#0.		
	A: 574	B: $\frac{7}{4}$
	C: $\frac{9}{4}$	D: $-\frac{4}{3\pi}$

#1.		
	A: 3	B: $\frac{-2}{e^2+2e}$
	C: 24	D: 19

#2.		
	A: 4π	B: $\frac{2\sqrt{2}}{45}$
	C: 20.8°	D: $5\pi + 19$

#3.		
	A: $12\sqrt{13}$	B: $\frac{2}{5}$
	C: 505	D: 11

#4.		
	A: 274	B: 6
	C: $\frac{3}{7}$	D: $\frac{1}{4}$

#5.		
	A: $-6\sqrt{5}$	B: 29
	C: $\frac{31\pi}{6}$	D: 464

#6.		
	A: $\sqrt{2}$	B: 89
	C: 34	D: 20

#7.		
	A: -990	B: $\sqrt[45]{10}$
	C: $\frac{\pi}{12}$	D: $5\sqrt{3}$

#8.		
	A: 58	B: $\frac{\pi^2}{8}$
	C: $\frac{\sqrt{14}}{2}$	D: $\frac{25}{256}$

#9.		
	A: $-\frac{3}{8}$	B: $\frac{55}{24}$
	C: 5	D: 6089

#10.		
	A: 1080	B: 7
	C: 6	D: 43

#11.		
	A: $2\sqrt{2}$	B: 46π
	C: $\frac{9}{7}$	D: $\frac{2}{3}$

#12.		
	A: 5	B: 35
	C: $\frac{11}{5}$	D: 19

#13.		
	A: 7	B: 128
	C: 286	D: $\frac{48}{143}$

0. A = 574 **B =** $\frac{7}{4}$ **C =** $\frac{9}{4}$ **D =** $-\frac{4}{3\pi}$

A) We compute $\mathbf{v}\cdot\mathbf{w} = 1\cdot 3 + 2\cdot 2 + 3\cdot x = 7 + 3x = 1729 \implies x = \boxed{574.}$

B) The center of the hyperbola is the intersection of the asymptotes $y = \frac{1}{4}$ and $x = \frac{3}{2}$, which is $\left(\frac{3}{2}, \frac{1}{4}\right)$. So, $x + y = \frac{3}{2} + \frac{1}{4} = \boxed{\frac{7}{4}}$

C) $\sin x$ is going to maximized at $x = \frac{\pi}{2}$ and minimized at $x = \frac{3\pi}{2}$. This yields a range of $\left[\frac{3}{4}, 3\right]$. This yields a length of $3 - \frac{3}{4} = \boxed{\frac{9}{4}.}$

D) The given function can be rewritten as $f(x) = 2 - 3\sin\left(3\pi(x + \frac{2}{3\pi})\right)$. This yields an amplitude of 3, phase shift of $-\frac{2}{3\pi}$, and a period of $\frac{2\pi}{3\pi} = \frac{2}{3}$. Hence, the product of these values is $3\cdot\frac{2}{3}\cdot\left(-\frac{2}{3\pi}\right) = \boxed{-\frac{4}{3\pi}}$

1. A = 3 **B =** $\dfrac{-2}{e^2+2e}$ **C =** 24 **D =** 19

A) Using double angle sine, we have:

$$r = \frac{5\sin\theta}{6\sin\theta + 18\sin\theta\cos\theta} \implies r = \frac{5}{6 + 18\cos\theta} \implies r = \frac{\frac{5}{6}}{1 + 3\cos\theta} \implies e = \boxed{3.}$$

B) $\displaystyle\sum_{n=1}^{\infty}\frac{(-2)^n}{e^{n+1}}$ is geometric with $a = -\dfrac{2}{e^2}$ and $r = -\dfrac{2}{e}$, therefore we have:

$$S = \frac{a}{1-r} \implies S = \frac{-\frac{2}{e^2}}{1 - (-\frac{2}{e})} = \boxed{-\frac{2}{e^2+2e}}$$

C) If we use grouping, sum-to-product, and then double angle sine formula, we can yield:

$$r = \sin 3\theta\cos 9\theta + \cos 3\theta\sin 3\theta = \sin 3\theta(\cos 9\theta + \cos 3\theta) = 2\sin 3\theta\cos 3\theta\cos 6\theta = \frac{1}{2}\sin 12\theta$$

Therefore, this a rose with $\boxed{24}$ petals.

