A) Consider the vectors $\mathbf{v} = \langle 1, 2, 3 \rangle$ and $\mathbf{w} = \langle 3, 2, x \rangle$. If $\mathbf{v} \cdot \mathbf{w} = 1729$, find the value of x.

B) Find $x + y$ given that (x, y) that is the center of the hyperbola $y = \dfrac{2x - 7}{8x - 12}$.

C) Find the length of interval that is the range of the function $f(x) = \sin^2 x + \sin x + 1$.

D) Find the product of the period, phase shift (within $-\frac{\pi}{2} < c < \frac{\pi}{2}$) and amplitude of the function: $f(x) = 2 - 3\sin(3\pi x + 2)$.

Note: Phase shifts right are considered positive, and left are considered negative.

MAθ National Convention 2025 - Alpha Bowl **Problem #0**

A) Consider the vectors $\mathbf{v} = \langle 1, 2, 3 \rangle$ and $\mathbf{w} = \langle 3, 2, x \rangle$. If $\mathbf{v} \cdot \mathbf{w} = 1729$, find the value of x.

B) Find $x + y$ given that (x, y) that is the center of the hyperbola $y = \dfrac{2x - 7}{8x - 12}$.

C) Find the length of interval that is the range of the function $f(x) = \sin^2 x + \sin x + 1$.

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Note: Phase shifts right are considered positive, and left are considered negative.

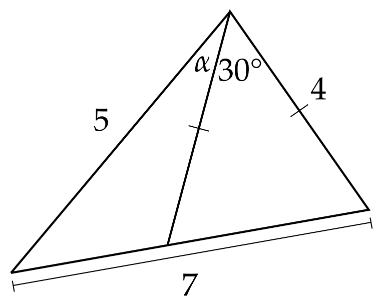
A) What is the eccentricity of the polar graph of $r = \dfrac{5\sin\theta}{6\sin\theta + 9\sin 2\theta}$, where it is defined?

B) Find the sum of the series: $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{e^{n+1}}$?

Note: Your answer should not contain negative exponents or nested fractions.

C) Find the number of petals are on the polar graph of $r = \sin 3\theta \cos 9\theta + \cos 3\theta \sin 3\theta$. If the graph is not a polar rose, give an answer of 1.

D) In the given figure below, if $\sin(\alpha) = \dfrac{a + b\sqrt{c}}{d}$ for positive integers a, b, c, and d, where a and d are relatively prime and c is not divisible by the square of any prime, find $a + b + c + d$.



MAθ National Convention 2025 - Alpha Bowl **Problem #1**

A) What is the eccentricity of the polar graph of $r = \dfrac{5\sin\theta}{6\sin\theta + 9\sin 2\theta}$, where it is defined?

B) Find the sum of the series: $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{e^{n+1}}$?

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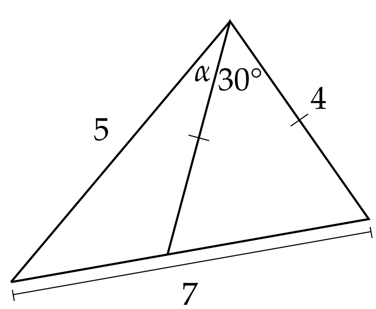

A) Find the sum of the solutions, where $x \in [0, 2\pi]$, of the equation $\sin^2(2x) \cdot \tan(x) \cdot \cot(2x) = 0$.

B) Suppose $f(x) = \dfrac{x}{\sqrt{1+x^2}}$ and let $f_n(x)$ be the n^{th} composition with of $f(x)$ with itself, i.e $f_1(x) = f(x), f_2(x) = f\big(f(x)\big)$ and so on. What is the value of $f_{253}(2\sqrt{2})$?

C) On planet Gandhi, the day is 30 hours, split into 15-hour intervals of A.M and P.M, similar to ours. However, their hour is also 75 minutes. Assuming minutes are the same as ours, what is the acute angle (in degrees) of the clock hands at 13:65 A.M.?

