Answer Key:

1. **C**
2. **B**
3. **D**
4. **D**
5. **B**
6. **A**
7. **D**
8. **E**
9. **D**
10. **E**
11. **C**
12. **C**
13. **B**
14. **A**
15. **B**
16. **D**
17. **B**
18. **A**
19. **B**
20. **D**
21. **D**
22. **B**
23. **D**
24. **B**
25. **D**
26. **B**
27. **C**
28. **A**
29. **D**
30. **C**

Solutions:

1. $\boxed{\textbf{C}}$: To evaluate $f(f(4))$ where $f(x) = (x-2)^2 + 1$:

Step 1: Find $f(4)$ $f(4) = (4-2)^2 + 1 = 2^2 + 1 = 4 + 1 = 5$

Step 2: Find $f(f(4))$, which means $f(5)$ $f(5) = (5-2)^2 + 1 = 3^2 + 1 = 9 + 1 = 10$

Therefore, $f(f(4)) = 10$

2. $\boxed{\textbf{B}}$: To find the value of $\sin\theta + \cos\theta$ where $\theta = -\frac{\pi}{4}$:

Step 1: Calculate $\sin\theta$
$\sin\left(-\frac{\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right)$ (using the property $\sin(-x) = -\sin(x)$)
$\sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$
Step 2: Calculate $\cos\theta$
$\cos\left(-\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right)$ (using the property $\cos(-x) = \cos(x)$)
$\cos\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$
Step 3: Find $\sin\theta + \cos\theta$
$\sin\left(-\frac{\pi}{4}\right) + \cos\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = 0$
Therefore, $\sin\theta + \cos\theta = 0$

3. $\boxed{\textbf{D}}$: From the problem statement: $r = (\text{cost for 3 months}) - 2600$

Since the cost for 3 months would be $3r$:
$r = 3r - 2600$

Solving for r:
$r - 3r = -2600$
$-2r = -2600$
$r = 1300$

So the monthly rent is $1300.

Therefore, the cost to rent the apartment for a year would be:
$12 \cdot 1300 = 15,600$

4. $\boxed{\textbf{D}}$: To evaluate $\cos^{-1}\left(\cos\left(-\frac{11\pi}{12}\right)\right)$:

Step 1: Simplify the inner expression using the even property of cosine.
$\cos\left(-\frac{11\pi}{12}\right) = \cos\left(\frac{11\pi}{12}\right)$ (since $\cos(-\theta) = \cos(\theta)$)

Step 2: Apply the inverse cosine function.
When applying $\cos^{-1}$ to $\cos(\theta)$, we get the principal value in the range $[0, \pi]$.

Since $\frac{11\pi}{12}$ is already in the range $[0, \pi]$, we have:
$\cos^{-1}\left(\cos\left(\frac{11\pi}{12}\right)\right) = \frac{11\pi}{12}$

Therefore:
$\cos^{-1}\left(\cos\left(-\frac{11\pi}{12}\right)\right) = \frac{11\pi}{12}$

5. $\boxed{\textbf{B}}$: To find the value of $A - B$ from the given sequences:

Step 1: Identify the patterns in the sequences.
Sequence A consists of the first 100 even numbers (from 2 to 200)
Sequence B consists of the first 100 natural numbers (from 1 to 100)

Step 2: Calculate the sum of sequence A.
For the sum of the first n even numbers, we can use the formula $n(n+1)$ where $n = 100$:
$A = 100 \cdot 101 = 10,100$

Step 3: Calculate the sum of sequence B.
For the sum of the first n natural numbers, we can use the formula $\frac{n(n+1)}{2}$ where $n = 100$:
$B = \frac{100\cdot 101}{2} = 5,050$

Step 4: Find the difference $A - B$.
$A - B = 10,100 - 5,050 = 5,050$

Alternatively, note that sequence A can be written as $2(1 + 2 + 3 + ... + 100) = 2B$
Therefore, $A - B = 2B - B = B = 5,050$

6. $\boxed{\textbf{A}}$: To find an angle that is coterminal with $-\frac{\pi}{6}$, identify which of the given options differs from $-\frac{\pi}{6}$ by an integer multiple of 2π.

Let me check each option:

$-\frac{433\pi}{6}$:
$-\frac{433\pi}{6} - \left(-\frac{\pi}{6}\right) = -\frac{433\pi}{6} + \frac{\pi}{6} = -\frac{432\pi}{6} = -72\pi = -36 \cdot 2\pi$
Since this difference is an integer multiple of 2π, option (A) is coterminal with $-\frac{\pi}{6}$.

$-\frac{103\pi}{6}$:
$-\frac{103\pi}{6} - \left(-\frac{\pi}{6}\right) = -\frac{103\pi}{6} + \frac{\pi}{6} = -\frac{102\pi}{6} = -17\pi$
Since -17π is not an integer multiple of 2π, this angle is not coterminal with $-\frac{\pi}{6}$.