D) By Law of Cosines, we have:

$$7^2 = 4^2 + 5^2 - 2\cdot 4\cdot 5\cdot\cos(\alpha + 30^\circ) \implies \cos(\alpha + 30^\circ) = -\tfrac{1}{5} \implies \alpha = \cos^{-1}\left(-\tfrac{1}{5}\right) + 30^\circ$$

By sum-and-difference, we have:

$$\sin\left(\cos^{-1}\left(-\tfrac{1}{5}\right) + 30\right) = \sin\left(\cos^{-1}\left(-\tfrac{1}{5}\right)\right)\cos(30^\circ) - \cos\left(\cos^{-1}(-\tfrac{1}{5})\right)\sin(30^\circ)$$

Evaluating, while being aware of typical restrictions of $\cos^{-1}$, we calculate:

$$\frac{2\sqrt{6}}{5}\cdot\frac{\sqrt{3}}{2}+\frac{1}{5}\cdot\frac{1}{2}=\frac{1+6\sqrt{2}}{10}$$

Therefore, $a+b+c+d=1+6+2+10=\boxed{19.}$

2. A $=4\pi$ **B** $=\frac{2\sqrt{2}}{45}$ **C** $=20.8°$ **D** $=5\pi+19$

A) We can manipulate this in a number of ways, being careful of the original domain.

$$\sin^2(2x)\cdot\tan(x)\cdot\cot(2x)=0 \implies \sin^2(2x)\cdot\tan(x)\cdot\frac{\cos(2x)}{\sin(2x)}=0 \implies \sin(2x)\cdot\cos(2x)\cdot\tan x=0$$

Since $\tan x$ is undefined where $\sin 2x=0$, we just consider where $\cos 2x=0$, giving solutions of $\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4}$ and $\frac{7\pi}{4}$. These sum to a total of $\boxed{4\pi.}$

B) Consider the composition:

$$f_2(x)=f\big(f(x)\big)=\frac{\frac{x}{\sqrt{1+x^2}}}{\sqrt{1+\left(\frac{x}{\sqrt{1+x^2}}\right)^2}}=\frac{x}{\sqrt{1+2x^2}}$$

This pattern persists to $f_n=\dfrac{x}{\sqrt{1+nx^2}}$. Hence, we have:

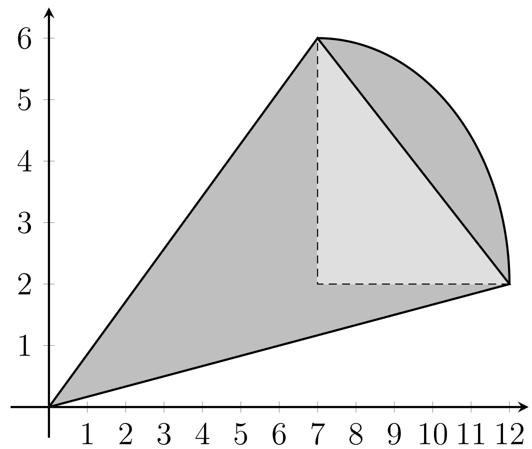
$$f_{253}(2\sqrt{2})=\frac{2\sqrt{2}}{\sqrt{1+253\cdot 8}}=\frac{2\sqrt{2}}{\sqrt{1+2024}}=\frac{2\sqrt{2}}{\sqrt{2025}}=\frac{2\sqrt{2}}{45}$$

C) Since there are 15-hour periods, the hour hand moves 24° per hour, and the minute hand moves 4.8° per minute for the 75 minutes. The hour hand also will move a proportion of the 24° per minute. So we have:

$$\begin{aligned}\textbf{Hour Angle} &= 13\cdot 24+\tfrac{65}{75}\cdot 24=332.8°\\ \textbf{Minute Angle} &= 65\cdot 4.8°=312°\\ \textbf{Angle Between} &= |332.8°-312°|=\boxed{20.8°}\end{aligned}$$

D) We consider $\cos t=\frac{x-7}{5}$ and $\sin t=\frac{y-2}{4}$. Using $\sin^2 t+\cos^2 t=1$, we can get the equation of an ellipse of $y=\frac{(x-7)^2}{25}+\frac{(y-2)^2}{16}$. The region forms a quarter ellipse of area of 5π, extending from the endpoints to the origin. The endpoints of the quarter ellipse are $(6,7)$ and $(12,2)$, which if we use shoelace, with $(0,0)$, we can find the area of the shaded triangle below as 29.