D) Find the first-quadrant area enclosed by the lines $y = \dfrac{6}{7}x$ and $y = \dfrac{1}{6}x$ and the curve defined by graph of the parametric equations $x(t) = 5\cos t + 7$ and $y(t) = 4\sin t + 2$ where $0 \le t \le \dfrac{\pi}{2}$.

MAθ National Convention 2025 - Alpha Bowl **Problem #2**

A) Find the sum of the solutions, where $x \in [0, 2\pi]$, of the equation $\sin^2(2x) \cdot \tan(x) \cdot \cot(2x) = 0$.

B) Suppose $f(x) = \dfrac{x}{\sqrt{1+x^2}}$ and let $f_n(x)$ be the n^{th} composition with of $f(x)$ with itself, i.e $f_1(x) = f(x), f_2(x) = f\big(f(x)\big)$ and so on. What is the value of $f_{253}(2\sqrt{2})$?

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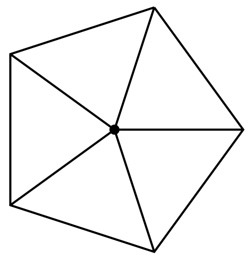
A) Find the area of the parallelogram with sides corresponding to the vectors $\langle 2, -2\sqrt{2}, 2\sqrt{3}\rangle$ and $\langle 1, 5\sqrt{2}, 3\sqrt{3}\rangle$.

B) Given a regular hexagon, we choose 4 random vertices. What is the probability that joining the lines of those vertices forms an isosceles trapezoid?

C) An ant starts in the center of the figure below, moves to an adjacent vertex 6 times by way of the drawn lines. How many ways can the ant end in the center?



D) Compute the number of positive integer divisors of 100000 which do not contain the digit 0.

MAθ National Convention 2025 - Alpha Bowl **Problem #3**

A) Find the area of the parallelogram with sides corresponding to the vectors $\langle 2, -2\sqrt{2}, 2\sqrt{3}\rangle$ and $\langle 1, 5\sqrt{2}, 3\sqrt{3}\rangle$.

B) Given a regular hexagon, we choose 4 random vertices. What is the probability that joining the lines of those vertices forms an isosceles trapezoid?

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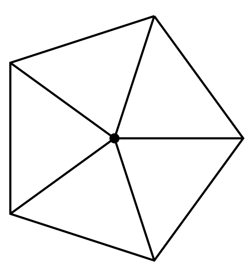

D) Compute the number of positive integer divisors of 100000 which do not contain the digit 0.

A) The Tribonacci numbers T_n are defined as follows: $T_0 = 0, T_1 = 1, T_2 = 1$. For all $n \geq 3$ we have $T_n = T_{n-1} + T_{n-2} + T_{n-3}$. Compute the smallest composite 3-digit Tribonacci number.

B) Given $\sqrt{x+1-4\sqrt{x-3}} + \sqrt{x+6-6\sqrt{x-3}} = 1$, how many integral solutions for x exist?

C) A game is played where a fair 7-sided die is rolled until a 5 occurs, and then the player stops and adds all their rolls up including that 5. What is the probability the sum is even?

D) Two complex numbers z and w are chosen at random such that $|z| = |w| = 1$. What is the probability that $\text{Re}(zw) > 0$ and $\text{Re}(\frac{z}{w}) > 0$? Note: $\text{Re}(z)$ indicates the real part of a complex number.

MAθ National Convention 2025 - Alpha Bowl **Problem #4**

A) The Tribonacci numbers T_n are defined as follows: $T_0 = 0, T_1 = 1, T_2 = 1$. For all $n \geq 3$ we have $T_n = T_{n-1} + T_{n-2} + T_{n-3}$. Compute the smallest composite 3-digit Tribonacci number.

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A) Liam writes an equation of an ellipse:

$$5x^2 - 10x + 9y^2 - 36y - 4 = 0$$

Lance comes along and adds the term kxy to the left side of the equation and turns it into a parabola. What is the least possible value of k that Lance could have added?

B) Daniel's favorite number is a positive two-digit integer. Daniel sums the integers from 5 to his favorite number, inclusive. Then, he sums the next 12 consecutive integers starting after his favorite number. If the two sums are consecutive integers and the second sum is greater than the first sum, what is Daniel's favorite number?

