Solutions	
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Question	Answer	Solution
1	24	A radius from the center of the circle perpendicular to the chord bisects the chord. Connecting the center to the end of the chord forms a right triangle with one leg equal to 7 cm (half the chord) and a hypotenuse equal to the radius, 25 cm. Using the Pythagorean Theorem, the missing leg is 24 cm which is the distance from the center to the chord.
2	28√2	The circle has radius of 7. If two diagonals of the square are drawn, four right triangles whose legs are radii are formed. These are 45-45-90 triangles, so the hypotenuse is $7\sqrt{2}$ and the perimeter of the square is 4(side) which is $28\sqrt{2}$.
3	148	This forms isosceles $\triangle ABD$ with $\angle B \cong \angle D$. Since there are 180° in a triangle, $m \angle A = 74^{\circ}$. $\angle A$ measures ½ the intercepted arc so minor arc BD measures 148.
4	2√91	Use the Law of Cosines to find the measure of s. $s^2 = 10^2 + 12^2 - 2(10)(12)(\cos 120) = 100 + 144 - 240(-1/2) = 364$ $s = \sqrt{364} = 2\sqrt{91}$
5	$\frac{\pi - 2}{\pi}$ or $1 - \frac{2}{\pi}$	We need the ratio of the area inside the circle but outside the square to the area of the circle for the probability. Let r be the radius of the circle. Then 2r is the diagonal of the square. The square has area = $\frac{1}{2}(2r)(2r) = 2r^2$ and the circle has area = πr^2 . Area inside circle and outside square = $\pi r^2 - 2r^2$ and we divide by πr^2 and reduce, dividing out the r ² .
6	156	There are 15 congruent exterior angles of the polygon and their sum is 360, so 360/15 gives exterior angles of 24. The interior angle is supplementary to the exterior angle so it is 180 – 24 or 156.
7	$\frac{15}{4}$ or 3.75	Adjacent sides of a rectangle are perpendicular so their slopes have a product of -1. The two slopes are -3/4 and 5/k. Thus $-15/4k = -1$ and $k = 15/4$.
8	4	The two lines are parallel and the second line has a y-intercept of (0, 1). We need to determine the distance from that point to the first line. Putting the first line into $Ax + By + C = 0$ form, we get $3x - 4y - 16 = 0$. Distance from point to line $= \frac{ Ax_1+By_1+C }{\sqrt{A^2+B^2}} = \frac{ 3(0)+(-4)(1)+(-16) }{\sqrt{3^2+(-4)^2}} = \frac{20}{5} = 4$
9	5√2	Traveling east-west, Ahbi goes 5 west and 10 east, ending up 5 miles east of the hotel. Traveling north-south, Ahbi goes 8 north and 3 south, ending up 5 miles north of the hotel. So Ahbi is 5 miles north and 5 miles east of his starting point which makes an isosceles right triangle. The hypotenuse is $5\sqrt{2}$ which is his straight-line distance from the hotel.
10	32π√3	The space diagonal of the cube is the diameter of the sphere. The cube has a space diagonal of $side\sqrt{3}$ which is $4\sqrt{3}$. This makes the radius of the sphere $2\sqrt{3}$. The sphere's volume is $(4/3) \pi (2\sqrt{3})^3 = (4\pi/3)(24\sqrt{3}) = 32\pi\sqrt{3}$.

11	AD	A A A A A B A C C C C A A B C C C A A B C B C C C A A B D A A B D A A B D A A B D A B D A C B D A A C B D A A C B D A A C B D A A C B D A A C B D A A A C B D A A A C B D A A A C B D A A A A A A A A B D A A A B D A A A A A B D A A A A A A B D A A A A A A A A A A A A A
12	32	A dodecahedron has 12 faces. A cube has 8 vertices. A hexagonal pyramid has 12 edges. 12 + 12 + 8 = 32
13	37.72 or 37 18/25 or 943/25	7-24-25 is a Pythagorean triple so the triangle is a right triangle and two of the altitudes are the legs, 7 and 24. The altitude to the hypotenuse may be found by using similar triangle ratios. Let h be the altitude to the hypotenuse. The two small right triangles are both similar to the large right triangle. Thus, $\frac{7}{h} = \frac{25}{24}$ and $h = \frac{7(24)}{25} = 6.72$ The sum of the altitudes is 7 + 24 + 6.72 = 37.72 or 37 18/25.