However, there is a portion (the dashed right triangle) that is being double counted by both the quarter ellipse and this triangle. So we have $5\pi + 29 - \frac{1}{2}(5)(4) = \boxed{5\pi + 19}$



3. **A =** $12\sqrt{13}$ **B =** $\dfrac{2}{5}$ **C =** 505 **D =** 11

A) If we note that the dot product of the vectors is 0, this is a right triangle in 3-space. We simply use $A = bh$:

$$\sqrt{2^2 + (-2\sqrt{2})^2 + (2\sqrt{3})^2} \cdot \sqrt{1^2 + (5\sqrt{2})^2 + (3\sqrt{3})^2} = 2\sqrt{6} \cdot \sqrt{78} = \boxed{12\sqrt{13}}$$

B) The vertices will only form a isosceles trapezoid if the four vertices are consecutive. There are 6 possible scenarios of vertices, and 15 possible choices for 4 vertices out of 6. This yields us $\frac{6}{15} = \boxed{\frac{2}{5}.}$

C) Consider a string of 6 characters, each being an O, I, or N. The character O stands for an out move (where the ant moves from the center of the pentagon to an outer point). The character I stands for an in move (where the ant moves from an outer point to the center of the pentagon). The character N stands for a next move (where the ant moves from an outer point to an adjacent outer point).

Each favorable path is defined of a string of letters 6 long, starting with O and ending with I. Every O we have 5 choices, every N we have two choices, while I only had one choice. We proceed with casework by the number of O's:

- **Case One:** One O $-$ This means that the four characters in the middle must be all be N. Therefore, we have $5 \cdot 2^4 = 80$ ways.
- **Case Two:** Two O's $-$ To get a second O, it needs to be preceded by an I. For example $ONNIOI$. So, we need to block the IO, and 3 choices to place that block among the other 4 characters. Those final 2 must be N's. That gives us $5^2 \cdot 2^2 \cdot 3 = 300$ ways.
- **Case Three:** Three O's $-$ $IOIOIOI$ This case is $5^3 = 125$ ways.

There are $80 + 300 + 125 = \boxed{505}$ ways total.

D) We note that $100000 = 2^5 \cdot 5^5$. All multiples of 10 end in zero, so consider the powers of 2 and the powers of 5. This gives us two sets:

$$2^a \in \{1, 2, 4, 8, 16, 32\}$$

$$5^a \in \{1, 5, 125, 625, 3125\}$$

Since they share the element of 1, we have $6 + 6 - 1 = \boxed{11}$ divisors.

4. **A** $=$ 274 **B** $=$ 6 **C** $= \dfrac{3}{7}$ **D** $= \dfrac{1}{4}$

A) The sequence of Tribonacci numbers are $0, 1, 1, 2, 4, 7, 13, 24, 44, 81, 149, 274$. 149 is prime though, so our answer is $\boxed{274.}$

B) We can note the following:

$$x + 1 - 4\sqrt{x-3} = \sqrt{x-3} - 4\sqrt{x-3} + 4 = (\sqrt{x-3} - 2)^2$$
$$x + 6 - 6\sqrt{x-3} = \sqrt{x-3} - 6\sqrt{x-3} + 9 = (\sqrt{x-3} - 2)^2$$

Hence, the equation simplifies to $\left|\sqrt{x-3} - 2\right| + \left|3 - \sqrt{x-3}\right| = 1$ which yields that $2 \leq \sqrt{x-3} \leq 3 \implies 7 \leq x \leq 12$. Therefore, there are $\boxed{6}$ integral solutions.

C) Using complementary counting, let p be the probability that the sum is odd, and $1 - p$ the probability the sum is even. We do case work based on the value and parity of the first roll:

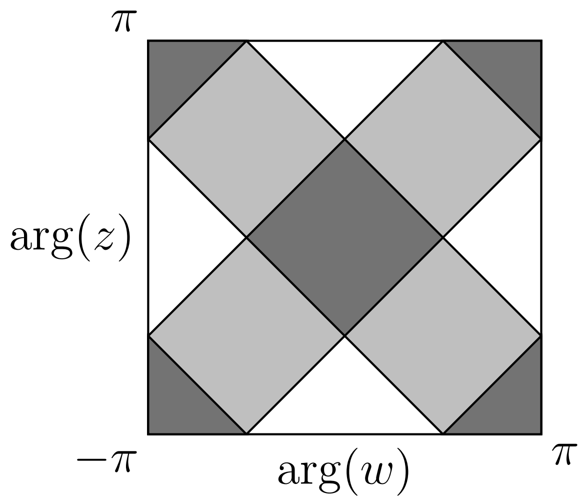
- If the first roll is a 5, the sum is odd and this occurs $\frac{1}{7}$ of the time.
- If the first roll is 2, 4, or 6, then rest of the rolls must sum to an odd value for the total sum to be odd. So the probability of this case is $\frac{3p}{7}$.
- If the first roll is a 1, 3 or 7, then the rest of the rolls have to sum to an even number, so the probability of this case is $\frac{3(1-p)}{7}$.

