Answer Key:

1. C
2. D
3. A
4. A
5. E

6. A
7. B
8. C
9. B
10. D

11. D
12. B
13. C
14. A
15. C

16. B
17. D
18. E
19. B
20. C

21. C
22. A
23. B
24. D
25. A

26. D
27. E
28. C
29. B
30. D

Solutions:

1. C: The cubes that aren't painted are all of the cubes on the interior of the larger prism. Therefore, by subtracting 1 from each edge should remove all the exterior cubes: $(17-2)(17-2)(11-2)=2025$.

2. D: Let *x* be the number of people who did not choose grapes. Then the numbers of people who did not choose a specific fruit is the sum of the numbers of people who chose the other three fruits. Therefore, by adding those numbers, we get three times the number of people who chose each fruit. Thus, $\frac{45+37+35+x}{3}=55 \Rightarrow x=48$.

3. A: Marv is paying out \$45 but receiving \$27, so to square him with Marg and Mark requires lowering his total amount by \$18, so this is the total he should pay out in the new scenario. Further, Marg is paying out \$31 but receiving \$45, so her total must increase by \$14. Finally, Mark is paying out \$27 but receiving \$31, so his total must increase by \$4. Therefore, to make everyone square, Marv should pay \$14 to Marg and \$4 to Mark.

4. A: $12=\frac{3}{y}+\frac{4}{x}=\frac{3x+4y}{xy}=\frac{240}{xy} \Rightarrow xy=20$

5. E: $0 \le \frac{x^2-2x-35}{x^2+4x-5}=\frac{(x-7)(x+5)}{(x-1)(x+5)}=\frac{x-7}{x-1}, x \ne -5$, and to get the quotient to be positive, we must have $x-7$ and $x-1$ both positive or both negative. Using sign analysis, in the first instance, $x>7$, and in the second instance, $x<1$. The only value that makes the quotient equal to 0 is $x=7$, so combining all of these restrictions on *x*, the solution is $(-\infty,-5)\cup(-5,1)\cup[7,\infty)$.

6. A: In standard position, the angle $-2\pi/3$ is in quadrant III. However, with a negative *r*-value, we must reflect through the origin, so this point appears in quadrant I.

7. B: $4x^2+4x+1=(2x+1)^2=(x+2)^2+(x+19)^2=(x^2+4x+4)+(x^2+38x+361)=2x^2+42x+365$ $\Rightarrow 0=2x^2-38x-364=2(x+7)(x-26)$, but since the quantities given are lengths, $x=26$. Therefore, the length of the hypotenuse is $2x+1=2(26)+1=53$.

8. C: Using either long or synthetic division, $\frac{3x^5+12x^4-17x^2+4x-9}{x^2+5x+2}=3x^3-3x^2+9x-56$ $+\frac{266x+103}{x^2+5x+2}$, so the remainder is $266x+103$.

9. B: $\vec{u}=\langle(-2)-3,1-2,0-(-1)\rangle=\langle-5,-1,1\rangle$ and $\vec{v}=\langle(-4)-3,3-2,3-(-1)\rangle=\langle-7,1,4\rangle$, so $\vec{u}\cdot\vec{v}$ $=(-5)(-7)+(-1)(1)+1(4)=35-1+4=38$.

10. D: $\vec{u}\times\vec{v}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ -5 & -1 & 1\\ -7 & 1 & 4\end{vmatrix}=-4\vec{i}-7\vec{j}-5\vec{k}-7\vec{k}-\vec{i}+20\vec{j}=-5\vec{i}+13\vec{j}-12\vec{k}=\langle -5,13,-12\rangle$

11. D: Plugging the three points into the sphere equation $(x-h)^2+(y-k)^2+(z-l)^2=r^2$ yields $r^2=(3-h)^2+(2-k)^2+(-1-l)^2=9-6h+h^2+4-4k+k^2+1+2l+l^2$, $r^2=(-2-h)^2+(1-k)^2+(0-l)^2$ $=4+4h+h^2+1-2k+k^2+l^2$, and $r^2=(-4-h)^2+(3-k)^2+(3-l)^2=16+8h+h^2+9-6k+k^2+9-6l$ $+l^2$. Since all three equations begin with r^2, setting the right sides of the first two equation equal to each other and simplifying yields $9=10h+2k-2l$. Doing the same thing with the first and third equation yields $20=-14h+2k+8l$, and subtracting these two equations from each other yields $11=-24h+10l$. Since $h=1$, $35=10l\Rightarrow l=3.5$.