$\frac{67\pi}{3}$:
$\frac{67\pi}{3} - \left(-\frac{\pi}{6}\right) = \frac{67\pi}{3} + \frac{\pi}{6} = \frac{134\pi+\pi}{6} = \frac{135\pi}{6} = \frac{45\pi}{2}$
Since $\frac{45\pi}{2}$ is not an integer multiple of 2π, this angle is not coterminal with $-\frac{\pi}{6}$.

$-\frac{383\pi}{6}$:
$-\frac{383\pi}{6} - \left(-\frac{\pi}{6}\right) = -\frac{383\pi}{6} + \frac{\pi}{6} = -\frac{382\pi}{6}$
To check if this is a multiple of 2π, I'll divide by 2π:
$-\frac{382\pi}{6} \div 2\pi = -\frac{382}{12} = -\frac{191}{6}$
Since $-\frac{191}{6}$ is not an integer, this angle is not coterminal with $-\frac{\pi}{6}$.

Therefore, $-\frac{433\pi}{6}$ is coterminal with $-\frac{\pi}{6}$.

7. $\boxed{\textbf{D}}$: To evaluate $(\vec{v} \cdot \vec{v}) \cdot (\vec{u} \cdot \vec{u})$ where $\vec{v} = \langle 4, 5, -6\rangle$ and $\vec{u} = \langle -3, 2, 1\rangle$:

Step 1: Calculate $\vec{v} \cdot \vec{v}$.
$\vec{v} \cdot \vec{v} = 4^2 + 5^2 + (-6)^2 = 16 + 25 + 36 = 77$

Step 2: Calculate $\vec{u} \cdot \vec{u}$.
$\vec{u} \cdot \vec{u} = (-3)^2 + 2^2 + 1^2 = 9 + 4 + 1 = 14$

Step 3: Calculate the final expression.
$(\vec{v} \cdot \vec{v}) \cdot (\vec{u} \cdot \vec{u}) = 77 \cdot 14 = 1078$

Therefore, $(\vec{v} \cdot \vec{v}) \cdot (\vec{u} \cdot \vec{u}) = 1078$

8. $\boxed{\textbf{E}}$: To evaluate $\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^3$:

Step 1: Express the complex number in polar form. For the given complex number $z = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$, we can recognize that $|z| = 1$ and $\arg(z) = \frac{3\pi}{4}$.

Therefore, $z = 1 \cdot (\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4})$

Step 2: Apply De Moivre's formula: $z^n = r^n(\cos(n\theta) + i\sin(n\theta))$
$z^3 = 1^3 \cdot \left(\cos\left(3 \cdot \frac{3\pi}{4}\right) + i \sin\left(3 \cdot \frac{3\pi}{4}\right)\right)$
$= \cos\left(\frac{9\pi}{4}\right) + i \sin\left(\frac{9\pi}{4}\right)$

Step 3: Simplify by noting that $\frac{9\pi}{4} = \frac{\pi}{4} + 2\pi$, and since the trig functions repeat every 2π:
$z^3 = \cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)$
$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$

Therefore, $\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^3 = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$

9. **D** : To find the probability that a person who thinks their job will be replaced by A.I. also thinks their job will be replaced by robots:

This is a conditional probability problem where we need to find P(Robots | A.I.).

Step 1: Identify the relevant counts from the survey data.

- Total people surveyed: 120
- People who think A.I. will replace their job: 60
- People who think both A.I. and robots will replace their job: 20

Step 2: Calculate the conditional probability using the formula:
P(Robots | A.I.) = $\frac{\text{P(Robots} \cap \text{A.I.)}}{\text{P(A.I.)}} = \frac{\text{Number who think both A.I. and robots}}{\text{Number who think A.I.}}$

Step 3: Substitute the values:
P(Robots | A.I.) = $\frac{20}{60} = \frac{1}{3}$

Therefore, the probability that a person who thinks their job will be replaced by A.I. also thinks their job will be replaced by robots is $\frac{1}{3}$.

10. **E** : To find the probability that a randomly selected person thinks that neither their job will be replaced by A.I. nor will be replaced by robots:

Step 1: Identify the relevant information from the survey data.

- Total people surveyed: 120
- People who think A.I. will replace their job: 60
- People who think robots will replace their job: 50
- People who think both A.I. and robots will replace their job: 20

Step 2: Calculate the total number of people who think at least one (A.I. or robots) will replace their job using the principle of inclusion-exclusion.
Number who think at least one will replace their job = (A.I.) + (Robots) - (Both)
Number who think at least one will replace their job = 60 + 50 - 20 = 90

Step 3: Calculate the number of people who think neither will replace their job.
Number who think neither will replace their job = Total - Number who think at least one will replace their job
Number who think neither will replace their job = 120 - 90 = 30

Step 4: Calculate the probability by dividing by the total number of people surveyed.
P(Neither) = $\frac{30}{120} = \frac{1}{4}$

Therefore, the probability that a randomly selected person thinks that neither their job will be replaced by A.I. nor by robots is $\frac{1}{4}$.

