For all questions below, the answer E) NOTA means "None of these answers". Good luck and have fun!

1. Bessie's eyes are 5.5 feet off the ground. She is 20 feet away from a tree and looks up to the top of a tree with an angle of elevation of 38°. You can assume the tree grows perpendicular to the ground. Which expression could be used to find the height of the tree, in feet?

A: $20\tan(38°) + 5.5$ B: $20\sin(38°) + 5.5$ C: $20\tan(38°) - 5.5$

D: $25.5\tan(38°)$ E: NOTA

2. Two Wi-Fi companies, *Cable R Us* and *Speed Warriors,* have competing monthly pricing plans. *Cable R Us* charge a flat fee of $20 per month plus $10 for every 100 gigabytes used. *Speed Warriors* charges $50 every month plus $2 for every 50 gigabytes used. How many gigabytes could be used in a month for both companies to charge the same total monthly prices?

A: 100 B: 250 C: 500 D: 750 E: NOTA

3. Jezebel takes a circular sector of central angle θ and radius R. She folds it to make a right cone without a base. Which expression represents the height of the cone?

A: $\frac{R}{4\pi}\sqrt{4\pi^2 - \theta^2}$ B: $\frac{R}{2\pi}\sqrt{4\pi^2 - \theta^2}$ C: $\frac{R}{4\pi}\sqrt{2\pi - \theta}$

D: $\frac{R}{2\pi}\sqrt{4\pi^2 + \theta^2}$ E: NOTA

4. A dog travels along the curve given by $x = 1 + 4\cos(2t)$ and $y = 4\sin(2t)$ for $0 \le t \le \frac{3\pi}{4}$. Which statement is true about the dog's path?

A: The dog travels a distance of 3π in a clockwise direction.

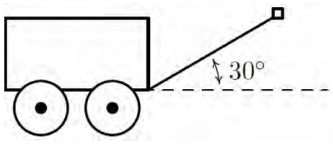
B: The dog travels a distance of 6π in a counterclockwise direction.

C: The dog travels a distance of 3π in a counterclockwise direction.

D: The dog travels a distance of 6π in a clockwise direction.

E: NOTA

5. Wendy exerts a force of 20 pounds to pull her wagon to the right. She pulls the wagon with the handle making an angle of $30°$ with the horizontal as shown. If Wendy exerted 600 foot-pounds of force, how far did the wagon go, in feet?



A: $10\sqrt{3}$ B: $20\sqrt{3}$ C: 10 D: 20 E: NOTA

6. In a 3-D coordinate system, a spider starts at $(-1, -2, 1)$ and moves in the direction of $(2, 1, 1)$ each second. A fly starts at $(13, 14, 11)$ and moves in the direction of $(-1, -2, -1)$ each second. Which statement is true?

A: The spider and fly will meet at the point $(7, 2, 5)$ after 4 seconds.

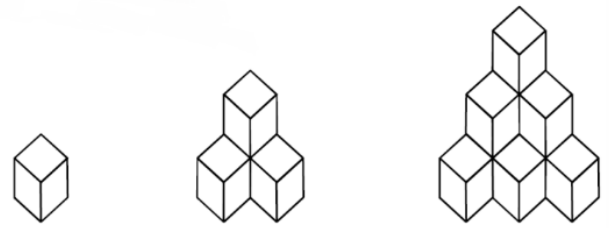
B: The spider and fly will meet at the point $(7, 2, 5)$ after 6 seconds.

C: The spider and fly will meet at the point $(9, 6, 7)$ after 6 seconds.

D: The spider and fly will meet at the point $(9, 6, 7)$ after 4 seconds.

E: NOTA

7. Nason has a lot of unit cubes in his toy chest. He makes the first three iterations of a stack of unit cubes as shown. He wonders how many unit cubes would be needed for the n^{th} stack, where $n \geq 1$. Which statement $P(n)$ would be correct that he could prove by induction?



A: $P(n)$: $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$

B: $P(n)$: $1 + 2 + 3 + \cdots + n = \frac{n^2(n+1)^2}{4}$

C: $P(n)$: $1 + 3 + 6 + \cdots + \frac{n(n+1)}{2} = \frac{n(n+1)(n+2)}{6}$

D: $P(n)$: $1 + 3 + 6 + \cdots + \frac{n(n+1)}{2} = \frac{n(n+1)(n+2)}{12}$

E: NOTA

8. Alfonso makes a triangle in the complex plane whose vertices are $A = 4\text{cis}\left(\frac{\pi}{6}\right), B = 4\text{cis}(\pi)$, $C = 4\text{cis}(\frac{7\pi}{2})$. What is the area of ΔABC?

