Unless otherwise specified, trigonometric functions can take any real input. Good luck, have fun, and as always: "NOTA" stands for "None of These Answers are correct."

1. Find the number of solutions to $\sin(x) = |\cos(x)|$ for $x \in [0, 2\pi]$.

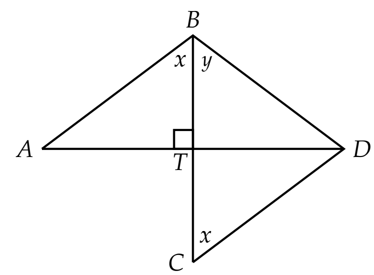
(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** NOTA

2. Solve: $\sin(-x) < |\cos(x)|$ for $x \in [0, 2\pi]$

(A) $\left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$ **(C)** $\left(\frac{7\pi}{4}, 2\pi\right)$ **(E)** NOTA

(B) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ **(D)** $\left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$

3. In the figure below, $AB = 16$, $\overline{BC} \perp \overline{AD}$, $\angle ABC = \angle BCD = x$ and $\angle CBD = y$. Find CD if $x = \sin^{-1}(\frac{2}{3})$ and $y = \sin^{-1}(\frac{1}{3})$.



(A) $2\sqrt{3}$ **(B)** $2\sqrt{10}$ **(C)** $4\sqrt{5}$ **(D)** $\frac{2\sqrt{10}}{3}$ **(E)** NOTA

4. For what value of a does the following system of equations have infinite solutions?

$$\begin{cases} 6x + y = 45 \\ ax + 45y = 2025 \end{cases}$$

(A) 6 **(B)** 270 **(C)** 45 **(D)** 2025 **(E)** NOTA

5. For how many positive integers k does the inequality $2023 < kx < 2025$ have an integer solution for x?

(A) 16 **(B)** 14 **(C)** 12 **(D)** 15 **(E)** NOTA

6. Find the smallest value of k such that

$$\cot\left(\frac{x}{4}\right) - \cot x = \frac{\sin(kx)}{\sin\left(\frac{x}{4}\right)\cdot\sin x}$$

(A) 2 **(B)** $\frac{1}{4}$ **(C)** $\frac{3}{4}$ **(D)** 2 **(E)** NOTA

7. Give the interval for all x where $x > \dfrac{1}{x}$?

(A) $(1,\infty)$ **(B)** $(-\infty,-1)$ **(C)** $x \neq 1$ **(D)** $(-\infty,-1)\cup(1,\infty)$ **(E)** NOTA

8. Find the number of real roots of $\sum_{n=0}^{2025} x^n$.

(A) 2025 **(B)** 2 **(C)** 1 **(D)** 0 **(E)** NOTA

9. For the orthogonal vectors below, find the sum of the distinct possible values of x.

$$\langle x^2, x-1, x+1\rangle \text{ and } \langle x-3, 2, 1\rangle$$

(A) -1 **(B)** 0 **(C)** 1 **(D)** 3 **(E)** NOTA

10. Given that $y = x^2 - 4x + 9$ intersects the line $y = 5x + b$ exactly once, then b can be written in the form $\frac{p}{q}$ for relatively prime integers p, q with $q > 0$. Find $p + q$.

(A) -41 **(B)** -43 **(C)** -45 **(D)** -47 **(E)** NOTA

11. What does the equation $x^2 + 9y^2 = 6xy + 2025$ form in the cartesian plane?

(A) An Ellipse
(B) Two Parabolas
(C) Two Lines
(D) A Hyperbola
(E) NOTA

12. Solve $x^4 - 6x^3 + 8x^2 < 0$

(A) $(-\infty, 0) \cup (2, 4)$ **(C)** $(4, \infty)$ **(E)** NOTA

(B) $(2, 4)$ **(D)** $(0, 2) \cup (4, \infty)$

13. Find $a + b$ given that $\frac{1}{a} + \frac{2}{b} = 20$ and $\frac{2}{a} + \frac{3}{b} = 25$.

(A) $-\dfrac{1}{30}$ **(B)** $\dfrac{1}{30}$ **(C)** $-\dfrac{1}{15}$ **(D)** $\dfrac{1}{15}$ **(E)** NOTA

14. Find the area in the Cartesian Plane of points satisfying $x^2 + y^2 < 25$ and $|x| + |y| > 5$.

(A) $25\pi - 25$ **(C)** 25π **(E)** NOTA

(B) $25\pi - 50$ **(D)** 25

15. The inequality $\sqrt{1 + \sin 2x} > \dfrac{\cos 2x}{\cos x + \sin x} + 1$ is satisfied over some interval (a, b) with $0 \leq a, b \leq \pi$. Find the length of this interval.