We have the equation:

$$p = \frac{1}{7} + \frac{3p}{7} + \frac{3(1-p)}{7} \implies p = \tfrac{4}{7}$$

Therefore, the probability of the sum being even is $1 - p = \boxed{\dfrac{3}{7}}$

D) Let $\arg(z)$ denote the argument of z. $\arg(zw) = \arg(z) + \arg(w)$ and $\arg(\frac{z}{w}) = \arg z - \arg w$. For $\mathrm{Re}(a) > 0$, we need $-\frac{\pi}{2} < \arg(a) < \frac{\pi}{2}$ or equivalent. Use geometric probability to get the following:



This area yields $\boxed{\frac{1}{4}.}$

5. **A** $= -6\sqrt{5}$ **B** $= 29$ **C** $= \frac{31\pi}{6}$ **D** $= 464$

A) For a rotated parabola, $B^2 - 4AC = 0$ for the general form of a conic. Here $k = B$, so we have:

$$k^2 - 4(5)(9) = 0 \implies k^2 - 180 = 0 \implies k = \pm 6\sqrt{5}$$

Since we are looking for the least value of k, the answer is $\boxed{-6\sqrt{5}.}$

B) Let n be Daniel's favorite number. The first sum is equal to $\frac{n(n+1)}{2} - \frac{4 \cdot 5}{2}$, which gives us the quadratic $\frac{1}{2}n^2 + \frac{1}{2}n - 10$. The second sum is equal to $12n + \frac{12 \cdot 13}{2} = 12n + 78$. Since the first sum is exactly one less than the second sum, we have:

$$\tfrac{1}{2}n^2 + \tfrac{1}{2}n - 10 = (12n + 78) - 1 \Longrightarrow \tfrac{1}{2}n^2 - \tfrac{23}{2}n - 87 = 0$$

Factoring yields $n = 29$ or $n = -6$, which lets us conclude Daniel's favorite number is $\boxed{29.}$

C) By DeMoivre's root theorem, we have $4\sqrt{3}\,\text{cis}\,\dfrac{7\pi + 12\pi k}{30}$, so the possible values of θ are $\frac{7\pi}{30}, \frac{19\pi}{30}, \frac{31\pi}{30}, \frac{43\pi}{30}$ and $\frac{55\pi}{30}$. The sum of these values is $\boxed{\frac{31\pi}{6}.}$

D) Consider by Vieta's, if the product of the roots is 20, then let p be the leading coefficient. We also let q be the linear coefficient to give us a quadratic of the form $px^2 + qx + 20p$. Therefore:

$$P(a) + P(b) = P(22) \implies pa^2 + aq + 20p + pb^2 + 20p = 22^2p + 22q + 20p$$

We factor the left side to get:

$$p(a^2+b^2)+q(a+b)+40p = 22^2p+22q+20p$$
$$p(a^2+b^2)+q(a+b)+20p = 22^2p+22q$$

Since $a+b=22$, we have:

$$p(a^2+b^2)+22q+20p = 22^2p+22q$$
$$p(a^2+b^2)+20p = 22^2p$$
$$a^2+b^2+20 = 22^2$$

Therefore $a^2+b^2 = 22^2-20 = \boxed{464.}$

6. A = $\sqrt{2}$ **B =** 89 **C =** 34 **D =** 20

A) Using the cofunction identity $\sin\theta = \cos(\frac{\pi}{2}-\theta)$, we can convert the equation into the following:

$$\frac{\sin\left(\frac{\pi}{24}\right)\sin\left(\frac{2\pi}{24}\right)\cdots\sin\left(\frac{5\pi}{24}\right)}{\cos\left(\frac{6\pi}{24}\right)\cos\left(\frac{7\pi}{24}\right)\cdots\cos\left(\frac{11\pi}{24}\right)} = \frac{\sin\left(\frac{\pi}{24}\right)\sin\left(\frac{2\pi}{24}\right)\cdots\sin\left(\frac{5\pi}{24}\right)}{\cos\left(\frac{6\pi}{24}\right)\sin\left(\frac{\pi}{24}\right)\sin\left(\frac{2\pi}{24}\right)\cdots\sin\left(\frac{5\pi}{24}\right)} = \frac{1}{\cos(\frac{\pi}{4})} = \boxed{\sqrt{2}}$$

B) Since **ROSEN** has 5 distinct letters, we know this has $5! = 120$ words on the list. Each letter would have 24 words where that letter starts the word. Counting from the bottom of the list, the start and end at lines 74 to 97. Counting back from 97, we have

$\boxed{89.}$ **ROSEN**	92. **RSEON**	95. **RSOEN**
90. **ROSNE**	93. **RSNEO**	96. **RSOEN**
91. **RSENO**	94. **RSNOE**	97. **SENOR**

