1. A: Let the height of the tree be h. The remaining height of the tree after subtracting Bessie's eye height is $h - 5.5$. Using right triangle trig, $\tan(38°) = \frac{h-5.5}{20}$. So $h = 20\tan(38°) + 5.5$.

2. C: Let x be measured in 100 gigabyte increments. Then *Cable R Us* charges $20 + 10x$ and *Speed Warriors* charges $50 + 4x$. Solve $20 + 10x = 50 + 4x$ to get $x = 5$. Thus, 500 gigabytes used make the two plans equal in monthly price.

3. B: The arc length $R\theta$ of the sector equals the circumference $2\pi r$ of the base of the cone, where r is the radius of the base of the cone. Thus, $R\theta = 2\pi r$, so $r = \frac{R\theta}{2\pi}$. Let h be the height of the cone. By the Pythagorean Theorem, $r^2 + h^2 = R^2$ or $\left(\frac{R\theta}{2\pi}\right)^2 + h^2 = R^2$. Multiply by $4\pi^2$ to get $R^2\theta^2 + 4\pi^2h^2 = 4\pi^2R^2$. Thus, $h = \frac{R}{2\pi}\sqrt{4\pi^2 - \theta^2}$.

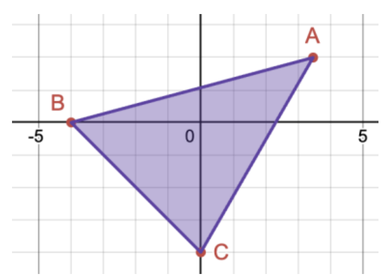
4. B: The period of the dog's circular path is only $\frac{2\pi}{2} = \pi$, so due to the range, the dog covers $\frac{3}{4}$ of a circle of equation $(x-1)^2 + y^2 = 16$. Since the circle has radius 4, the full circumference is $2\pi(4) = 8\pi$. But since it only covers $\frac{3}{4}$ of the circle, the distance traveled is $\frac{3}{4}(8\pi) = 6\pi$. Finally, the dog travels counterclockwise since x decreases as y increases for initial values of t.

5. B: The force vector can be written as $\boldsymbol{F} =< 20\cos 30°, 20\sin 30° > = < 10\sqrt{3}, 10 >$. Let the displacement vector be $\boldsymbol{d} =< x, 0 >$ since it moves to the right. The work done is given by the dot product $\boldsymbol{F} \cdot \boldsymbol{d} = 10\sqrt{3}x = 600$ so $x = \frac{600}{10\sqrt{3}} = 20\sqrt{3}$ feet.

6. E: The position of the spider can be written parametrically as $s(t) =< -1, -2, 1 > +t < 2, 1, 1 >$ or $s(t) =< -1 + 2t, -2 + t, 1 + t >$. The fly's position is $f(t) =< 13, 14, 11 > +t < -1, -2, -1 >$ or $f(t) =< 13 - t, 14 - 2t, 11 - t >$. Each coordinate component must be equal for the same *t.* Set the x-components equal: $-1 + 2t = 13 - t$ to find $t = \frac{14}{3}$. However, $-2 + \frac{14}{3} \neq 14 - 2\left(\frac{14}{3}\right)$ so the spider and fly are not at the same location at the same time.

7. C: From the picture, the first three stacks have 1, $1 + 3 = 4$, and $1 + 3 + 6 = 10$ unit cubes. In general, the n^{th} stack adds a layer of $\frac{n(n+1)}{2}$ cubes to the previous stack. Thus, the total number of unit cubes in the n^{th} stack is $1 + 3 + 6 + \cdots + \frac{n(n+1)}{2}$. Note that for $n = 1, n = 2$, and $n = 3$, the formula $\frac{n(n+1)(n+2)}{6}$ correctly produces outputs of 1, 4, and 10. Thus, the statement to prove for all $n \geq 1$ is $P(n)\colon 1 + 3 + 6 + \cdots + \frac{n(n+1)}{2} = \frac{n(n+1)(n+2)}{6}$.

8. A: Plot in the complex plane and convert to rectangular using $x = r\cos\theta$ and $y = r\sin\theta$. Thus, $A = (2\sqrt{3}, 2)$, $B = (-4, 0)$ and $C = (0, -4)$, as shown. The segment AC intersects the x-axis at D. Note $y = \sqrt{3}x - 4$ represents the equation of this side, which has an x-intercept of $\left(\frac{4\sqrt{3}}{3}, 0\right)$. Now break into two triangles: $[BCD] = \frac{1}{2}\left(\frac{4\sqrt{3}}{3} + 4\right)(4) = \frac{8\sqrt{3}}{3} + 8$ and $[ABD] = \frac{1}{2}\left(\frac{4\sqrt{3}}{3} + 4\right)(2) = \frac{4\sqrt{3}}{3} + 4$. Add to find the total area $[ABC] = 4\sqrt{3} + 12$.