11. **C** : To find the coefficient of the x term in the expansion of $(x^3 + \frac{2}{x})^7$:

Step 1: Use the binomial theorem to expand $(x^3 + \frac{2}{x})^7$:
$(x^3 + \frac{2}{x})^7 = \sum_{k=0}^{7} \binom{7}{k} (x^3)^{7-k} (\frac{2}{x})^k$

Step 2: Find which term has the power of x^1 by solving:
$(x^3)^{7-k}(\frac{2}{x})^k = x^1 \Rightarrow 3(7-k) + (-1)k = 1$ (from exponent rules)

$21 - 4k = 1$
$-4k = -20$
$k = 5$

Step 3: Calculate the coefficient when $k = 5$:
$\binom{7}{5}(x^3)^2(\frac{2}{x})^5 = \binom{7}{5} \cdot 2^5 \cdot x^1$

Step 4: Evaluate this coefficient:
$\binom{7}{5} = \frac{7!}{5!(7-5)!} = \frac{7!}{5!2!} = \frac{7\cdot6}{2\cdot1} = 21$

$\binom{7}{5} \cdot 2^5 = 21 \cdot 32 = 672$

Therefore, the coefficient of the x term in the expansion of $(x^3 + \frac{2}{x})^7$ is 672.

12. **C** : To find the domain of $f(g(x))$ if $f(x) = \sqrt{x^2 - 64}$ and $g(x) = \ln(x-2)$:

Step 1: Find the domain of $g(x) = \ln(x-2)$.
For the natural logarithm to be defined, we need:
$x - 2 > 0$
$x > 2$
So the domain of g is $(2, \infty)$

Step 2: Find the domain of $f(x) = \sqrt{x^2 - 64}$.
For the square root to be defined, we need:
$x^2 - 64 \geq 0$
$x^2 \geq 64$
$|x| \geq 8$
$x \leq -8$ or $x \geq 8$
So the domain of f is $(-\infty, -8] \cup [8, \infty)$

Step 3: For the composition $f(g(x))$, we need:

- x must be in the domain of g, so $x > 2$
- $g(x)$ must be in the domain of f, so either:
  - $g(x) \leq -8$ or $g(x) \geq 8$
  - That is: $\ln(x-2) \leq -8$ or $\ln(x-2) \geq 8$

Step 4: Solve these inequalities:
For $\ln(x-2) \geq 8$:
$x - 2 \geq e^8$
$x \geq e^8 + 2$

For $\ln(x-2) \leq -8$:
$x - 2 \leq e^{-8}$
$x \leq e^{-8} + 2$

Since we already know $x > 2$, this gives us a very narrow range $(2, e^{-8} + 2]$.

Therefore, the complete domain of $f(g(x))$ is $(2, e^{-8} + 2] \cup [e^8 + 2, \infty)$

13. **B** : To identify an equation that represents the graph shown:

Given the graph with these characteristics:

- Horizontal lines at $y = 0$, $y = 1$, and $y = 3$
- At $x = 0$, the graph value is $y = 1$
- The slope of the graph at x=0 is positive
- The crests of the graph align perfectly along $y = 1$ and $y = 3$
- The entire graph lies above the x-axis

Whichever option matches all these features is the correct graph.

Step 1: Analyze the value at $x = 0$ for each function.

- If a function gives $y = 1$ at $x = 0$, it remains a candidate
- Any function that gives a different value at $x = 0$ can be eliminated

Step 2: Check the slope at $x = 0$.

- The slope must be positive at $x = 0$
- This can be determined by finding the derivative and evaluating it at $x = 0$

Step 3: Verify the maximum and minimum values.

- The function should have crests at $y = 3$
- The function should have smaller crests at $y = 1$
- The function should never go below the x-axis

Step 4: For functions with absolute values, carefully analyze how the absolute value affects the range.

Let's evaluate $|2\sin x + 1|$:

- At $x = 0 : |2\sin(0) + 1| = |0 + 1| = 1$✓
- Slope at $x = 0$: The derivative is $2\cos(0) = 2 > 0$ (since $2\sin(0) + 1 > 0$) ✓
- Crest value: When $\sin x = 1$, we get $|2(1) + 1| = |3| = 3$ ✓
- Smaller crest value: When $\sin x = -1$, we get $|2(-1) + 1| = |-1| = 1$ ✓
- Range is, always above x-axis ✓

Therefore, $|2\sin x + 1|$ matches all the characteristics of the given graph.

14. **A** : To find the length of side YZ in triangle XYZ:

Given:

- $XY = 8$ units
- $XZ = 6$ units
- Angle $X = 120°$

Using the Law of Cosines to find the unknown side length:

$YZ^2 = XY^2 + XZ^2 - 2(XY)(XZ)\cos(\angle X)$

Substituting the given values:
$YZ^2 = 8^2 + 6^2 - 2(8)(6)\cos(120)$
$YZ^2 = 64 + 36 - 2(48)(-0.5)$
$YZ^2 = 100 + 48$
$YZ^2 = 148$
$YZ = \sqrt{148}$
$YZ = \sqrt{4 \cdot 37}$
$YZ = 2\sqrt{37}$

Therefore, the length of side YZ is $2\sqrt{37}$ units.

15. **B** : To find the area of triangle XYZ:

The area of a triangle can be calculated using the formula:
$A = \frac{1}{2} \cdot ab \cdot \sin(C)$

Where a and b are two sides, and C is the angle between them.

