Important Instructions For This Test (Do not skip): Assume all angles are in radians unless a degrees symbol is used. Take $i = \sqrt{-1}$. "NOTA" means "None of The Answers A, B, C, D are correct." Good luck and have fun!

1. Express the polar coordinates $\left(4, \frac{2\pi}{3}\right)$ in rectangular form.

(A) $(2, 2\sqrt{3})$ **(B)** $(-2, 2\sqrt{3})$ **(C)** $(2\sqrt{3}, 2)$ **(D)** $(2\sqrt{3}, -2)$ **(E)** NOTA

2. Which of the following is equal to $1 - \cos 2\theta$?

(A) $\cos^2\theta$ **(B)** $\sin^2\theta$ **(C)** $2\cos^2\theta$ **(D)** $2\sin^2\theta$ **(E)** NOTA

3. For how many angles x in the interval $[0, 2\pi)$ does the following equation hold?

$$\tan(x) + \cot(x) = 3$$

(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** NOTA

4. Given that $\sin\theta = \frac{2}{3}$ and $\cot\theta = -\frac{\sqrt{5}}{2}$, find $\sec\theta$.

(A) $-\frac{\sqrt{5}}{3}$ **(B)** $\frac{\sqrt{5}}{3}$ **(C)** $-\frac{3\sqrt{5}}{5}$ **(D)** $\frac{3\sqrt{5}}{5}$ **(E)** NOTA

5. Let $x = 0.0001$. Which of the following values is the largest?

(A) $2\sin\frac{x}{2}$ **(B)** $\frac{1}{2}\sin 2x$ **(C)** $2\tan\frac{x}{2}$ **(D)** $\frac{1}{2}\tan 2x$ **(E)** NOTA

6. Find $\tan\left(\sin^{-1}\left(\frac{13}{5}\right)\right)$.

(A) $\frac{5}{12}$ **(B)** $\frac{5}{13}$ **(C)** $\frac{12}{13}$ **(D)** $\frac{12}{5}$ **(E)** NOTA

7. Triangle ABC has $\angle A = 30^\circ$, $\angle B = 45^\circ$, and $\angle C = 105^\circ$. If $BC = 18$, find CA.

(A) $9\sqrt{2}$ **(B)** 18 **(C)** $18\sqrt{2}$ **(D)** $36\sqrt{2}$ **(E)** NOTA

8. Given that $\cos\frac{2\pi}{5} = \frac{-1+\sqrt{5}}{4}$, find the area of a regular pentagon inscribed in the unit circle.

(A) $\frac{5}{8}\sqrt{10+2\sqrt{5}}$ **(B)** $\frac{5}{4}\sqrt{10+2\sqrt{5}}$ **(C)** $\frac{5}{8}\sqrt{10-2\sqrt{5}}$ **(D)** $\frac{5}{4}\sqrt{10-2\sqrt{5}}$ **(E)** NOTA

9. In standard position, the angle 2θ is in the first quadrant and 3θ is in the third quadrant. The angle θ must belong to which quadrant?

(A) I **(B)** II **(C)** III **(D)** IV **(E)** NOTA

10. For how many values of x in $[0, 2\pi)$ is the following expression zero or undefined?

$$\sin x + \sin^2 x + \sin^3 x + \sin^4 x + ...$$

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** NOTA

11. Which of the following values is in the range of $\sin x + \csc x$?

(A) 0 **(B)** 0.5 **(C)** 1 **(D)** 2 **(E)** NOTA

12. What is the maximum possible value of $\operatorname{arcsec} x$?

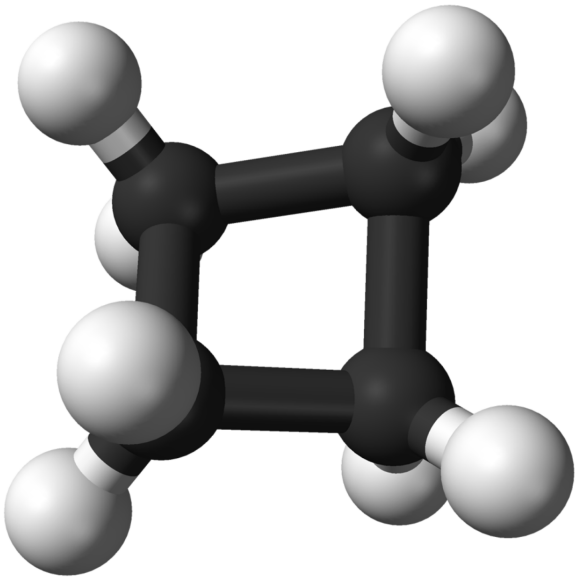
(A) 0 **(B)** $\frac{\pi}{2}$ **(C)** π **(D)** 2π **(E)** NOTA

13. With $\exp(x) = e^x$, find the real part of

$$\exp\left(\frac{\pi}{4}\sqrt{2}\exp\left(\frac{\pi}{4}i\right)\right).$$

(A) 0 **(B)** $\frac{e^{\frac{\pi}{4}}}{\sqrt{2}}$ **(C)** $e^{\frac{\pi}{4}\sqrt{2}}\cos\left(\frac{\pi}{4}\sqrt{2}\right)$ **(D)** 1 **(E)** NOTA

14. The molecule *cyclobutane* contains four carbon atoms in a ring (here shown in black). Due to extreme strain, two of the atoms "pucker" slightly out of the plane, such that the angle between any two adjacent carbon-carbon bonds is 88°. Given that the carbon-carbon bond length is 1.5 Angstroms, find the distance between two diagonally-placed carbon atoms in Angstroms.