C) Since $\dfrac{2}{5} < 1$, we know that $a<b$ and $c<d$. Also, since these are factorials, both fractions will reduce to some value $\dfrac{1}{n}$. So we suppose we have $\dfrac{1}{p}+\dfrac{1}{q}=\dfrac{2}{5}$, we have:

$$\frac{1}{p}+\frac{1}{q}=\frac{2}{5} \Longrightarrow 2pq-5p-5q=0$$

By SFFT, we can factor this, starting with doubling the equation:

$$4pq-10p-10q=0 \Longrightarrow 2p(2q-5)-10q+25=25$$

$$\Longrightarrow 2p(2q-5)-5(2q-5)=25 \Longrightarrow (2p-5)(2q-5)=25$$

To maximize this, we want p and q to have the largest possible difference, this is accomplished if $2p-5=1$ and $2q-5=25$, giving us $p=3$ and $q=15$. Therefore, we have;

$$\frac{1}{3}+\frac{1}{15}=\frac{a!}{b!}+\frac{c!}{d!} \Longrightarrow \frac{1}{3}+\frac{1}{15}=\frac{2!}{3!}+\frac{14!}{15!}$$

Therefore $a+b+c+d = 2+3+14+15 = \boxed{34.}$

D) If we exponentiate both sides, we get that $a^d = \overline{b2c}$. We then do casework on the value of d, and we only have to find one value to work for us to consider it as potential solution. We know $d \notin \{0, 1, 2\}$ as $a \leq 9$, and $9^2 = 81$, which is clearly not 3 digits.

- If $d = 3, 5^3 = 125$ this works.
- If $d = 4, 5^4 = 625$, so this also works.
- If $d = 5$, 2^5 is too small, $3^5 = 243$ and 4^5 is too big, so this doesn't work.
- If $d = 6$, $3^6 = 729$, so this works.
- If $d = 7$, $2^7 = 128$.

Neither $d = 8$ or $d = 9$ works, so we have possible values of d being $3 + 4 + 6 + 7 = \boxed{20.}$

7. **A =** -990 **B =** $\sqrt[45]{10}$ **C =** $\dfrac{\pi}{12}$ **D =** $5\sqrt{3}$

A) Suppose the roots if the Pandya function are r_1, r_2, r_3, r_4 and r_5. From the given statements and Vieta's, we know:

$$p(1) = a + b + c + 1 = -\frac{a}{5} = -c$$
$$p(0) = c = 3$$

So, we can infer that $c = 3$ and $a = 15$. From this, we have $18 + b + 1 = -3$, so $b = -22$. Hence $a \cdot b \cdot c = 35 \cdot 7 \cdot (-20) = \boxed{-990}$

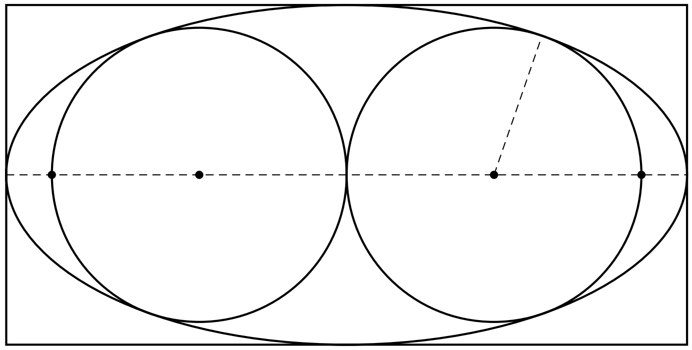
B) Consider by rules of logs, we have:

$$\begin{aligned}
\log_a(10) + \log_a(10^2) + \cdots + \log_a(10^9) &= 2025 \\
\log_a(10 \cdot 10^2 \cdots 10^9) &= 2025 \\
\log_a(10^{1+2+3+\cdots+9}) &= 2025 \\
\log_a(10^{45}) = 45 \log_a(10) &= 2025 \\
\log_a(10) &= 45 \\
10 = a^{45} \implies a &= \boxed{\sqrt[45]{10}}
\end{aligned}$$

C) Given $\sin\alpha = -\frac{\sqrt{2}}{2}$, α is any coterminal angel equal to $\frac{5\pi}{4}$ or $\frac{7\pi}{4}$, and $\alpha - \beta$ must be coterminal with $\frac{\pi}{3}$ or $\frac{5\pi}{3}$. Trying to get the greatest difference between α and β to minimize β, we let $\alpha = \frac{7\pi}{4}$ to find that is β must be a multiple of $\boxed{\frac{\pi}{12},}$ the smallest positive β being $\frac{\pi}{12}$, itself.

D) We can defined that the ellipse is horizontal without a loss of generality, giving it the equation of $\frac{x^2}{100} + \frac{y^2}{25} = 1$, as the dimensions of the rectangle will correspond to the major and minor axes of the ellipse.