Substituting the given values:
$A = \frac{1}{2} \cdot 8 \cdot 6 \cdot \sin(120)$
$A = \frac{1}{2} \cdot 48 \cdot \sin(120)$

Since $\sin(120) = \sin(180 - 60) = \sin(60) = \frac{\sqrt{3}}{2}$:
$A = 24 \cdot \frac{\sqrt{3}}{2}$
$A = 12\sqrt{3}$

Therefore, the area of triangle XYZ is $12\sqrt{3}$ square units.

16. **D** : To find the oblique asymptote of the rational function $\frac{2x^3-14x^2+3x-5}{x^2+6}$:

Step 1: Since the degree of the numerator (3) exceeds the degree of the denominator (2) by exactly 1, this rational function has an oblique asymptote of the form $y = mx + b$.

Step 2: Perform polynomial long division to find the quotient and remainder.
Dividing $2x^3 - 14x^2 + 3x - 5$ by $x^2 + 6$:

$2x^3 \div (x^2 + 6) = 2x$ (first term of quotient)
$2x(x^2 + 6) = 2x^3 + 12x$
$(2x^3 - 14x^2 + 3x - 5) - (2x^3 + 12x) = -14x^2 - 9x - 5$

$-14x^2 \div (x^2 + 6) = -14$ (second term of quotient)
$-14(x^2 + 6) = -14x^2 - 84$
$(-14x^2 - 9x - 5) - (-14x^2 - 84) = -9x + 79$ (remainder)

Step 3: Write the function in the form of quotient plus remainder:
$\frac{2x^3-14x^2+3x-5}{x^2+6} = 2x - 14 + \frac{-9x+79}{x^2+6}$

Step 4: Identify the oblique asymptote from the quotient:
As $x \to \pm\infty$, the term $\frac{-9x+79}{x^2+6} \to 0$

Therefore, the oblique asymptote of the given rational function is $y = 2x - 14$.

17. **B** : To determine on which domain the function $f(x) = x^3 - 4x$ is odd:

Step 1: Check if the function is algebraically odd by verifying if $f(-x) = -f(x)$.
$f(-x) = (-x)^3 - 4(-x) = -x^3 + 4x = -(x^3 - 4x) = -f(x)$

Since $f(-x) = -f(x)$, this function is algebraically odd.

Step 2: For a function to be odd on a specific domain, two conditions must be met:

- The domain must be symmetric about the origin
- Both x and $-x$ must be in the domain for all x in the domain

Step 3: Analyze each given domain:

(A) $(-1, 6)$
This domain is not symmetric about the origin. For example, 5 is in the domain, but -5 is not.

(B) $(-8, 8)$
This domain is symmetric about the origin. For any x in $(-8, 8)$, $-x$ is also in $(-8, 8)$.

(C) $(2, 43)$
This domain contains only positive numbers (except for 0), so it's not symmetric about the origin.

(D) $(-\infty, 500)$
This domain is not symmetric about the origin. For example, 499 is in the domain, but -499 is in the domain while 499 is not.

Therefore, the function $f(x) = x^3 - 4x$ is odd on the domain $(-8, 8)$.

18. **A** : To determine on which domain the function $f(x) = x^3 - 4x$ is monotonically decreasing:

Step 1: Identify the three intervals created by the vertices:

- Interval 1: $x < -\frac{2}{\sqrt{3}}$
- Interval 2: $-\frac{2}{\sqrt{3}} < x < \frac{2}{\sqrt{3}}$
- Interval 3: $x > \frac{2}{\sqrt{3}}$

Step 2: Analyze the behavior of the function in each interval.
To determine if the function is increasing or decreasing in each interval, evaluate the function at different points.

Interval 1: $x < -\frac{2}{\sqrt{3}}$
Test points: $x = -2$ and $x = -3$
$f(-2) = (-2)^3 - 4(-2) = -8 + 8 = 0$
$f(-3) = (-3)^3 - 4(-3) = -27 + 12 = -15$
Since $f(-3) < f(-2)$, the function is increasing in this interval.

Interval 2: $-\frac{2}{\sqrt{3}} < x < \frac{2}{\sqrt{3}}$
Test points: $x = -1$ and $x = 0$
$f(-1) = (-1)^3 - 4(-1) = -1 + 4 = 3$
$f(0) = 0^3 - 4(0) = 0$
Since $f(0) < f(-1)$, the function is decreasing in this interval.

Interval 3: $x > \frac{2}{\sqrt{3}}$
Test points: $x = 2$ and $x = 3$
$f(2) = 2^3 - 4(2) = 8 - 8 = 0$
$f(3) = 3^3 - 4(3) = 27 - 12 = 15$
Since $f(2) < f(3)$, the function is increasing in this interval.

Step 3: Examine each given domain to determine where the function is monotonically decreasing.

(A) $(-1, 0)$: Since $-1 > -\frac{2}{\sqrt{3}}$ and $0 < \frac{2}{\sqrt{3}}$, this domain is entirely within Interval 2, where the function is decreasing.

