Important Instructions for this Test: Good luck, have fun, and as always: "NOTA" stands for "None of These Answers is correct."

1. A rectangular prism with dimensions $17\times17\times11$ is painted red, then cut into 3179 $1\times1\times1$ cubes. How many of these $1\times1\times1$ cubes have no sides painted?

A: 1853 B: 1989 C: 2025 D: 2560 E: NOTA

2. Some number of people were surveyed as to what was their favorite fruit. Every person surveyed responded with exactly one of the following: blueberries, bananas, strawberries, and grapes. 45 people did not choose blueberries, 37 people did not choose bananas, and 35 people did not choose strawberries. If 55 people were surveyed, how many people did not choose grapes?

A: 7 B: 23 C: 32 D: 48 E: NOTA

3. Each of three people owes money to one of the other people. Marv owes \$45 to Marg; Marg owes \$31 to Mark; and Mark owes \$27 to Marv. In order to make things less complicated, Marv agrees to pay both Marg and Mark, which would settle all debts. What amounts should Marv pay out?

A: \$14 to Marg, \$4 to Mark
B: \$12 to Marg, \$6 to Mark
C: \$16 to Marg, \$2 to Mark
D: \$13 to Marg, \$5 to Mark
E: NOTA

4. If $3x+4y=240$ and $\frac{3}{y}+\frac{4}{x}=12$, find the product xy.

A: 20 B: 16 C: 12 D: 10 E: NOTA

5. Solve the inequality: $\frac{x^2-2x-35}{x^2+4x-5}\geq 0$

A: $(-\infty,1)\cup[7,\infty)$
B: $(-\infty,-1)\cup[5,\infty)$
C: $(-\infty,-5)\cup(1,\infty)$
D: $(-\infty,-1]\cup(7,\infty)$
E: NOTA

6. A point with polar coordinates $\left(-4,-2\pi/3\right)$ appears in which quadrant?

A: I B: II C: III D: IV E: the point is on an axis

7. A right triangle has legs of length $x+2$ and $x+19$ and hypotenuse of length $2x+1$. Find the numerical length of the triangle's hypotenuse.

A: 85 B: 53 C: 45 D: 26 E: NOTA

8. Find the remainder when $3x^5+12x^4-17x^2+4x-9$ is divided by x^2+5x+2.

A: $50x+7$ C: $266x+103$ E: NOTA
B: $3996x+1475$ D: $678x+239$

Please Use the Following Information to Answer Questions 9 to 11:
Let $A(3,2,-1)$, $B(-2,1,0)$, and $C(-4,3,3)$ be three Cartestian points in space, and let $\vec{u}=\overrightarrow{AB}$ and $\vec{v}=\overrightarrow{AC}$.

9. Find $\vec{u}\cdot\vec{v}$.

A: -38 B: 38 C: -32 D: 32 E: NOTA

10. Find $\vec{u}\times\vec{v}$.

A: $\langle -3,-27,2\rangle$ C: $\langle 5,-13,12\rangle$ E: NOTA
B: $\langle 3,27,-2\rangle$ D: $\langle -5,13,-12\rangle$

11. A sphere passes through *A*, *B*, and *C*, and the *x*-coordinate of the center of the sphere is 1. Find the *z*-coordinate of the center of the sphere.

A: -3 B: 3 C: -3.5 D: 3.5 E: NOTA

12. If $\dfrac{\sin x}{1+\cos x}=\dfrac{7}{3}$, find the value of $\tan x$.

A: $21/20$ B: $-21/20$ C: $20/21$ D: $-20/21$ E: NOTA

13. Find the *y*-coordinate of the hole on the graph of $y=\dfrac{2x^3-3x^2-50x-24}{3x^3+19x^2+22x-24}$.

A: 1 B: $2/3$ C: 5 D: -2 E: NOTA

14. Two parallel lines with slope $-3/4$ have *y*-intercepts that are 5 units apart. Find the distance between the two parallel lines.

A: 4 B: 3 C: $32/7$ D: $24/7$ E: NOTA

15. Which of the following can never equal $\tan\theta$ for some angle θ?

A: $\sin\theta$ B: $\cos\theta$ C: $\sec\theta$ D: $\cot\theta$ E: NOTA

16. Find the sum of all radian-measure solutions to the equation $\frac{\cos(5x)-\cos(3x)}{\sin(5x)+\sin(3x)}=\sqrt{3}$ in the interval $[-2\pi,2\pi]$.