12. B: $\frac{7}{3}=\frac{\sin x}{1+\cos x}=\tan\frac{x}{2}\Rightarrow \tan x=\frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}=\frac{2\left(\frac{7}{3}\right)}{1-\left(\frac{7}{3}\right)^2}=\frac{\frac{14}{3}}{-\frac{40}{9}}=-\frac{21}{20}$

13. C: $y=\frac{2x^3-3x^2-50x-24}{3x^3+19x^2+22x-24}=\frac{(2x+1)(x-6)(x+4)}{(3x-2)(x+3)(x+4)}$, so the hole occurs at $x=-4$. To find the y-coordinate for the hole, plug this x-value into the simplified rational function:
$y=\frac{(2(-4)+1)((-4)-6)}{(3(-4)-2)((-4)+3)}=\frac{(-7)(-10)}{(-14)(-1)}=\frac{70}{14}=5$.

14. A: Any two lines that meet the criteria should have the same distance between them. Give the lines equations $3x+4y=0$ and $3x+4y-20=0$—both lines has slope $-\frac{3}{4}$, and the y-intercepts of the two lines are 0 and 5, respectively. Since the origin is a point on the first line, the distance between the origin and the second line is the distance between the two lines; that distance is
$\frac{|3(0)+4(0)-20|}{\sqrt{3^2+4^2}}=\frac{20}{5}=4$.

15. C: $\tan\theta=\sin\theta$ when $\theta=0$, and $\tan\theta=\cot\theta$ when $\theta=\frac{\pi}{4}$. If $\tan\theta=\cos\theta$, multiplying by $\cos\theta$ and using identities yields $\sin\theta=\cos^2\theta=1-\sin^2\theta\Rightarrow 0=\sin^2\theta+\sin\theta-1$
$\sin\theta=\frac{-1\pm\sqrt{1^2-4(1)(-1)}}{2(1)}=\frac{-1\pm\sqrt{5}}{2}$, but $\frac{-1-\sqrt{5}}{2}<-1$, this is not possible. However, $-1<\frac{-1+\sqrt{5}}{2}<1$, and since $\cos\theta\neq 0$ for this case, there is a solution to this equation. For $\tan\theta=\sec\theta$, $\frac{\sin\theta}{\cos\theta}=\frac{1}{\cos\theta}$, this seems possible on the surface as long as $\sin\theta=1$; however, if this

is the case, $\cos\theta = 0$, which makes both of these quantities undefined. Therefore, $\tan\theta \neq \sec\theta$ for any angle θ.

16. B: Use sum and difference formulas: $\sqrt{3} = \dfrac{\cos(5x)-\cos(3x)}{\sin(5x)+\sin(3x)} = \dfrac{\cos(4x+x)-\cos(4x-x)}{\sin(4x+x)+\sin(4x-x)}$

$$= \frac{\cos(4x)\cos x - \sin(4x)\sin x - \cos(4x)\cos x - \sin(4x)\sin x}{\sin(4x)\cos x + \cos(4x)\sin x + \sin(4x)\cos x - \cos(4x)\sin x} = \frac{-2\sin(4x)\sin x}{2\sin(4x)\cos x} = -\tan x$$

$\Rightarrow \tan x = -\sqrt{3}$, and the solutions in the interval to this equation are $x = -\dfrac{4\pi}{3}$, $-\dfrac{\pi}{3}$, $\dfrac{2\pi}{3}$, and $\dfrac{5\pi}{3}$. None of these values make $\sin(4x)$ or $\cos x$ equal to 0, so all four solutions work, and the sum of these solutions is $\left(-\dfrac{4\pi}{3}\right)+\left(-\dfrac{\pi}{3}\right)+\dfrac{2\pi}{3}+\dfrac{5\pi}{3}=\dfrac{2\pi}{3}$.