9. D: The equation $r = 4\cos\theta + 4$ graphs a cardioid, not a circle.

10. B: Given $P(S) = \frac{2}{5}$, $P(C) = \frac{3}{4}$, and $P(C \cup S) = \frac{9}{10}$. Then $P(C \cap S) = \frac{2}{5} + \frac{3}{4} - \frac{9}{10} = \frac{1}{4}$. Thus, $P(C|S) = \frac{P(C\cap S)}{P(S)} = \frac{\frac{1}{4}}{\frac{2}{5}} = \frac{5}{8}$.

11. D: At 4 pm, the angle between the hands is $120°$. By Law of Cosines, setup the following: $10^2 = 4^2 + (4+k)^2 - 2\cdot 4(4+k)\cos(120°)$, where $\cos(120°) = -\frac{1}{2}$. Expand and simplify to: $k^2 + 12k - 52 = 0$. By the Quadratic Formula, $k = \frac{-12+\sqrt{12^2+4\cdot 52}}{2} = -6 + 2\sqrt{22}$. Since $\sqrt{22} < 5$, then this value is less than 4. To exceed a distance of 10 inches, the least $k = 4$.

12. B: $m = P(0) = \frac{1000}{1+4e^{-2(0)}} = \frac{1000}{1+4} = 200$ and $n = \lim_{t\to\infty} P(t) = \frac{1000}{1+0} = 1000$ since $e^{-2t} \to 0$ as $t \to \infty$. Then $n - m = 1000 - 200 = 800$.

13. C: To find the smallest ellipse, we must consider the dimensions of the rectangular in relation to the ellipse's axes**.** The vertices of the rectangle must lie on the ellipse, at $(\pm 4, \pm 5)$, and also be points on the ellipse. Considering, (4,5), we have: $\frac{4^2}{a^2} + \frac{5^2}{b^2} = 1$, and need to minimize ab where neither is 0. To make a and b as small as possible, we make $\frac{16}{X} + \frac{25}{Y} = 1$, where X and Y are close as possible to each other. This results in $X = 32$ and $Y = 50$. So, the smallest possible ellipse will be $\frac{x^2}{32} + \frac{y^2}{50} = 1$ which yields $16x^2 + 25y^2 = 800$.

14. D: By the Distance Formula, $\left((t-2) - (-t-1)\right)^2 + \left((2t+1) - (t+4)\right)^2 = 5^2$. Simplify to $(2t-1)^2 + (t-3)^2 = 25$ or $5t^2 - 10t - 15 = 0$. Thus, $5(t-3)(t+1) = 0$ so $t = 3$ or $t = -1$. The sum of possible values of t is $3 - 1 = 2$.

15. D: The probability is $\frac{26\cdot 25\cdot 24\cdot 10\cdot 9\cdot 8}{26^3\cdot 10^3} = \frac{108}{169}$ when reduced.

16. E: The total distance the ball drops is $20 + 20r + 20r^2 + \cdots = \frac{20}{1-r}$. The total distance the ball rebounds upward is $20r + 20r^2 + \cdots = \frac{20r}{1-r}$. Thus, $\frac{20}{1-r} + \frac{20r}{1-r} = \frac{20+20r}{1-r} = 140$. Solve to find $r = \frac{3}{4}$.

17. C: This follows a binomial distribution. The probability of making exactly two free throws is: $\binom{4}{2} p^2(1-p)^2 = \frac{3}{8}$. This simplifies to $16p^2(1 - 2p + p^2) = 1$ or $16p^4 - 32p^3 + 16p^2 - 1 = 0$. By the Rational Root Theorem, the possible rational roots are $p = \pm 1, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm\frac{1}{8}, \pm\frac{1}{16}$. Synthetically divide to find $p = \frac{1}{2}$ is a root: $(2p-1)(8p^3 - 12p^2 + 2p + 1)$. Synthetically divide again to find $p = \frac{1}{2}$ is again a root: $(2p-1)(2p-1)(4p^2 - 4p - 1)$. The roots to the final quadratic do not make sense since they are outside the interval $[0, 1]$. Thus, $p = \frac{1}{2}$. The probability that Tom makes at least one shot is $1 - \binom{4}{0} p^0(1-p)^4 = 1 - \left(\frac{1}{2}\right)^4 = 1 - \frac{1}{16} = \frac{15}{16}$.

18. A: The z −score is defined as $z = \frac{x-\mu}{\sigma}$, where μ is the mean and σ is the standard deviation. Then $z_1 = \frac{60-50}{5} = 2$ and $z_2 = \frac{60-40}{15} = \frac{4}{3}$. Since $z_1 > z_2$, Julius did relatively better on the first test.