A: $\pi/3$ B: $2\pi/3$ C: $4\pi/3$ D: $7\pi/3$ E: NOTA

17. Evaluate: $\begin{vmatrix} -4 & 7 & -5 \\ 3 & -3 & 0 \\ 1 & 2 & 3 \end{vmatrix}$

A: 57 B: 72 C: −57 D: −72 E: NOTA

18. Find the coefficient of x^3 in the expansion of $\left(x^2-x+2\right)^6$.

A: 400 B: −400 C: 720 D: −720 E: NOTA

19. If $m^{\log_2 5}=8$, find the value of $m^{(\log_2 5)^2}$.

A: 625 B: 125 C: 64 D: 32 E: NOTA

20. An infinite geometric sequence has first term 270 and fifth term $10/3$. For all such possible sequences, find the sum all possible sums of the series associated with these sequences.

A: 405 B: 1215 C: $1215/2$ D: $1215/4$ E: NOTA

21. In how many distinct locations in the coordinate plane do the graphs of $r=2+5\cos\theta$ and $r=4\sin\theta$ intersect?

A: 1 B: 2 C: 3 D: 4 E: NOTA

22. Solve for matrix A: $A\begin{bmatrix} -4 & 3 \\ 2 & -1 \end{bmatrix}=\begin{bmatrix} -2 & 4 \\ 8 & -5 \end{bmatrix}$.

A: $\begin{bmatrix} 3 & 5 \\ -1 & 2 \end{bmatrix}$ C: $\begin{bmatrix} 5 & -2 \\ -3 & 1 \end{bmatrix}$ E: NOTA

B: $\begin{bmatrix} 11 & -11/2 \\ 14 & -6 \end{bmatrix}$ D: $\begin{bmatrix} -11 & 11/2 \\ -14 & 6 \end{bmatrix}$

Please Use the Following Information to Answer Questions 23 and 24:
Consider the ellipse with equation $16x^2+25y^2+32x-150y-159=0$.

23. Find the length of the latus rectum of the ellipse.

A: $16/5$ B: $32/5$ C: $24/5$ D: $48/5$ E: NOTA

24. Which set of parametric equations, using t as the parameter, describes clockwise motion along the ellipse, with $t=0$ corresponding to the lower co-vertex, and requiring a t-interval of length π to complete exactly one circuit on the ellipse?

A: $\begin{cases} x=-1-5\cos\left(\frac{t}{2}\right) \\ y=3-4\sin\left(\frac{t}{2}\right) \end{cases}$ C: $\begin{cases} x=-1-5\sin\left(\frac{t}{2}\right) \\ y=3-4\cos\left(\frac{t}{2}\right) \end{cases}$ E: NOTA

B: $\begin{cases} x=-1-5\cos(2t) \\ y=3-4\sin(2t) \end{cases}$ D: $\begin{cases} x=-1-5\sin(2t) \\ y=3-4\cos(2t) \end{cases}$

25. Simplify: $\dfrac{\sin^2 x+\cos^2 x+\tan^2 x}{\csc^2 x-\cot^2 x+\tan^2 x}$

A: 1 B: $\sin^2 x$ C: $\sec^2 x$ D: $\tan^2 x$ E: NOTA

26. The function $y=a\cdot x^b$, where a and b are real numbers, passes through the points $(5,10)$ and $(20,30)$. If $\log_5\left(\frac{a}{2}\right)=\log_4 c$, find the value of c.

A: $4/5$ B: $3/5$ C: $3/4$ D: $4/3$ E: NOTA

27. Suppose that three bags contain solid white and solid black marbles only. The first bag initially contains exactly 9 marbles: 5 white and 4 black. The second bag initially contains exactly 10 marbles: 4 white and 6 black. The third bag initially contains exactly 11 marbles: 6 white and 5 black. A marble is selected at random from the first bag and transferred to the second bag without looking to see what color it is. After that, a marble is selected at random from the second bag and transferred to the third bag without looking to see what color it is. Finally, a marble is selected at random from the third bag. If the marble drawn from the third bag is white, what is the probability that the marble transferred from the first bag to the second bag was also white?

A: $5/9$ B: $25/41$ C: $287/635$ D: $343/635$ E: NOTA

28. If $f(x)=3x^2-7x+2$, find the value of $\lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$.

A: 1 B: $3x-7$ C: $6x-7$ D: $-7x+2$ E: NOTA

29. Find the sum: $\sum_{j=1}^{2025}\left(\sum_{k=1}^{2025}\left(\frac{2^k}{2^j+2^k}\right)\right)$

A: 2049300
B: 2050312.5
C: 2051325
D: 2052337.5
E: NOTA

30. Find the vertex of the parabola given by the equation $x^2+4xy+4y^2-30x-90y+450=0$.

A: $\left(21\sqrt{5}/5, 3\sqrt{5}/10\right)$
B: $\left(25\sqrt{2}/2, 15\sqrt{2}/2\right)$
C: $\left(4\sqrt{5}, 3\sqrt{5}/5\right)$
D: $\left(18/5, 87/10\right)$
E: NOTA