17. D: $\begin{vmatrix} -4 & 7 & -5 \\ 3 & -3 & 0 \\ 1 & 2 & 3 \end{vmatrix} = 36+0+(-30)-15-0-63=-72$

18. E: The expansion consists of some number of x^2's, $-x$'s, and 2's, a total of 6 in all. Therefore, the only ways to do this are:

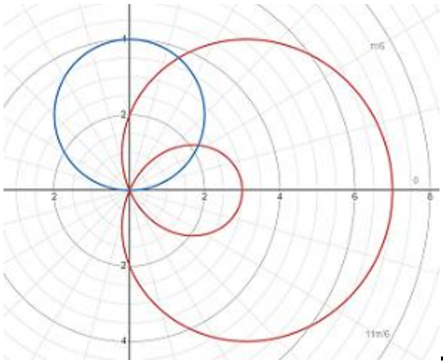
- 1 x^2, 1 $-x$, and 4 2's
- 3 $-x$'s and 3 2's

For the first scenario, the term is $\dfrac{6!}{4!1!1!}\left(x^2\right)^1(-x)^1(2)^4 = -480x^3$; for the second scenario, the term is $\dfrac{6!}{3!3!0!}\left(x^2\right)^0(-x)^3(2)^3 = -160x^3$, so the coefficient of x^3 is $(-480)+(-160)=-640$.

19. B: $m^{(\log_2 5)^2} = \left(m^{\log_2 5}\right)^{\log_2 5} = (8)^{\log_2 5} = (5)^{\log_2 8} = 5^3 = 125$

20. C: Given the first and fifth terms, $\dfrac{10}{3} = a_5 = a_1 r^{5-1} = 270r^4 \Rightarrow r^4 = \dfrac{1}{81} \Rightarrow r = \pm\dfrac{1}{3}$. Therefore, for all possible series, all term magnitudes are the same, but the terms in even positions are positive in one series and negative in the other, so those terms will cancel out when both sums are added together. Therefore, the sum of all possible sums is $2\left(270+30+\dfrac{10}{3}+\ldots\right) = 2\left(\dfrac{270}{1-\frac{1}{9}}\right) = 2(270)\left(\dfrac{9}{8}\right)$ $= \dfrac{1215}{2}$.

21. C: You could try to solve this algebraically, but the numbers are not ideal with which to work. Instead, just think about what the graphs of each look like, which is the following:



. Therefore, there are three intersection points.

22. A: The inverse of $\begin{bmatrix} -4 & 3 \\ 2 & -1 \end{bmatrix}$ is $\dfrac{1}{(-4)(-1)-2(3)}\begin{bmatrix} -1 & -3 \\ -2 & -4 \end{bmatrix} = \begin{bmatrix} 1/2 & 3/2 \\ 1 & 2 \end{bmatrix}$, so to find A, multiply by this inverse on the right side of both sides of the equation: $A = \begin{bmatrix} -2 & 4 \\ 8 & -5 \end{bmatrix}\begin{bmatrix} 1/2 & 3/2 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ -1 & 2 \end{bmatrix}$.

23. B: The standard form of the ellipse is $0 = 16x^2 + 25y^2 + 32x - 150y - 159 = 16(x+1)^2$ $+25(y-3)^2 - 400 \Rightarrow \dfrac{(x+1)^2}{25} + \dfrac{(y-3)^2}{16} = 1$, so the latus rectum has length $\dfrac{2b^2}{a} = \dfrac{2(16)}{5} = \dfrac{32}{5}$.