19. A: Let the height of the UFO off the ground be h, the distance along the ground from Cory to the base of the UFO be x, and the distance along the ground from Craig to the base of the UFO be y. Then $\tan(a) = \frac{h}{x}$ so $x = h\cot(a)$ and $\tan(b) = \frac{h}{y}$ so $y = h\cot(b)$. Since $d = x + y$, then $d = h(\cot(a) + \cot(b))$ and $h = \frac{d}{\cot(a)+\cot(b)}$. Multiply numerator and denominator by $\tan(a)\tan(b)$ to get $h = \frac{d\tan(a)\tan(b)}{\tan(a)+\tan(b)}$.

20. B: Solve $10 = -\frac{1}{2}x + 12$ to find $x = 4$. Then $C = \frac{\sqrt{4}}{2} + 5 = 6$.

21. C: Let Ron and Nicole's speeds in km/hour be denoted by R and N where $R = N + 1$. It takes Nicole $\frac{4}{N}$ hours and Ron $\frac{4}{R} = \frac{4}{N+1}$ hours. We are given $\frac{4}{N} - \frac{4}{N+1} = \frac{1}{60}$. Multiply by $60N(N+1)$ to get $240(N+1) - 240N = N(N+1)$ or $N^2 + N - 240 = 0$. Thus, $(N+16)(N-15) = 0$ and $N = 15$.

22. A: To be a triangle, the Triangle Inequality states that the sum of two side lengths must be greater than the third. Use this to find the 7 possible triples of sides are $(2,3,4), (2,4,5), (2,5,6), (3,4,5), (3,4,6), (3,5,6), (4,5,6)$. To see if a triangle is obtuse, $c^2 > a^2 + b^2$ for sides $a \le b \le c$. Notice two of the pairs do not form an obtuse triangle since $5^2 = 3^2 + 4^2$ and $6^2 < 4^2 + 5^2$. The probability is $\frac{5}{7}$.

23. B: By inspection, note $T(1) = 0$ so the average temperature is 0 in January. Factor to write $T(m) = (m-1)(\frac{1}{2}m^2 - 7m + 20)$. This factors further as $T(m) = \frac{1}{2}(m-1)(m-4)(m-10)$. The average temperature is also 0 in April ($m = 4$) and October ($m = 10$), but it is not in February ($m = 2$).

24. E: The maximum value of $y = \sin x$ is 1 when $x = \frac{\pi}{2}$. Thus, $P^* = 500(1) + 100 = 600$. Solve $2\theta^* = \frac{\pi}{2}$ to find $\theta^* = \frac{\pi}{4}$. Thus, $\theta^* \cdot P^* = 600\left(\frac{\pi}{4}\right) = 150\pi$.

25. C: Let the third side length be x. Then the semiperimeter $s = \frac{9+10+x}{2}$ so the perimeter is $2s = 19 + x$. The area is $\sqrt{s(s-9)(s-10)(s-x)}$. Since the area and perimeter are equal, then $2s = \sqrt{s(s-9)(s-10)(s-x)}$ or $4s^2 = s(s-9)(s-10)(s-x)$. Since $s \neq 0$ then $4s = (s-9)(s-10)(s-x)$. Substituting s gives $38 + 2x = \frac{1}{8}(x+1)(x-1)(19-x)$. Thus, $304 + 16x = (x^2-1)(19-x) = -x^3 + 19x^2 + x - 19$ or $x^3 - 19x^2 + 15x + 323 = 0$. The only divisors of 323 are 17 and 19. Use synthetic division to find $x = 17$ is a root so $(x-17)(x^2 - 2x - 19) = 0$. The other quadratic has non-integer roots so $x = 17$. The area then is the same as the perimeter, which is $9 + 10 + 17 = 36$.

26. A: $T = M \cdot P = \begin{bmatrix} 50 & 48 \\ 135 & 126 \end{bmatrix}$. Summing by column, City A required $50 + 135 = 185$ total minutes and City B required $48 + 126 = 174$ total minutes, showing City A took 11 more minutes than City B.

27. C: Solve $75 = 12 + 3(n-1)$ to find there are $n = 22$ total rows. Using the arithmetic series formula, there are total of $(12 + 75)\left(\frac{22}{2}\right) = 957$ total seats.

28. C: The maximum value is $M = 50 - 40(-1) = 90$ and the minimum value is $N = 50 - 40(1) = 10$. Thus, $M - N = 90 - 10 = 80$.

29. C: The rose petal curve as n petals for n odd and $2n$ petals for n even. Also note that when $\theta = 0$, $r = 10$. Thus, $r = 10\cos(4\theta)$ produces 8 petals, each with a length of 10, oriented as desired.

30. B: The average rate of change is $\frac{f(9)-f(2)}{9-2}$. Notice $f(9) = 120\log_2 8 + 5 = 120(3) + 5 = 365$ and $f(2) = 120\log_2 1 + 5 = 120(0) + 5 = 5$. Thus, the average rate of change is $\frac{365-5}{7} = \frac{360}{7}$. Since $f(m)$ was measured in lily pads per month, then its rate of change is measured in lily pads per month per month.