Now, to place the congruent circles, consider that the the largest circle to be inscribed would be concentric with the ellipse. However, since we have two of these circles, and they cannot overlap, we have to move their centers equally away from the center of the ellipse.



Consider the circles will have centers at $(\pm r, 0)$, we consider that for $r > 0$, we can relate the equation of the ellipse with that of the circles:

$$y = \sqrt{r^2 - (x-r)^2} \text{ and } y = \sqrt{25 - \tfrac{x^2}{4}}$$
$$r^2 - (x-r)^2 = 25 - \tfrac{x^2}{4} \implies 2rx - x^2 = 25 - \tfrac{x^2}{4}$$
$$0 = \tfrac{3}{4}x^2 - 2rx + 25 \xRightarrow{\text{since } r=x} -\tfrac{4}{3}r^2 + 25 = 0 \implies r^2 = \tfrac{75}{4} \implies r = \tfrac{5\sqrt{3}}{2}$$

Hence, this proves that the distance between the two centers is $\frac{5\sqrt{3}}{2} \cdot 2 = \boxed{5\sqrt{3},}$ which just so happens to equal c for the ellipse.

8. A $= 58$ **B** $= \dfrac{\pi^2}{8}$ **C** $= \dfrac{\sqrt{14}}{2}$ **D** $= \dfrac{25}{256}$

A) Let our matrix be M. Rather than finding them individually, we remember that the trace of the matrix is equal to the sum the eigenvalues. This will mean that $\text{TR}(M^2)$ will be the sum of the squares of the eigenvalues. The diagonal entries of M^2 are 18, 19 and 21, yielding us a sum of $\boxed{58.}$

B) As angles, $\sqrt{2}\cos x$ and $\sqrt{2}\sin x$ must lie in the first quadrant as x lies in the first quadrant. Consider that if $\tan(\sqrt{2}\cos(x)) = \frac{1}{\tan\left(\sqrt{2}\cos\left(\frac{\pi}{2}-x\right)\right)}$, then by the nature of the first quadrant angles, we have $\sqrt{2}\sin x + \sqrt{2}\cos x = \frac{\pi}{2}$. Squaring both sides of this yields:

$$2 + 4\sin x\cos x = \frac{\pi^2}{4} \implies 2\sin 2x = \frac{\pi^2}{4} - 2 \implies \sin 2x = \frac{\pi^2}{8} - 1 \implies \sin 2x + 1 = \boxed{\frac{\pi^2}{8}}$$

C) Note the first plane can be rewritten as $3x - 2y - z + 5 = 0$. An easy to find point on the second plane is $(1, 0, 1)$, which we can then do point-to-plane distance:

$$d = \frac{|Ax + By + Cz - D|}{\sqrt{A^2 + B^2 + C^2}} \implies \frac{|3 - 0 - 1 + 5|}{\sqrt{9 + 4 + 1}} = \boxed{\frac{\sqrt{14}}{2}}$$

D) Since the prime factorization of 216 is $2^3 \cdot 3^3$, 216 has 16 factors. Therefore, the denominator of our desired probability is $16^3 = 2^{12}$. Each divisor of 216 can be represented as an ordered pair (a, b), which correspond to a number $2^a \cdot 3^b$ where $a, b \in \{0, 1, 2, 3\}$. We suppose the divisors we draw are $(a_1, b_1), (a_2, b_2)$ and (a_3, b_3), this becomes a stars and bars problem. We desire $a_1 + a_2 + a_3 \leq 3$ as well as $b_1 + b_2 + b_3 \leq 3$. This is equivalent to the number of solutions of $a_1 + a_2 + a_3 + s = 3$, where s accounts for when a_1, a_2, a_3 add to less than 3. By stars-and-bars, this is $\binom{6}{3} = 20$, which is squared to account for b_1, b_2 and b_3. Therefore, our answer is $\frac{20^2}{2^{12}} = \boxed{\frac{25}{256}}.$

9. A = $-\frac{3}{8}$ **B =** $\frac{55}{24}$ **C =** 5 **D =** 6089

A) We multiply by the conjugate, and proceed:

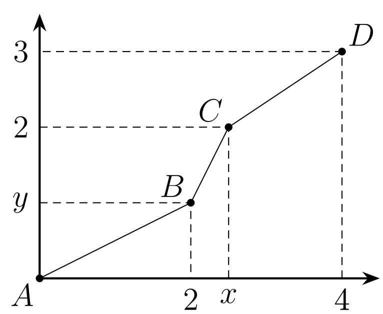
$$\lim_{x\to-\infty} \frac{x^2 + \frac{3}{4}x - x^2}{\sqrt{x^2 + \frac{3}{4}x} + \sqrt{x^2}} = \frac{\frac{3}{4}x}{\sqrt{x^2 + \frac{3}{4}x} + \sqrt{x^2}}$$