24. D: To parametrize an ellipse, use the equations $(x \text{ or } y) = d \pm e(\sin(mt) \text{ or } \cos(mt))$, where d represents the center and e is another real value; therefore, $d = -1$ for the x-term and $d = 3$ for the y-term. Further, since the equation of the ellipse is $1 = \dfrac{(x+1)^2}{25} + \dfrac{(y-3)^2}{16} = \left(\dfrac{x+1}{5}\right)^2 + \left(\dfrac{y-3}{4}\right)^2$, then one of $\dfrac{x+1}{5}$ and $\dfrac{y-3}{4}$ is $\pm\cos(mt)$ while the other is $\pm\sin(mt)$; additionally, since the time interval for one circuit completion is π, $\dfrac{2\pi}{m} = \pi \Rightarrow m = 2$. Now, since the lower co-vertex is at the point $(-1,-1)$, which corresponds to $t = 0$, we must have $\sin(2t)$ go with the y-term since $\dfrac{(-1)+1}{5} = 0 = \pm\sin(2\cdot 0)$ (we just don't know if it is $+$ or $-$ yet), and we must also have $-\cos(2t)$ go with the x-term since $\dfrac{(-1)-3}{4} = -1 = -\cos(2\cdot 0)$. Lastly, since the period is π and the orientation is clockwise, $t = \dfrac{\pi}{4}$ corresponds to the left vertex, which is at the point $(-6,3)$; this means it is $-\sin(2t)$ with the x-term since $\dfrac{(-6)+1}{5} = -1 = -\sin\left(2\cdot\dfrac{\pi}{4}\right)$. Therefore, the parametric equations needed for the given specifications are $\dfrac{x+1}{5} = -\sin(2t) \Rightarrow x = -1 - 5\sin(2t)$ and $\dfrac{y-3}{4} = -\cos(2t) \Rightarrow y = 3 - 4\cos(2t)$.

25. A: $\dfrac{\sin^2 x+\cos^2 x+\tan^2 x}{\csc^2 x-\cot^2 x+\tan^2 x}=\dfrac{1+\tan^2 x}{1+\tan^2 x}=1$

26. D: Based on the given points, we have the two relationships $30=a(20)^b$ and $10=a(5)^b$.

Dividing the quantites in each equation by each other yields $3=\dfrac{30}{10}=\dfrac{a(20)^b}{a(5)^b}=4^b\Rightarrow b=\log_4 3$.

Plugging this back in to the second equation gives $10=a(5)^{\log_4 3}\Rightarrow a=\dfrac{10}{(5)^{\log_4 3}}=2(5)^{1-\log_4 3}$

$=2(5)^{\log_4 4-\log_4 3}=2(5)^{\log_4\left(\frac{4}{3}\right)}\Rightarrow\dfrac{a}{2}=(5)^{\log_4\left(\frac{4}{3}\right)}$. Therefore, $\log_4 c=\log_5\left(\dfrac{a}{2}\right)=\log_5(5)^{\log_4\left(\frac{4}{3}\right)}=\log_4\left(\dfrac{4}{3}\right)$

$\Rightarrow c=\dfrac{4}{3}$.

27. E: There are two options for the transferred marble, either white or black, and those have probabilities of $5/9$ and $4/9$, respectively. If the first transferred marble was white, then the probability of drawing a white marble from the second bag is $5/11$ and the probability of drawing a black marble is from the second bag is $6/11$; if the first transferred marble was black, then the probability of drawing a white marble from the second bag is $4/11$ and the probability of drawing a black marble is from the second bag is $7/11$. Lastly, if the second transferred marble was white, then the probability of drawing a white marble from the third bag is $7/12$; if the second transferred marble was black, then the probability of drawing a white marble from the third bag is $6/12$. The sequences in which the third marble is white are WWW, WBW, BWW, and BBW, so the probability we are looking for is $\dfrac{P(W\_W)}{P(\_\ \_W)}=\dfrac{\frac{5}{9}\cdot\frac{5}{11}\cdot\frac{7}{12}+\frac{5}{9}\cdot\frac{6}{11}\cdot\frac{6}{12}}{\frac{5}{9}\cdot\frac{5}{11}\cdot\frac{7}{12}+\frac{5}{9}\cdot\frac{6}{11}\cdot\frac{6}{12}+\frac{4}{9}\cdot\frac{4}{11}\cdot\frac{7}{12}+\frac{4}{9}\cdot\frac{7}{11}\cdot\frac{6}{12}}$

$=\dfrac{355/1188}{635/1188}=\dfrac{355}{635}=\dfrac{71}{127}$.