C) Given $\sqrt[5]{-6 - 2i\sqrt{3}}$ can be written in exponential form $re^{i\theta}$ where $r > 0, 0 \le \theta \le 2\pi$ and r, θ are real numbers, find the sum of all possible values of θ.

D) Let $P(x)$ be a quadratic polynomial such that the product of the roots of P is 20. Real numbers a and b satisfy $a + b = 22$, $P(a) + P(b) = P(22)$. Find $a^2 + b^2$.

MAθ National Convention 2025 - Alpha Bowl **Problem #5**

A) Liam writes an equation of an ellipse:

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A) What is the value of $\frac{\sin\left(\frac{\pi}{24}\right)\sin\left(\frac{2\pi}{24}\right)\cdots\sin\left(\frac{5\pi}{24}\right)}{\cos\left(\frac{6\pi}{24}\right)\cos\left(\frac{7\pi}{24}\right)\cdots\cos\left(\frac{11\pi}{24}\right)}$?

B) Suppose we list of all 5 letter possible strings from the set $\{E, N, O, R, S\}$ (where each element is used once) in alphabetical order, starting first with **ENORS**. Which element of the list would the word **ROSEN** be?

C) Let a, b, c and d be positive integers. If

$$\frac{a!}{b!} + \frac{c!}{d!} = \frac{2}{5}$$

then what is the largest possible value of $a + b + c + d$?

D) Chloe writes an equation on the board. However, Jimmy comes along and erases some of the digits, replacing them with the variables a, b, c and d. On the board remains:

$$\log_a \overline{b2c} = d$$

where $a \neq 1$ and $b \neq 0$,. Find the sum of all possible values of d.

MAθ National Convention 2025 - Alpha Bowl **Problem #6**

A) What is the value of $\frac{\sin\left(\frac{\pi}{24}\right)\sin\left(\frac{2\pi}{24}\right)\cdots\sin\left(\frac{5\pi}{24}\right)}{\cos\left(\frac{6\pi}{24}\right)\cos\left(\frac{7\pi}{24}\right)\cdots\cos\left(\frac{11\pi}{24}\right)}$?

B) Suppose we list of all 5 letter possible strings from the set $\{E, N, O, R, S\}$ (where each element is used once) in alphabetical order, starting first with **ENORS**. Which element of the list would the word **ROSEN** be?

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where $a \neq 1$ and $b \neq 0$,. Find the sum of all possible values of d.

A) The Pandya function, denoted as $p(x) = x^5 + ax^4 + bx + c$ has the property that the arithmetic mean of its roots equal to the product of the roots, and both are equal to $p(1)$. If the y-intercept of the Pandya function is 3, then what is the value of $a \cdot b \cdot c$?

B) Let a be a positive real number $a \neq 1$, such that:

$$\log_a(10) + \log_a(10^2) + \cdots + \log_a(10^9) = 2025$$

What is the value of a?

C) If $\sin\alpha = -\frac{\sqrt{2}}{2}$ and $\cos(\alpha - \beta) = \frac{1}{2}$ with $\beta > 0$, and both angles are in radians, what is the minimum value of β?

D) An ellipse $\mathcal{E}$ is inscribed inside a rectangle of side lengths 10 and 20 such that the axes of the ellipse are parallel to the sides of the rectangle. Two congruent, non-overlapping circles, of maximum area are inscribed inside the ellipse. What is the distance between the centers of the two circles?

MAθ National Convention 2025 - Alpha Bowl **Problem #7**

A) The Pandya function, denoted as $p(x) = x^5 + ax^4 + bx + c$ has the property that the arithmetic mean of its roots equal to the product of the roots, and both are equal to $p(1)$. If the y-intercept of the Pandya function is 3, then what is the value of $a \cdot b \cdot c$?

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What is the value of a?

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A) Find the sum of the squares of the eigenvalues of the matrix below:

$$\begin{bmatrix} 2 & 3 & 1 \\ 4 & 2 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

B) Let x be an angle in the first quadrant. If $\tan(\sqrt{2}\cos x) = \cot(\sqrt{2}\sin x)$, compute $1 + \sin 2x$.