As the limit decreases without bound, $\sqrt{x^2 + \frac{3}{4x}x} + \sqrt{x^2} = 2|x|$, hence, we have:

$$\lim_{x\to-\infty} \frac{\frac{3}{4}x}{2|x|} = \boxed{-\frac{3}{8}}$$

B) Since the area is 4π, we have $\pi\left(r + \frac{3}{4}\right)^2 - \pi r^2 = 4\pi$, which yields $\frac{3}{2}r + \frac{9}{16} = 4$ and $r = \boxed{\frac{55}{24}}.$

C) Each of the parts of the equation seem to be an evaluated distance formula, so consider for points $A = (0, 0)$, $B = (2, y), C = (x, 2), D = (4, 3)$:



So the function $f(x, y)$ is the sum of $AB + BC + CD$, which has a minimum value of the line of the length of the line AD. If $x = \frac{8}{3}$ and $y = \frac{3}{2}$, then we achieve this minimum distance of $\boxed{5.}$

D) The sequence is $2025, 15, 4, 3, 2, 2, 2, ...$ with all remaining entries begin 2. So the sum of the 2025 terms would be $2025 + 15 + 4 + 3 + 2 \cdot 2021 = \boxed{6089}$

10. **A =** 1080 **B =** 7 **C =** 6 **D =** 43

A) Since $2025 = 45^2 = 3^2 \cdot 5$, we have $\varphi(2025) = 2025\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right) = 2025\left(\frac{2}{3}\right)\left(\frac{4}{5}\right) = \boxed{1080.}$

B) Expanding to a long sum, we have:

$$\sum_{k=0}^{3}\varphi(a\cdot 10^k) = \varphi(a)+\varphi(10a)+\varphi(100a)+\varphi(1000a) = 2670$$

Since a and 10^k are relatively prime, $\varphi(10^k a) = \varphi(10^k)\varphi(a)$, so we have:

$$\varphi(a)+\varphi(10)\varphi(a)+\varphi(100)\varphi(a)+\varphi(1000)\varphi(a) = \varphi(a)(1+\varphi(10)+\varphi(100)+\varphi(1000)) = 2670$$

This yields $445\cdot\varphi(a) = 2670, \varphi(a) = 6$. Noting that is p is prime then $\varphi(p) = p-1$, we have the smallest possible a is $\boxed{7.}$

C) Note that $\varphi(7) = 6$, so $a^7 \equiv a \pmod{n}$ for all a. Thus, the second sum has the same residue modulo 7 as the first sum, which is $\boxed{6.}$

D) We reframe this question in the terms of $43^{2025} \pmod{100}$, and note that $\varphi(100) = 40$. Since 43 and 100 are relatively prime, we can reduce the exponent modulo 40: $2025 - 40\cdot 50 = 2025 - 2000 = 25$. So $43^{2025} \equiv 43^{25} \pmod{100}$ We can then use successive squaring to yield the final two digits:

$$\begin{aligned}
43^2 = 1849 &\implies 49 \pmod{100}\\
43^4 \equiv 49^2 \pmod{100} &\implies 1 \pmod{100}\\
(43^4)^6 \equiv 43^{24} \equiv 1^6 \pmod{100} &\implies 1 \pmod{100}
\end{aligned}$$

Hence, $43^{25} \equiv \boxed{43} \pmod{100}$.

11. **A =** $2\sqrt{2}$ **B =** 46π **C =** $\dfrac{9}{7}$ **D =** $\dfrac{2}{3}$

A) We can remove a scalar of 3 from the first vector to make our calculations easier. From angle between vectors, we have:

$$\cos\theta = \frac{\langle -1,2,2\rangle\cdot\langle 3,4,0\rangle}{\|\langle -1,2,2\rangle\|\cdot\|\langle 3,4,0\rangle\|} = \frac{1}{3}$$

Since $\sin\theta = \pm\sqrt{1-\cos^2\theta} = \pm\frac{2\sqrt{2}}{3}$. Since we are looking for the smaller angle, we use positive sine, and we get:

$$\tan\theta = \frac{\frac{2\sqrt{2}}{3}}{\frac{1}{3}} = \boxed{2\sqrt{2}}$$

B) Let the shaded area be R. Let the large circle's area be C_{10}, and let the small circle's area be C_A. Then by symmetry, $2R + C_A = C_{10}$. Therefore our value is $A = \frac{1}{2}(100\pi - 8\pi) = \boxed{46\pi.}$

C) So, although it is a relay-style question, you can break the cycle here. If we set up that

$$B + 2\sum_{n=1}^{\infty} B\left(\frac{1}{C}\right)^n = 8B \implies \sum_{n=1}^{\infty}\left(\frac{1}{C}\right)^n = \frac{7}{2}$$