A: $4\sqrt{3} + 12$ B: $4\sqrt{3} + 8$ C: $6\sqrt{3} + 12$ D: $6\sqrt{3} + 8$ E: NOTA

9. Sally absolutely loves circles! Which Polar equation will <u>not</u> graph a circle?

A: $r = 4\cos\theta$ B: $r = 2\sin\theta - 2\cos\theta$ C: $r = 4$

D: $r = 4\cos\theta + 4$ E: NOTA

10. The probability that a randomly-chosen student plays soccer is $\frac{2}{5}$ and the probability that a randomly-chosen student plays cello $\frac{3}{4}$. The probability that a randomly-chosen student plays soccer or cello is $\frac{9}{10}$. Given that a student plays soccer, what is the probability they play cello?

A: $\frac{1}{2}$ B: $\frac{5}{8}$ C: $\frac{1}{3}$ D: $\frac{4}{9}$ E: NOTA

11. On an analog clock, the hour hand and minute hands are 4 inches and $4 + k$ inches long. What is the least positive integer k so that the distance between the tips of these hands at 4 pm exceeds 10 inches?

A: 1 B: 2 C: 3 D: 4 E: NOTA

12. The function $P(t) = \frac{1000}{1+4e^{-2t}}$ represents the population of a town after t years. Let m be the initial population of the town and let $n = \lim_{t\to\infty} P(t)$. Find $n - m$.

A: 1000 B: 800 C: 750 D: 600 E: NOTA

13. Jayden has an 8" by 10" rectangular photo that he wants to inscribe in an elliptical picture frame. If x and y are measured in inches, what is the equation of the ellipse centered at the origin with smallest area that can completely inscribe the rectangular photo?

A: $4x^2 + 5y^2 = 20$ B: $25x^2 + 16y^2 = 400$ C: $25x^2 + 16y^2 = 800$

D: $16x^2 + 25y^2 = 1600$ E: NOTA

14. Kallyn and Alina are 5 units apart. Kallyn is at $(t - 2, 2t + 1)$ and Alina is at $(-t - 1, t + 4)$. What is the sum of all possible values of t?

A: 3 B: -1 C: 4 D: 2 E: NOTA

15. A passcode consists of 3 letters (A to Z) followed by 3 digits (0 to 9). What is the probability that the passcode will have no repeated characters?

A: $\frac{94}{169}$ B: $\frac{118}{169}$ C: $\frac{18}{169}$ D: $\frac{108}{169}$ E: NOTA

16. Nicole drops a ball that always rebounds a certain fraction r of the previous height. The initial height from which the ball is dropped is 20 feet. If the *total distance* the ball travels (both up and down) is 140 feet, what is r?

A: $\frac{1}{4}$ B: $\frac{1}{2}$ C: $\frac{2}{3}$ D: $\frac{4}{5}$ E: NOTA

17. Tom has probability p of making a free throw in basketball. The probability that he makes exactly two free throws out of 4 shots is $\frac{3}{8}$. What is the probability that he makes at least one shot in 4 shots?

A: $\frac{1}{8}$ B: $\frac{1}{16}$ C: $\frac{15}{16}$ D: $\frac{5}{8}$ E: NOTA

18. Julius took two math contests. Julius scored a 60 points on the first test with an average of 50 points and a standard deviation of 5 points. On the second test, Julius scored a 60 points with an average of 40 points and a standard deviation of 15 points. Let z_1 and z_2 be his z-scores on tests 1 and 2, respectively. Both tests were out of 100 points.

A: Julius did relatively better on the first test since $z_1 = 2$, $z_2 = \frac{4}{3}$, and $z_1 > z_2$.

B: Julius did relatively better on the second test since $z_1 = 10$, $z_2 = 20$, and $z_1 < z_2$.

A: Julius did relatively better on the second test since $z_1 = 5$, $z_2 = 15$, and $z_1 < z_2$.

A: Julius did relatively better on the first test since $z_1 = 1$, $z_2 = \frac{1}{2}$, and $z_1 > z_2$.

E: NOTA

19. Cory and Craig decide to go look for UFOs. Cory looks up to the UFO at an angle of elevation of a. Craig is d miles down the road from Cory and spots the UFO at angle of elevation of b. What is the height of the UFO off the ground, in terms of α, β , and d?

A: $\frac{d\tan(a)\tan(b)}{\tan(a)+\tan(b)}$ B: $\frac{d\sin(a)\sin(b)}{\sin(a)+\sin(b)}$ C: $\frac{\sin(a)+\sin(b)}{\sin(a)\sin(b)d}$

D: $\frac{\tan(a)+\tan(b)}{\tan(a)\tan(b)d}$ E: NOTA

20. The selling price p, in dollars, of selling x units of bubble tea is given by $p = -\frac{1}{2}x + 12$ where $0 \leq x \leq 24$. The cost C, in collars, of producing x units of bubble tea is given by $C = \frac{\sqrt{x}}{2} + 5$. Find the cost if the selling price of bubble tea \$10.