28. C: $\lim\limits_{h\to 0}\left(\dfrac{f(x+h)-f(x)}{h}\right)=\lim\limits_{h\to 0}\left(\dfrac{\left(3(x+h)^2-7(x+h)+2\right)-\left(3x^2-7x+2\right)}{h}\right)$

$=\lim\limits_{h\to 0}\left(\dfrac{3x^2+6xh+3h^2-7x-7h+2-3x^2+7x-2}{h}\right)=\lim\limits_{h\to 0}\left(\dfrac{6xh+3h^2-7h}{h}\right)=\lim\limits_{h\to 0}\left(\dfrac{h(6x+3h-7)}{h}\right)$

$=\lim\limits_{h\to 0}(6x+3h-7)=6x+3(0)-7=6x-7$

29. B: $\sum_{j=1}^{2025}\left(\sum_{k=1}^{2025}\left(\frac{2^k}{2^j+2^k}\right)\right)=\sum_{j=1}^{2025}\left(\frac{2^1}{2^j+2^1}+\frac{2^2}{2^j+2^2}+\ldots+\frac{2^{2025}}{2^j+2^{2025}}\right)=\left(\frac{2^1}{2^1+2^1}+\frac{2^2}{2^1+2^2}+\ldots+\frac{2^{2025}}{2^1+2^{2025}}\right)$ $+\left(\frac{2^1}{2^2+2^1}+\frac{2^2}{2^2+2^2}+\ldots+\frac{2^{2025}}{2^2+2^{2025}}\right)+\ldots+\left(\frac{2^1}{2^{2025}+2^1}+\frac{2^2}{2^{2025}+2^2}+\ldots+\frac{2^{2025}}{2^{2025}+2^{2025}}\right)$. Notice that if the two powers of 2 in the denominator of a term are different, then there are exactly two terms in this series with that denominator: one with one of the powers of 2 in the numerator, the other with the other power of 2 in the numerator. Thus, those terms sum to 1, and there are exactly $\binom{2025}{2}=2049300$ of those terms. If the powers of 2 in the denominator of a term are the same, then the numerator is that same power of 2, so those terms are all equal to $\frac{1}{2}$, and there are 2025 of them. Therefore, the sum is $2049300+2025\left(1/2\right)=2050312.5$.

30. D: The angle of rotation θ satisfies $\tan 2\theta=\frac{4}{1-4}=-\frac{4}{3}\Rightarrow 2\theta=\pi-\tan^{-1}\left(\frac{4}{3}\right)\Rightarrow\theta=\frac{\pi}{2}$ $-\frac{1}{2}\tan^{-1}\left(\frac{4}{3}\right)$, so $\tan\theta=\cot\left(\frac{1}{2}\tan^{-1}\left(\frac{4}{3}\right)\right)=\frac{1+\cos\left(\tan^{-1}\left(\frac{4}{3}\right)\right)}{\sin\left(\tan^{-1}\left(\frac{4}{3}\right)\right)}=\frac{1+\frac{3}{5}}{\frac{4}{5}}=2$. This means that there is a line with slope 2 that intersects the parabola only at its vertex—let this line have equation $y=2x+c$. To find the intersection of this line with the parabola, plug in to the parabola equation: $0=x^2+4x(2x+c)+4(2x+c)^2-30x-90(2x+c)+450=x^2+8x^2+4cx+16x^2+16cx+4c^2-30x$ $-180x-90c+450=25x^2+(20c-210)x+\left(4c^2-90c+450\right)$. For this to have only one intersection, the discriminant must equal 0: $0=(20c-210)^2-4(25)\left(4c^2-90c+450\right)=600c-900\Rightarrow c=3/2$. Further, the x-coordinate would be $x=-\frac{20\left(3/2\right)-210}{2(25)}=\frac{18}{5}$, and the y-coordinate would be $y=2\left(\frac{18}{5}\right)+\frac{3}{2}=\frac{87}{10}$, making the vertex $\left(18/5, 87/10\right)$.