So by the geometric series sum formula, we have:

$$\frac{\frac{1}{C}}{1-\frac{1}{C}} = \frac{7}{2} \implies \frac{C}{C-1} = \frac{7}{2} \implies C = \boxed{\frac{9}{7}}$$

D) From the statement, we have $16D - 5(1-D) = 7C \xRightarrow{C=\frac{9}{7}} 16D - 5 + 5D = 9 \implies D = \boxed{\frac{2}{3}}$

12. **A** $= 5$ **B** $= 35$ **C** $= \dfrac{11}{5}$ **D** $= 19$

A) Consider $243 \equiv 1 \pmod{11}$ so $100A + 60 + B \equiv 10 \pmod{11}$. Since 60 is already equivalent to $5 \pmod{11}$, then $100A + B = 99A + A + B \equiv A + B \equiv \boxed{5} \pmod{11}$.

B) If all of the other elements of S are greater than 8, then the median is 8. If all of the other elements are less than 2, then the median is 2. No three integers exist such that 1, 13, or any other integer less than 2 or greater than 8 can be the median. However, any integer between 2 and 8 (inclusive) can be the median of S, so the answer is $\boxed{35}$

C) The sum of the roots is $\frac{12}{10} = \frac{6}{5} = \sin\alpha + \cos\alpha$, which if we square we get:

$$\tfrac{36}{25} = (\sin\alpha + \cos\alpha)^2 \implies \tfrac{36}{25} = 1 + 2\sin\alpha\cos\alpha \implies \tfrac{11}{50} = \sin\alpha\cos\alpha$$

We are looking to find k, which $\dfrac{k}{10} = \sin\alpha\cos\alpha$, so $\dfrac{k}{10} = \dfrac{11}{50} \implies k = \boxed{\dfrac{11}{5}}$

D) Consider the proportions of books that are remaining are 1, $1 - \frac{m}{n}$, and $1 - \frac{m}{n} - \frac{1}{m}(1 - \frac{m}{n})$. Since this sequence is arithmetic, the differences must be equal:

$$1 - \left(1 - \frac{m}{n}\right) = 1 - \frac{m}{n} - \left(1 - \frac{m}{n} - \frac{1}{m}\left(1 - \frac{m}{n}\right)\right) \implies \frac{m}{n} = \frac{1}{m} - \frac{1}{n}$$

Multiplying by mn, we get $n = m^2 + m$ and therefore $\dfrac{m}{n} = \dfrac{1}{m+1}$. Therefore, there the number of solutions for m is equivalent to the number of factors where $\dfrac{240}{m+1}$ is an integer, which yields $\boxed{19}$ as $m \neq 0$.

13. A $=7$ **B** $=128$ **C** $=286$ **D** $=\frac{48}{143}$

A) Note that no permutation of $(1,2,3)$ works. Try the numbers $1,2,4$ since 4 is a square number and likely to work. The expression $\sqrt{2\sqrt{4\sqrt{1}}}$ gives the integer 2. Therefore the sum is $1+2+4=\boxed{7}$

B) Take $\log_2$ of every single equation. The first one simplifies to $\log_2 a+b=3$. The second becomes $b\log_2 a=1$. The last equation becomes $(\log_2 a)^2+b^2$. Note that this is equal to $(\log_2 a+b)^2-2\cdot b\log_2 a$ which equals 7. $\log_2$ of the desired answer is 7, so it must be $\boxed{128}$

C) By Vieta's, the sum of the roots is zero. If three prime numbers (assuming negative integers count in this case) add to zero, one of them must equal 2 or -2. This is because all primes other than two are odd. $O+O=E$, so the remaining prime must be even in order for the triple sum to be zero. Using Vieta's again gives that the sum of the squares of the roots is $0-2\cdot(-147)=294$. Since squares are positive regardless of the sign of the roots, we simply need to find two prime integers a and b such that $a^2+b^2=294-2^2=290$. The numbers 11 and 13 are the desired answers because for the 1 and 17 solution, 1 is not prime. Since the question asks for $|c|$, the sign doesn't matter. The product $2\cdot 11\cdot 13$ gives the answer $\boxed{286}$

D) This problem is equivalent to solving for the probability that no man is standing diametrically opposite to another man. Place the 1st man anywhere on the circle, now place the 2nd man somewhere around the circle such that he is not diametrically opposite to the first man. This can happen with a probability of $\frac{12}{13}$ because there are 13 available spots, and 12 of them are not opposite to the first man. Continue similarly for every other man.

$$\frac{12}{13}\cdot\frac{10}{12}\cdot\frac{8}{11}\cdot\frac{6}{10}=\boxed{\frac{48}{143}}$$