C) Find the shortest distance between the planes $6x - 4y - 2z + 10 = 0$ and $3x - 2y + z = 4$.

D) Jeffrey writes all positive divisors of the number 216 on separate slips of paper, then places the slips into a hat. He randomly selects three slips from the hat, with replacement. What is the probability that the product of the numbers on the three slips Jeffrey selects is also divisor of 216?

MAθ National Convention 2025 - Alpha Bowl **Problem #8**

A) Find the sum of the squares of the eigenvalues of the matrix below:

$$\begin{bmatrix} 2 & 3 & 1 \\ 4 & 2 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

B) Let x be an angle in the first quadrant. If $\tan(\sqrt{2}\cos x) = \cot(\sqrt{2}\sin x)$, compute $1 + \sin 2x$.

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A) Evaluate: $\lim_{x\to-\infty}\left(\sqrt{x^2+\frac{3}{4}x}-\sqrt{x^2}\right)$

B) Two concentric circles have radii that differ by $\frac{3}{4}$ and region between the two circles has an area of 4π. What is the radius of the inner circle?

C) Let x and y be real numbers. What is the minimum value of the function below:

$$f(x,y)=\sqrt{4+y^2}+\sqrt{(x-2)^2+(2-y)^2}+\sqrt{(4-x)^2+1}$$

D) Suppose a sequence has $a_1 = 2025$ and $a_n = \sigma(a_{n-1})$ for $n \geq 2$, where $\sigma(n)$ is the number of positive integer divisors of n. What is $\sum_{n=1}^{2025} a_n$?

MAθ National Convention 2025 - Alpha Bowl **Problem #9**

A) Evaluate: $\lim_{x\to-\infty}\left(\sqrt{x^2+\frac{3}{4}x}-\sqrt{x^2}\right)$

B) Two concentric circles have radii that differ by $\frac{3}{4}$ and region between the two circles has an area of 4π. What is the radius of the inner circle?

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$$f(x,y)=\sqrt{4+y^2}+\sqrt{(x-2)^2+(2-y)^2}+\sqrt{(4-x)^2+1}$$

D) Suppose a sequence has $a_1 = 2025$ and $a_n = \sigma(a_{n-1})$ for $n \geq 2$, where $\sigma(n)$ is the number of positive integer divisors of n. What is $\sum_{n=1}^{2025} a_n$?

Euler's Totient function, $\varphi(n)$ counts the number of positive integers up to the given integer that are relatively prime to n. Given a prime factorization $n = p_1^{e_1} \cdot p_2^{e_2} \cdot p_3^{e_3} \cdots p_n^{e_n}$ the totient of n is given by the formula:

$$\varphi(n) = n\left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right)\cdots\left(1 - \frac{1}{p_n}\right)$$

A) Find $\varphi(2025)$.

B) Suppose we have a series such that $\sum_{k=0}^{3} \varphi(a \cdot 10^k) = 2670$, where a is a positive integer that is relatively prime to 10. Find the smallest possible positive integer value of a.

An application of the Totient function is Euler's Theorem, which states that for relatively prime, positive integers, a and n, then $a^{\varphi(n)} \equiv 1 \pmod{n}$

C) Let $a_1, a_2, a_3, \ldots, a_{2025}$ be a strictly increasing sequence of positive integers. If it is given that $\sum_{i=1}^{2025} a_i = 202520262027$, then find the remainder when $\sum_{i=1}^{2025} a_i^7$ is divided by 7.

D) Find the last two digits of 43^{2025}.

MAθ National Convention 2025 - Alpha Bowl **Problem #10**

Euler's Totient function, $\varphi(n)$ counts the number of positive integers up to the given integer that are relatively prime to n. Given a prime factorization $n = p_1^{e_1} \cdot p_2^{e_2} \cdot p_3^{e_3} \cdots p_n^{e_n}$ the totient of n is given by the formula:

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A) Find $\varphi(2025)$.