(A) $\dfrac{5\pi}{12}$ **(B)** $\dfrac{7\pi}{12}$ **(C)** $\dfrac{\pi}{2}$ **(D)** $\dfrac{\pi}{4}$ **(E)** NOTA

16. Given that the quartic $ax^4 + bx^3 + cx^2 + bx + a$ with real coefficients a, b, c, has at least one positive real root, find the minium sum of the positive real roots (double roots count as two roots).

(A) 4 **(B)** 3 **(C)** 2 **(D)** 1 **(E)** NOTA

17. How many ordered pairs of positive integers (a, b) are there that satisfy $3a + ab = 2b + 2031$?

(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** NOTA

18. How many complex numbers z satisfy the equation $z^2 = \overline{z}$, where $\overline{z}$ is the complex conjugate of z?

(A) 3 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** NOTA

19. Let $f(x)$ be the monic cubic polynomial such that $f(21) = f(37) = f(71) = 2025$. Find the sum of the coefficients of $f(x)$.

(A) -50400 **(B)** -52425 **(C)** -48325 **(D)** -48375 **(E)** NOTA

20. Six numbers a, b, c, d, e, f are chosen with replacement from the set $\{1, 2, 3, 4, 5, 6\}$. Find the probability that the system of equations $ax + by = c, dx + ey = f$ has exactly one solution.

(A) $\frac{22}{25}$ **(B)** $\frac{4}{5}$ **(C)** $\frac{9}{10}$ **(D)** $\frac{114}{125}$ **(E)** NOTA

21. Find the sum of the real roots of $x^3 + 6x^2 + 12x - 1$.

(A) -6 **(B)** $\sqrt[3]{7} - 1$ **(C)** $\sqrt[3]{9} - 2$ **(D)** $\sqrt[3]{11} - 2$ **(E)** NOTA

22. A certain function with integral domain satisfies the following for $0 \le a \le n$:

$$f(n) = f(a) + f(n-a) = a(n-a).$$

Given that $f(2) = 1.5$, evaluate $\sum_{i=0}^{20} f(i)$.

(A) 1995 **(B)** 2025 **(C)** 1785 **(D)** 1330 **(E)** NOTA

23. Find the expected number of rounds played of a three-way rock-paper-scissors match. Players select their option randomly, and if one player chooses a losing option, they are eliminated and a sub-match will be played between the remaining two players. This system will occur until one player is left standing. In the case where all three players lose, it is considered a tie and the players replay.

(A) 2 **(B)** $\frac{11}{4}$ **(C)** $\frac{9}{4}$ **(D)** $\frac{5}{2}$ **(E)** NOTA

24. A random point is chosen on the unit circle that is not on the x-axis. The line segment from this point to the origin serves as the hypotenuse of a right triangle, and the line segment drawn straight down to the x-axis is one of its legs. Find the probability that the area of this triangle is greater than the positive difference between the length of its hypotenuse and its longer leg.

(A) $\frac{1}{3}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{3}{4}$ **(D)** 1 **(E)** NOTA

25. Find the number of real solutions to $\sin(2025x) = 2025x$

(A) 1 **(B)** 2025 **(C)** 3 **(D)** 5 **(E)** NOTA

26. Which of the following equations has the least real solution?

(A) $x^{21} + 2025x^{25} + 1 = 0$

(B) $x^{19} + 2025x^{25} + 1 = 0$

(C) $x^{21} + 2025x^{27} + 1 = 0$

(D) $x^{19} + 2025x^{27} + 1 = 0$

(E) $2025x^2 - 1 = 0$

27. Find the minimum value of $\dfrac{x \cdot 3^{2x} \cdot 2^x + 5x^2 \cdot 6^x}{54^x}$ for real x.

(A) $-\dfrac{1}{10}$ **(B)** $-\dfrac{1}{20}$ **(C)** $-\dfrac{1}{30}$ **(D)** $-\dfrac{1}{40}$ **(E)** NOTA

28. The solutions to the equation $(x^3 - 2x^2)^4 = x^4(x-2)$ are plotted as points in the complex plane. These points can be connected to form the vertices of two distinct convex regular polygons, with no point acting as a vertex of both polygons. Find the area of overlap of these two polygons.

(A) $\dfrac{\sqrt{2}}{2}$ **(B)** $\sqrt{2}$ **(C)** $\sqrt{3}$ **(D)** $\dfrac{\sqrt{3}}{2}$ **(E)** NOTA

29. The equation $x^4 + 13x^2 + 8x + 33 = 0$ has four complex solutions. Find the sum of the squares of the magnitudes of the imaginary components for each of these solutions.

(A) -13 **(B)** -23 **(C)** -25 **(D)** -27 **(E)** NOTA

30. When does $x > 0$ *imply* $x > 1$?

(A) Always

(B) Never

(C) Sometimes

(D) Not Enough Information

(E) NOTA