A: 5.50 B: 6.00 C: 6.50 D: 7.00 E: NOTA

21. Ron and Nicole both ride their bicycles a distance of 4 km. Nicole starts 1 minute before Ron and Ron travels 1 km/hour faster than Nicole. If they start at the same time, what is Nicole's speed, in km/hour?

A: 8 B: 12 C: 15 D: 16 E: NOTA

22. Shannon wants to make an obtuse triangle. To do so, she selects three integers (without replacement) from the set {1, 2, 3, 4, 5, 6} for the side lengths. Given that Shannon makes a valid triangle, what is the probability it is obtuse triangle?

A: $\frac{5}{7}$ B: $\frac{6}{7}$ C: $\frac{2}{3}$ D: $\frac{5}{6}$ E: NOTA

23. Mohammed used the equation $T(m) = \frac{1}{2}m^3 - \frac{15}{2}m^2 + 27m - 20$ to model the average temperature (in degrees Celsius) at month m, where $m = 1$ represents January. In which month is the average temperature <u>not</u> 0 degrees Celsius?

A: January B: February C: April D: October E: NOTA

24. Ellie the engineer designs a solar panel where the power generated is given by $P(\theta) = 500\sin(2\theta) + 100$ where $0 \leq \theta \leq \frac{\pi}{2}$ in radians represents the angle of the solar panel relative to the horizontal. Let θ^* determine the angle that provides the maximum power P^*. Find the value of $\theta^* \cdot P^*$.

A: 100π B: 250π C: 300π D: 500π E: NOTA

25. Pete the pirate wants to bury his gold treasure. He starts at point A and walks 9 yards then turns and walks 10 more yards, stopping at point B. He then walks some integer amount of yards to point C. He completes his trip by walking back to point A. Being a good mathematical pirate, he realizes that the area contained by ΔABC is numerically equal to the perimeter of ΔABC, both of which are integers. What is the area of ΔABC, in square yards?

A: 24 B: 30 C: 36 D: 72 E: NOTA

26. A drone company delivers two types of packages – lightweight and heavyweight – to cities A and B. Each lightweight package takes 1 minute to load and 3 minutes to deliver. Each heavyweight package takes 2 minutes to load and 5 minutes to deliver. In one day, city A received 20 lightweight and 15 heavyweight packages, while city B received 12 lightweight and 15 heavyweight packages. Let $M = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$ and $P = \begin{bmatrix} 20 & 12 \\ 15 & 18 \end{bmatrix}$. Compute $T = M \cdot P$. Which statement is true?

A: City A required 11 more minutes of total time than City B.

B: City A required 174 total minutes.

C: City B required 11 more minutes of total time than City A.

D: City B required 164 total minutes.

E: NOTA

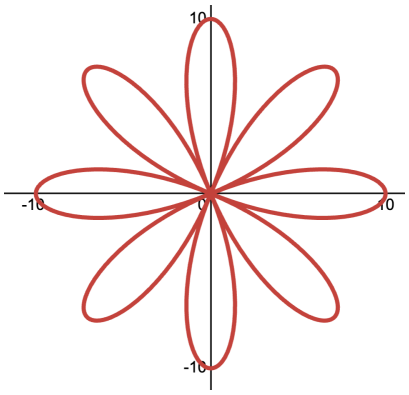
27. An auditorium has 12 seats in the first row and each row thereafter has 3 more seats than the prior row. The last row has 75 seats. How many total seats are in the auditorium?

A: 638 B: 913 C: 957 D: 1914 E: NOTA

28. Landen uses the model $f(t) = 50 - 40\cos\left(\frac{\pi}{12}(t-1)\right) +$ to represent his energy level through the day, where t is measured in hours after midnight. Let M be his maximum energy level and N be his minimum energy level. What is $M - N$?

A: 100 B: 90 C: 80 D: 10 E: NOTA

29. An designer wants to create the design shown with 8 equally-spaced petals, each of length 10. Which polar function could the designer use to create this design?



A: $r = 10\sin(4\theta)$ B: $r = 10\cos(8\theta)$ C: $r = 10\cos(4\theta)$

D: $r = 8\sin(10\theta)$ E: NOTA

30. The function $f(m) = 120\log_2\big((m-1)\big) + 5$ represents the rate of change of the number of lily pads per month in a pond, where m is measured in months. What is the average rate of change of $f(m)$ over the interval $2 \leq m \leq 9$?

A: $\frac{240}{7}$ lilypads per month

B: $\frac{360}{7}$ lily pads per month per month

C: $\frac{240}{7}$ lily pads per month per month

D: $\frac{360}{7}$ lilypads per month

E: NOTA