$, but on the domain of $(-\infty, 0) \cup (0, \infty)$. $f'(6)$ can be expressed as $a\pi + b\sqrt{c}$, where a, b and c are positive integers and c is prime. Find $a + b + c + d$.

A) 298 **B) 305** **C) 364** **D) 419** **E) *NOTA***

22) Estimate $f(2)$ using Euler's Method, the fact that for all x, $f'(x) = 2\big(x + f(x)\big) - 3$, the starting point (1,4), and two steps of equal length.

Speaking of steps, Amy is practicing clog dancing. Since she gets tired as she dances, the rate a which she clogs in $steps/second$ can be represented as $r(x) = \frac{3t+4}{t+1}$, where t is the number of seconds since she began. How many steps has she completed when t equals the estimation of $f(2)$ from above? Assume Amy can complete fractional amounts of steps.

A) $12 + \ln(5)$ **B) $20 + 2\ln(3)$** **C) $45 + 4\ln(2)$** **D) $60 + \ln(2)$** **E) *NOTA***

23) Let m equal the number of items from the following list that can prove divergence. Let n be the number of items from the following list that can prove convergence. List: Nth Term Test, Integral Test, P Series Test, Direct Comparison Test, Limit Comparison Test, Alternating Series Theorem, Ratio test.

Use an m^{th} degree MacLaurin polynomial to estimate e^{n-3}. Let the estimation be your answer.

A) $\frac{5557}{280}$ **B) $\frac{1553}{80}$** **C) $\frac{331}{45}$** **D) $\frac{155}{21}$** **E) *NOTA***

24) Eddie is playing a 3-minute-long song on the piano. Let his volume level be $f(t)$, a continuous, differentiable except at $t = 0$ and $t = 3$, first-quadrant function of t, the time since he began the song. Since he wants the music to "flow," the absolute value of his instantaneous rate of change in volume cannot exceed $4t$ at any time. If $f(0) = 1$ and $f(3) = 14$, let the interval $[a, b]$ be the possible volumes of his playing at $x = 2$.

Find $\int_a^b \frac{1}{2x^{\frac{3}{2}} - 4x + 4x^{\frac{1}{2}}} dx$.

A) 0 **B) $\arctan(9)$** **C) $\arctan(2) - \frac{\pi}{4}$.** **D) $\arctan(6) + \frac{\pi}{3}$** **E) *NOTA***

25) William's hottub is in the shape of the solid with a base in the shape of a circle of diameter 8 ft. All of its cross sections taken perpendicular to the base and parallel to eachother are half regular hexagons with two vertices lying on the circumference of the circle. The volume it can hold can be expressed as $a\sqrt{b}$, where a and b are positive integers and b is not divisible by the square of any prime. Let $M = 4a + 11b - 1$.

William starts filling the tub at time $t = 0$ with a faucet that runs at a rate of $(t^3 + 2t - 4\,)ft^3/min$, where a is a constant. At time $t = 2$, he accidentally opens the drain and leaves it opened, which drains the water at $1\ ft^3/min$. He continues with the faucet running and the drain open. At time $t = x$ he has filled a volume of $(\frac{M}{4} - 10)ft^3$. Let your answer be x.

A) 3 **B) 4** **C) 6** **D) 10** **E) *NOTA***

26) Andrew is trying to read an unnecessarily-long 845 word apps question, but his glasses are dirty. As a result, he can only read at a rate of 3 $words/sec$. Every time he wipes his glasses, he uses 5 $seconds$ of time but increases his reading rate by 2 $words/sec$. Let x equal the whole number of times he should wipe his glasses in order to read most quickly.

Andrew learns his lesson and switches to contacts for his next topic test. A contact is in the shape of the shell of a cap of height x of a sphere of radius 10 mm. What maximum volume, in mm^3, of contact solution can a contact hold?

A) $\frac{625\pi}{3}$ **B)** $\frac{625\pi}{9}$ **C)** $\frac{25\pi}{3}$ **D)** $\frac{2000\pi}{3}$ **E) *NOTA***

27) Zach makes jello in the shape of the figure formed when the region bounded by $y = -\frac{x}{2} + 2$, the x-axis, and $x = 2$ is rotated 360° about the y-axis.

Then, he melts the jello (its volume is preserved) and begins to pour it into a cup shaped like an inverted cone of height 10 and radius $10\sqrt{2}$ at a constant rate of $16\pi\ units^3/sec$. At the instant when Zach is pouring the last of the jello in, at what rate, in $units/sec$, is the height of the jello increasing?

A) 2 **B) 4** **C) 6** **D) 8** **E) *NOTA***

28) Kasra evaluates $\int sec(x)dx$. Milaan evaluates $\int sec^3(x)dx$. Find the sum of their results.

A) $\frac{\sec(x)\tan(x)+ln|\sec(x)+\tan(x)|}{2} + C$ **B)** $\frac{\sec(x)\tan(x)+3ln|\sec(x)+\tan(x)|}{2} + C$

C) $\frac{\sec(x)\tan(x)+ln|\sec(x)+\tan(x)|}{4} + C$ **D)** $\frac{3\sec(x)\tan(x)+ln|\sec(x)+\tan(x)|}{4} + C$

E) *NOTA*

29) Let $L = \lim_{n\to\infty} \sum_{i=0}^{n} \left(\sqrt[n]{\frac{e^i}{n^n}} + \frac{1}{n} \right)$.

Let $M = \lim_{h\to 0} \frac{L^{2+h} - L^2}{h}$. Let your answer be M.

A) $4\ln(2)$ **B) $9\ln(3)$** **C) $25\ln(5)$** **D) 0** **E) *NOTA***

30) Let a be the number of letters in the correct word to fill in the blank ("MacLaurin," "Taylor," or the word "both" if they could both fill in the blank): A _____ series can be centered at $x = 3$.

Let b be the number of letters in the correct word to fill in the blank ("absolutely," "conditionally," or "not"): The series $\sum_{n=1}^{\infty} a_n$ is _____ convergent when $\sum_{n=1}^{\infty} |a_n|$ converges.

Let your answer be the number of positive integer factors of $2ab$.

A)8 **B) 12** **C) 16** **D) 18** **E) *NOTA***