(B) $(0, 2)$: Since $\frac{2}{\sqrt{3}} < 2$, this domain crosses from Interval 2 (decreasing) to Interval 3 (increasing).

(C) $(2, \infty)$: Since $2 > \frac{2}{\sqrt{3}}$, this domain is entirely within Interval 3, where the function is increasing.

(D) $(-\sqrt{2}, 500)$: Since $-\sqrt{2} > -\frac{2}{\sqrt{3}}$ and $500 > \frac{2}{\sqrt{3}}$, this domain crosses multiple intervals.

Therefore, the function $f(x) = x^3 - 4x$ is monotonically decreasing on the domain $(-1, 0)$.

19. **B** : To find the volume of the 3D shape created by rotating region H about the y-axis:

The region H is bounded by the lines $y = 2x + 2$, $y = -2x + 2$, and the x-axis ($y = 0$), forming a triangle with vertices at $(-1, 0)$, $(0, 2)$, and $(1, 0)$. When rotated about the y-axis, this triangular region creates a cone.

For this cone:

- The height is the y-coordinate of the top vertex: $h = 2$ units
- The radius of the base is the distance from the y-axis to the vertices on the x-axis: $r = 1$ unit

Substituting values into the formula for the volume of a cone:
$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(1)^2(2) = \frac{2\pi}{3}$

Therefore, the volume of the 3D shape created by rotating region H about the y-axis is $\frac{2\pi}{3}$ cubic units.

20. **D** : To find the smallest value such that $(x^2 + 4x - 14)^2 - (x - 4)^2 = 0$:

Step 1: Set up the equation.
$(x^2 + 4x - 14)^2 - (x - 4)^2 = 0$

Step 2: This can be solved by factoring or using algebraic transformations.
Let $u = x^2 + 4x - 14$ and $v = x - 4$
Then the equation becomes $u^2 - v^2 = 0$
Which gives us $u^2 = v^2$
Therefore $u = \pm v$

Step 3: Solve both cases.
Case 1: $u = v$
$x^2 + 4x - 14 = x - 4$
$x^2 + 3x - 10 = 0$
$x = \frac{-3\pm\sqrt{9+40}}{2} = \frac{-3\pm\sqrt{49}}{2} = \frac{-3\pm 7}{2}$
This gives $x = 2$ or $x = -5$

Case 2: $u = -v$
$x^2 + 4x - 14 = -(x - 4)$
$x^2 + 4x - 14 = -x + 4$
$x^2 + 5x - 18 = 0$
$x = \frac{-5\pm\sqrt{25+72}}{2} = \frac{-5\pm\sqrt{97}}{2}$
This gives $x = \frac{-5+\sqrt{97}}{2}$ or $x = \frac{-5-\sqrt{97}}{2}$

Step 4: Compare all solutions to find the smallest value.
The solutions are: $x = -5$, $x = 2$, $x = \frac{-5+\sqrt{97}}{2}$, and $x = \frac{-5-\sqrt{97}}{2}$

Since $\sqrt{97} > 0$, the smallest value is $x = \frac{-5-\sqrt{97}}{2}$

Therefore, the smallest value of x that satisfies the given equation is $\frac{-5-\sqrt{97}}{2}$.

21. **D** : To determine which sets of vectors are linearly independent, we will analyze each option:

(A) Vectors: $\begin{bmatrix}1\\2\\3\end{bmatrix}$ and $\begin{bmatrix}2\\4\\6\end{bmatrix}$

Notice that the second vector is twice the first vector:
$\begin{bmatrix}2\\4\\6\end{bmatrix} = 2 \cdot \begin{bmatrix}1\\2\\3\end{bmatrix}$
Therefore, these vectors are linearly dependent.

(B) Vectors: $\begin{bmatrix}6\\2\\8\end{bmatrix}$ and $\begin{bmatrix}0\\0\\0\end{bmatrix}$

Any set of vectors containing the zero vector is automatically linearly dependent, since we can express $0 = 0 \cdot v_1 + 1 \cdot 0$.

(C) Vectors: $\begin{bmatrix}4\\9\\-3\end{bmatrix}$, $\begin{bmatrix}-1\\2\\3\end{bmatrix}$, and $\begin{bmatrix}3\\11\\0\end{bmatrix}$

With a quick check, we can see that the third vector equals the sum of the first two vectors:
$\begin{bmatrix}4\\9\\-3\end{bmatrix} + \begin{bmatrix}-1\\2\\3\end{bmatrix} = \begin{bmatrix}3\\11\\0\end{bmatrix}$
Since one vector can be expressed as a linear combination of the others, these vectors are linearly dependent.

(D) Vectors: $\begin{bmatrix}1\\0\\0\end{bmatrix}$, $\begin{bmatrix}1\\1\\0\end{bmatrix}$, and $\begin{bmatrix}1\\1\\1\end{bmatrix}$

The matrix formed by these vectors is:
$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$
This is an upper triangular matrix with non-zero diagonal elements, which means it has full rank and the vectors are linearly independent.

Therefore, option (D) is the only set of linearly independent vectors.