(A) $\sqrt{4.50 - 3\cos 88^\circ}$
(B) $\sqrt{4.50 - 2\cos 88^\circ}$
(C) $\sqrt{4.50 - 4.5\cos 88^\circ}$
(D) $\sqrt{4.50 - 1.5\cos 88^\circ}$
(E) NOTA

15. Find the sum of all solutions to

$$\frac{1+\tan x}{1+\cot x} = \frac{1+\cot x}{1+\tan x}$$

on $x \in [0, 2\pi)$.

(A) $\frac{3\pi}{2}$ **(B)** 2π **(C)** $\frac{5\pi}{2}$ **(D)** 4π **(E)** NOTA

16. Which of the following values is closest to $x = \left(\sin(\frac{\pi}{2}) - \sin(\frac{\pi}{4})\right)\left(\tan(\frac{\pi}{4}) + \tan(\frac{\pi}{3})\right)$.

(A) 0.4 **(B)** 0.8 **(C)** 1.2 **(D)** 1.6 **(E)** NOTA

17. Which of the following is **closest** to $\cot(3.14)$?

(A) $-10\,000$ **(B)** $-1\,000$ **(C)** -100 **(D)** -10 **(E)** NOTA

18. How many intersections are there between the graphs $y = \frac{1}{2}\tan x$ and $x = \frac{1}{2}\tan y$ inside the region $0 \le x \le y \le \frac{7\pi}{2}$?

(A) 9 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** NOTA

19. The below system has two distinct solutions that can be expressed as $(\tan x_1, \tan y_1)$ and $(\tan x_2, \tan y_2)$. Find $\tan(x_1 + x_2) + \tan(y_1 + y_2)$.

$$\begin{cases} x^2 - y^2 - 2x &= \cos\frac{2\pi}{3} \\ 2xy - 2y &= \sin\frac{2\pi}{3} \end{cases}$$

(A) $\frac{4}{3}$ **(B)** 2 **(C)** $\frac{8}{3}$ **(D)** 4 **(E)** NOTA

20. There exists a real number x such that

$$\arctan(x) = 3\arctan(1) - 2\arctan(2) + 1\arctan(3).$$

Find x.

(A) -5.5 **(B)** -4.5 **(C)** 4.5 **(D)** 5.5 **(E)** NOTA

21. Find the amplitude of $f(x) = \cos(x) + 6\sin(x + \frac{\pi}{6})$.

(A) $\sqrt{31}$ **(B)** $\sqrt{35}$ **(C)** $\sqrt{37}$ **(D)** $\sqrt{43}$ **(E)** NOTA

22. Simplify

$$\sum_{n=0}^{90} \sin^4(n^\circ).$$

(A) $\frac{131}{4}$ **(B)** $\frac{133}{4}$ **(C)** $\frac{135}{4}$ **(D)** $\frac{137}{4}$ **(E)** NOTA

23. Suppose that $\sin x + \cos x = \frac{1}{2}$. Then $\sin^3 x + \cos^3 x$ can be expressed as $\frac{m}{n}$ for relatively prime positive integers m and n. Find $m + n$.

(A) 9 **(B)** 17 **(C)** 21 **(D)** 27 **(E)** NOTA

24. Which of the following polynomials have a root that is equal to $\tan \frac{2\pi}{5}$? *Hint:* $\tan\left(\frac{4\pi}{5}\right) = \tan\left(-\frac{6\pi}{5}\right)$

(A) $x^4 - 10x^2 + 5$
(B) $x^4 - 10x^2 - 5$
(C) $x^4 - 7x^2 + 4$
(D) $x^4 - 7x^2 - 4$
(E) NOTA

25. Find the smallest positive solution to the equation

$$\frac{1 + \cot x}{\tan x + \cot x} = \frac{\frac{3}{4} - \sin^2 x}{\cos x - \sin x}.$$

(A) $\frac{\pi}{5}$ **(B)** $\frac{2\pi}{5}$ **(C)** $\frac{\pi}{7}$ **(D)** $\frac{2\pi}{7}$ **(E)** NOTA

26. What is the smallest integer value of r such that the following system of equations has real solutions?

$$\begin{cases} \cos^2 x + \cos^2 y = 1 \\ x^2 + y^2 = r \end{cases}$$

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** NOTA

27. In triangle ABC, $AB = 10$, $BC = 17$, and $CA = 21$. Given that $\sin \frac{1}{2}\angle C = \frac{1}{\sqrt{17}}$, find $\cos \frac{1}{2}(\angle A - \angle B)$.

(A) $\frac{1}{\sqrt{17}}$ **(B)** $\frac{8}{5\sqrt{17}}$ **(C)** $\frac{19}{5\sqrt{17}}$ **(D)** $\frac{24}{5\sqrt{17}}$ **(E)** NOTA

28. Simplify the following expression. *Hint: use your half-angle identities!*

$$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2+\sqrt{2}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}{2} \cdots$$

(A) $\frac{\pi}{4}$ **(B)** $\frac{\pi}{2}$ **(C)** $\frac{2}{\pi}$ **(D)** $\frac{4}{\pi}$ **(E)** NOTA

29. Find

$$\sum_{n=0}^{\infty} \frac{\cos\left(2^n \frac{2\pi}{5}\right)}{2^n}.$$

(A) $-\frac{1}{2} + \frac{\sqrt{5}}{6}$ **(B)** $-\frac{1}{6} + \frac{\sqrt{5}}{2}$ **(C)** $-\frac{3}{10} + \frac{\sqrt{5}}{10}$ **(D)** $-\frac{1}{10} + \frac{3\sqrt{5}}{10}$ **(E)** NOTA

30. The last question is usually easy. What is $\cos 90$?

(A) -1 **(B)** 0 **(C)** 1 **(D)** $\frac{\sqrt{2}}{2}$ **(E)** NOTA