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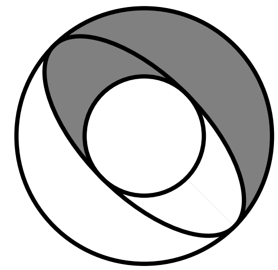
D) Find the last two digits of 43^{2025}.

This question is in a relay-style. The values of A, B, C and D will be **both** the answers to the parts as well as maintaining their values in the consecutive parts.

A) Let A be the value of the tangent of the acute angle between the vectors $\langle -3, 6, 6\rangle$ and $\langle 3, 4, 0\rangle$.

B) A circle with radius 10, a circle with radius A, and an ellipse with semi-minor axis A and semi-major axis 10 are concentric, as shown in the diagram. Let B be the area of the shaded region.



C) Julia drops a bouncing ball from a height of B feet, and the bouncing ball travels infinitely, with the ability to rebound to back to $\frac{1}{C}$ of its previous height. The total amount of that the ball travels is $8B$, what is C?

D) You're playing an odd game where have your probability of winning is equal to D. If you win, you get 16 points and if you lose, you lose 5 points for your team. If the expected value of your winnings is $7C$ points, what is D?

MAθ National Convention 2025 - Alpha Bowl **Problem #11**

This question is in a relay-style. The values of A, B, C and D will be **both** the answers to the parts as well as maintaining their values in the consecutive parts.

A) Let A be the value of the tangent of the acute angle between the vectors $\langle -3, 6, 6\rangle$ and $\langle 3, 4, 0\rangle$.

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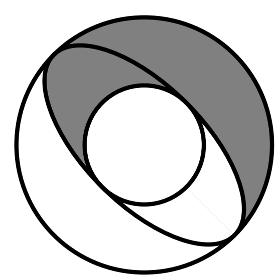

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A) The three-digit numbers $\overline{243}$ and $\overline{A6B}$ sum to a multiple of 11, where A and B are digits. What is the smallest possible value of $A + B$?

B) Let S be a set of nine distinct integers. Six of the elements of the set are 1, 2, 3, 5, 8, and 13. Compute the sum of possible values for the median of S.

C) The roots of $10x^2 - 12x + k$ are $\cos\alpha$ and $\sin\alpha$ for some angle α. Compute k.

D) Charlene has a collection of $P = 240$ books. She takes $\frac{m}{n}$ of the books for positive integers $m < n$, leaving Q books in the collection. Then she takes $\frac{1}{m}$ of the remaining books, leaving R books in the collection. P, Q, and R form a decreasing arithmetic sequence of positive integers. Find the number of all possible values of m.

MAθ National Convention 2025 - Alpha Bowl **Problem #12**

A) The three-digit numbers $\overline{243}$ and $\overline{A6B}$ sum to a multiple of 11, where A and B are digits. What is the smallest possible value of $A + B$?

B) Let S be a set of nine distinct integers. Six of the elements of the set are 1, 2, 3, 5, 8, and 13. Compute the sum of possible values for the median of S.

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A) Suppose a, b, and c are distinct positive integers such that $\sqrt{a\sqrt{b\sqrt{c}}}$ is an integer. Compute the least possible value of $a+b+c$.

B) If a and b are real numbers such that

$$a \cdot 2^b = 8 \text{ and } a^b = 2$$

Compute $a^{\log_2 a} \cdot 2^{b^2}$.

C) The absolute values of the roots of $x^3 - 147x + c$ are all prime integers. Find $|c|$.

D) Five men and nine women stand equally spaced around a circle in random order. Find the probability that every man stands diametrically opposite a woman.

MAθ National Convention 2025 - Alpha Bowl **Problem #13**

A) Suppose a, b, and c are distinct positive integers such that $\sqrt{a\sqrt{b\sqrt{c}}}$ is an integer. Compute the least possible value of $a+b+c$.

B) If a and b are real numbers such that

$$a \cdot 2^b = 8 \text{ and } a^b = 2$$

Compute $a^{\log_2 a} \cdot 2^{b^2}$.

C) The absolute values of the roots of $x^3 - 147x + c$ are all prime integers. Find $|c|$.

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