22. $\boxed{\textbf{B}}$: To solve this problem about Victor the alien on a Ferris wheel:

Since Victor starts at the peak height, we will use cosine to model his height.

Let's model Victor's height as:
$h(t) = A\cos(\frac{2\pi t}{T}) + d$

Where A is the amplitude and d is the vertical shift.

Step 1: Find A

At the peak height, we have:
$A + d = 168$ (equation 1)

At the trough (when Victor is at height 20 feet), the cosine equals -1:
$-A + d = 20$ (equation 2)

Solving these equations:
$A + d = 168$
$-A + d = 20$

Subtract the second one from the first one:
$2A = 148$
$A = 74$

Step 2: Find the possible values of T.

At $t = 118$, Victor is at the trough, so:
$\cos(\frac{2\pi \cdot 118}{T}) = -1$

This happens when $\frac{2\pi \cdot 118}{T} = \pi + 2\pi n$ for integer n
$\frac{118}{T} = \frac{1+2n}{2}$
$T = \frac{236}{1+2n}$

For integer values of T, $1 + 2n$ must divide 236 evenly with n being an integer.

Since 236 = $2^2 \cdot 59$, its divisors are: 1, 2, 4, 59, 118, 236

For $1 + 2n$ to equal one of these divisors with integer n:

- 1: $1 + 2n = 1$, so $n = 0$, giving $T = 236$
- 59: $1 + 2n = 59$, so $n = 29$, giving $T = 4$

The sum of A and all possible integer values of T is:
$74 + 4 + 236 = 314$

23. $\boxed{\textbf{D}}$: To identify the type of limaçon represented by the polar equation $r = 7 + 4\sin\theta + 6\cos\theta$:

Step 1: Convert the equation to standard limaçon form $r = a + b\cos(\theta - \alpha)$ or $r = a + b\sin(\theta - \alpha)$.

The terms $4\sin\theta + 6\cos\theta$ can be rewritten as a single sinusoidal term using the identity:
$a\sin\theta + b\cos\theta = R\sin(\theta + \phi)$ or $R\cos(\theta - \phi)$
where $R = \sqrt{a^2 + b^2}$ and $\phi = \arctan(b/a)$ or $\arctan(a/b)$

Step 2: Calculate the amplitude R:
$R = \sqrt{4^2 + 6^2} = \sqrt{16 + 36} = \sqrt{52}$

Step 3: Rewrite the equation in standard form:
$r = 7 + \sqrt{52}\cos(\theta - \alpha)$ where α is some angle

Step 4: Classify the limaçon based on the relationship between the constant term a and the coefficient b:

- If $a > b$: convex limaçon
- If $a = b$: cardioid
- If $0 < a < b$: limaçon with inner loop
- If $a < 0$, $|a| < b$: dimpled limaçon

Since $7 < \sqrt{52}$ (or $a < b$), this is a limaçon with an inner loop.

Therefore, the answer is Inner loop.

24. $\boxed{\textbf{B}}$: To evaluate $\cos(3x)$:

Step 1: Express $\cos(3x)$ as $\cos(2x + x)$ and use sum-to-product identities:
$\cos(3x) = \cos(2x + x) = \cos(2x)\cos(x) - \sin(2x)\sin(x)$

Step 2: Substitute known identities for $\cos(2x)$ and $\sin(2x)$:
$\cos(2x) = 2\cos^2(x) - 1$
$\sin(2x) = 2\sin(x)\cos(x)$

This gives:
$\cos(3x) = (2\cos^2(x) - 1)\cos(x) - 2\sin(x)\cos(x)\sin(x)$
$\cos(3x) = 2\cos^3(x) - \cos(x) - 2\sin^2(x)\cos(x)$

Step 3: Use the Pythagorean identity $\sin^2(x) = 1 - \cos^2(x)$ to simplify:
$\cos(3x) = 2\cos^3(x) - \cos(x) - 2(1 - \cos^2(x))\cos(x)$
$\cos(3x) = 2\cos^3(x) - \cos(x) - 2\cos(x) + 2\cos^3(x)$
$\cos(3x) = 4\cos^3(x) - 3\cos(x)$

Step 4: Compare with the given options and identify the equivalent expression:
$\cos(3x) = 4\cos^3(x) - 3\cos(x)$

Therefore, $\cos(3x) = -3\cos x + 4\cos^3 x$

25. $\boxed{\textbf{D}}$: To solve how much less likely a player is to have a second roll under the new rule:

Step 1: Calculate the probability of getting a second roll under the original rule.
Under the old rule, a player gets a second roll if both dice show the same number.

- Total possible outcomes with 2 dice: $6^2 = 36$
- Number of doubles (1,1), (2,2), ..., (6,6): 6
- Probability of getting a double: $\frac{6}{36} = \frac{1}{6}$

Step 2: Calculate the probability of getting a second roll under the new rule (4 dice). Under the new rule, a player gets a second roll if at least 3 of the 4 dice show the same number.

- Total possible outcomes with 4 dice: $6^4 = 1296$
- Probability of exactly 3 matching dice:
  - Ways to choose the value: 6 choices
  - Ways to choose which 3 dice show this value: $\binom{4}{3} = 4$
  - Ways to choose a different value for remaining die: 5 choices
  - Total favorable outcomes: $6 \cdot 4 \cdot 5 = 120$
- Probability of all 4 matching dice: 6 outcomes
- Total favorable outcomes: $120 + 6 = 126$
- Probability of getting a second roll: $\frac{126}{1296} = \frac{7}{72}$

Step 3: Calculate how much lower the probability that a player gets a second roll under the new rule is.
Subtract the new probability from the old probability:
$\frac{1}{6} - \frac{7}{72} = \frac{5}{72}$
Therefore, the probability that a player gets a second roll under the new rule is $\frac{5}{72}$ lower compared to the original rule.

26. $\boxed{\text{B}}$: To find the velocity vector of the third piece when a cup breaks into three equal pieces:

Step 1: Use the principle of conservation of momentum.
Initial momentum = Final momentum

Step 2: Express this mathematically.
Since the cup with mass $m = 30g$ breaks into three equal pieces (each with mass $m/3 = 10g$):
$m\vec{v} = \frac{m}{3}\vec{v}_1 + \frac{m}{3}\vec{v}_2 + \frac{m}{3}\vec{v}_3$

Step 3: Simplify to solve for $\vec{v}_3$.
$3\vec{v} = \vec{v}_1 + \vec{v}_2 + \vec{v}_3$
$\vec{v}_3 = 3\vec{v} - \vec{v}_1 - \vec{v}_2$

Step 4: Substitute the known values.
$\vec{v} = \langle 4, 2\rangle$
$\vec{v}_1 = \langle -6, 3\rangle$
$\vec{v}_2 = \langle 5, 0\rangle$

Step 5: Calculate $\vec{v}_3$.
$\vec{v}_3 = 3\langle 4, 2\rangle - \langle -6, 3\rangle - \langle 5, 0\rangle$
$\vec{v}_3 = \langle 12, 6\rangle + \langle 6, -3\rangle - \langle 5, 0\rangle$
$\vec{v}_3 = \langle 13, 3\rangle$

Therefore, the velocity vector of the third piece is $\vec{v}_3 = \langle 13, 3\rangle$.

27. $\boxed{\text{C}}$: To find the horizontal range of a projectile fired at 40m/s at an angle of 15° above the horizon with g = 10m/s^2, use the horizontal range formula.

The horizontal range formula can be derived through the following steps:

Step 1: Resolve the initial velocity into components:

- Horizontal component: $v_{0x} = v_0 \cos\theta$
- Vertical component: $v_{0y} = v_0 \sin\theta$

Step 2: Find the time of flight (when the projectile returns to the same height):
For vertical motion: $y = v_{0y}t - \frac{1}{2}gt^2$
When $y = 0$ (returning to launch height):
$0 = v_0 \sin\theta \cdot t - \frac{1}{2}gt^2$
$\frac{1}{2}gt^2 = v_0 \sin\theta \cdot t$
Solving for time (ignoring $t = 0$ which is the starting point):
$t = \frac{2v_0 \sin\theta}{g}$

Step 3: Calculate the horizontal distance using constant horizontal velocity:
$R = v_{0x} \cdot t = v_0 \cos\theta \cdot \frac{2v_0 \sin\theta}{g}$
$R = \frac{2v_0^2 \cos\theta \sin\theta}{g}$
Using the trigonometric identity $2\sin\theta\cos\theta = \sin(2\theta)$:
$R = \frac{v_0^2 \sin(2\theta)}{g}$
This is the standard horizontal range formula for projectile motion. Now we will use it to solve the problem.

Given:

- Initial velocity: $v_0 = 40$ m/s

- Launch angle: $\theta = 15$
- Acceleration due to gravity: $g = 10$ m/s²

Calculating the horizontal range:
$R = \frac{v_0^2 \sin(2\theta)}{g}$
$R = \frac{(40)^2 \sin(2 \cdot 15)}{10}$
$R = \frac{1600 \cdot \sin(30)}{10}$
$R = \frac{1600 \cdot 0.5}{10}$
$R = \frac{800}{10}$
$R = 80$
Therefore, the horizontal range of the projectile is 80 meters.

28. **A** : To find the total amount of cake Jaden brings in:
Step 1: Identify the series pattern.

- First: 1 complete cake
- Second: 2 half cakes = 1 cake
- Third: 3 quarter cakes = $\frac{3}{4}$ cake
- Fourth: 4 eighth cakes = $\frac{1}{2}$ cake
- And so on...

Step 2: Express the total as a sum.
The pattern forms an arithmetico-geometric series where the general term is $(n+1) \cdot \frac{1}{2^n}$
Total $= \sum_{n=0}^{\infty}(n+1) \cdot \frac{1}{2^n} = \sum_{n=0}^{\infty} \frac{n}{2^n} + \sum_{n=0}^{\infty} \frac{1}{2^n}$
Step 3: Evaluate $\sum_{n=0}^{\infty} \frac{1}{2^n}$.
This is a standard geometric series with first term $a = 1$ and ratio $r = \frac{1}{2}$
$\sum_{n=0}^{\infty} \frac{1}{2^n} = \frac{1}{1-\frac{1}{2}} = 2$
Step 4: Evaluate $\sum_{n=0}^{\infty} \frac{n}{2^n}$
This is an arithmetico-geometric series. To find its sum algebraically:
Let $S = \sum_{n=1}^{\infty} \frac{n}{2^n}$ (starting from $n = 1$ for simplicity)
Multiply both sides by $\frac{1}{2}$: $\frac{S}{2} = \sum_{n=1}^{\infty} \frac{n}{2^{n+1}} = \sum_{n=1}^{\infty} \frac{n}{2 \cdot 2^n}$ Subtract the second equation from the first:

$$S - \frac{S}{2} = \sum_{n=1}^{\infty} \frac{n}{2^n} - \sum_{n=1}^{\infty} \frac{n}{2^{n+1}}$$
$$\frac{S}{2} = \sum_{n=1}^{\infty} \frac{n}{2^n} - \sum_{n=1}^{\infty} \frac{n}{2^{n+1}}$$
$$\frac{S}{2} = \sum_{n=1}^{\infty} \frac{n}{2^n} - \frac{1}{2} \sum_{n=1}^{\infty} \frac{n}{2^n}$$
$$\frac{S}{2} = \sum_{n=1}^{\infty} \frac{1}{2^n}$$
$$\frac{S}{2} = \frac{1/2}{1-1/2} = \frac{1/2}{1/2} = 1$$
$$S = 2$$

This calculation for $\sum_{n=0}^{\infty} \frac{n}{2^n}$ can be simplified by using the formula $\frac{r}{(1-r)^2}$ where for this case $r = \frac{1}{2}$. Sum = $\frac{\frac{1}{2}}{(1-\frac{1}{2})^2}$ = $\frac{\frac{1}{2}}{\frac{1}{4}}$ = 2

Step 5: Calculate the total.
Total $= 2 + 2 = 4$

Therefore, the total amount of cake Jaden brings in is 4 complete cakes.

29. **D** : To rotate the conic $7x^2 + 6\sqrt{3}xy + 13y^2 = 16$ to eliminate the xy term:

Step 1: Find the angle of rotation. For a conic section $Ax^2 + Bxy + Cy^2 = D$, the rotation angle θ satisfies:
$\tan(2\theta) = \frac{B}{A-C}$
Substituting our coefficients:
$\tan(2\theta) = \frac{6\sqrt{3}}{7-13} = \frac{6\sqrt{3}}{-6} = -\sqrt{3}$
This gives $2\theta = 120$ or $\theta = 60$.

Step 2: Calculate the new coefficients using the rotation formulas:
$A' = A\cos^2\theta + B\cos\theta\sin\theta + C\sin^2\theta$
$C' = A\sin^2\theta - B\cos\theta\sin\theta + C\cos^2\theta$

With $\cos(60) = \frac{1}{2}$ and $\sin(60) = \frac{\sqrt{3}}{2}$:

$A' = 7(\frac{1}{4}) + 6\sqrt{3}(\frac{1}{2})(\frac{\sqrt{3}}{2}) + 13(\frac{3}{4})$
$A' = \frac{7}{4} + \frac{6\cdot 3}{4} + \frac{39}{4} = \frac{64}{4} = 16$

$C' = 7(\frac{3}{4}) - 6\sqrt{3}(\frac{1}{2})(\frac{\sqrt{3}}{2}) + 13(\frac{1}{4})$
$C' = \frac{21}{4} - \frac{18}{4} + \frac{13}{4} = \frac{16}{4} = 4$

Step 3: Write the rotated equation:
$16x^2 + 4y^2 = 16$

Dividing by 16:
$x^2 + \frac{y^2}{4} = 1$

Therefore, the answer is $x^2 + \frac{y^2}{4} = 1$.

30. **C** : To evaluate $\lim_{x\to 2} \frac{x^3-8}{x^2-4}$:

Step 1: Note that this is an indeterminate form $\frac{0}{0}$ when $x = 2$, as:
$x^3 - 8 = 2^3 - 8 = 0$ and $x^2 - 4 = 2^2 - 4 = 0$

Step 2: Factor both numerator and denominator.
For the numerator: $x^3 - 8 = x^3 - 2^3 = (x-2)(x^2 + 2x + 4)$
For the denominator: $x^2 - 4 = x^2 - 2^2 = (x-2)(x+2)$

Step 3: Simplify the expression by canceling the common factor $(x - 2)$:
$\lim_{x\to 2} \frac{x^3-8}{x^2-4} = \lim_{x\to 2} \frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)} = \lim_{x\to 2} \frac{x^2+2x+4}{x+2}$

Step 4: Evaluate the limit by direct substitution:
$\lim_{x\to 2} \frac{x^2+2x+4}{x+2} = \frac{2^2+2(2)+4}{2+2} = \frac{4+4+4}{4} = \frac{12}{4} = 3$

Therefore, $\lim_{x\to 2} \frac{x^3-8}{x^2-4} = 3$