14	18	$A = \left(\pm \frac{1}{2}\right) \begin{vmatrix} -2 & 5 & 1\\ 3 & 1 & 1\\ -1 & -3 & 1 \end{vmatrix} = \left(\pm \frac{1}{2}\right) (-2 - 5 - 9 + 1 - 6 - 15) = \left(-\frac{1}{2}\right) (-36)$ $= 18$
15	5√17	Draw in radii AX and BY. Draw a segment from B that is parallel to XY and intersects AX at point C. Consider \triangle ABC which is a right triangle with BC = 20 and AC = 5. Apply the Pythagorean Theorem to find AB. $AB^2 = 25 + 400 = 425$ So $AB = \sqrt{425} = 5\sqrt{17}$
16	90π	The base of the original cone has a radius of 10. The small cone that was cut off is similar to the original cone. The radii have a 6:10 or 3:5 ratio so the heights must also have that same ratio. Let h be the height of the small cone. $\frac{3}{5} = \frac{h}{h+5}$ which leads to h = 15/2 or 7.5. For the smaller cone, $V = \frac{1}{3}\pi r^2 h = \frac{\pi}{3}(6)^2(7.5) = 90\pi$
17	3√5 - 3	The shortest distance will be a part of the segment joining the centers of the circles and containing points A and B. The circle with point A has a radius of $2\sqrt{5}$ and the circle with point B has a radius of 3. The segment joining the centers, $(2, -5)$ and $(7, 5)$, has a length of $d = \sqrt{(7-2)^2 + (5+5)^2} = \sqrt{25 + 100} = \sqrt{125} = 5\sqrt{5}$. Subtract the two radii from this segment for the shortest distance between A and B. $5\sqrt{5} - 3 - 2\sqrt{5} = 3\sqrt{5} - 3$

18	800	A radius increased by 200% is 3 times as long as the original radius. The ratio of linear distances is 1:3 so the areas have that ratio squared 1:9. That is an 800% increase.
19	80	$\angle Z$ is inscribed in the circle so its intercepted arc \widehat{WY} measures 100°. $\angle X$ is an angle formed by tangents and is $\frac{1}{2}(\widehat{WZY} - \widehat{WY}) = \frac{1}{2}(260 - 100) = 80$
20	(5/2, -1/2)	Plotting the 3 points and forming a triangle, you can check the slopes of two sides that appear perpendicular and find that they are -3/5 and 5/3 which makes the legs perpendicular. The center of a circumscribed circle will then be the midpoint of the hypotenuse of the right triangle which is (5/2, -1/2)
21	33	Rectangle area is length times width so $x(2x - 3) = 65$ $2x^2 - 3x - 65 = 0$ which factors into $(2x - 13)(x + 5) = 0$ with solutions of 13/2 and -5. Only the positive root works as a length, so $x = 13/2$. The perimeter of the rectangle = $2(L + W) = 2(x + 2x - 3) = 2(3x - 3) = 6x - 6$ Substituting for x gives $6(13/2) - 6 = 39 - 6 = 33$
22	224	Area of a rhombus is half the product of its diagonals. $\frac{1}{2}(16)(28) = 224$
23	- 4	The altitude must be perpendicular to \overline{BC} and contain point A. The slope of \overline{BC} is $\frac{10-0}{0-8} = -\frac{5}{4}$ so the altitude must have a slope of $\frac{4}{5}$ and contain (-4, 0). $y = mx + b$ gives $0 = \frac{4}{5}(-4) + b$ so $b = \frac{16}{5}$ $y = \frac{4}{5}x + \frac{16}{5} \implies 5y = 4x + 16 \implies 4x - 5y = -16$ RS - T = 4(-5) - (-16) = -4
24	2	If two chords intersect inside a circle, the product of their parts must be equal. AE (EC) = BE(DE) \Rightarrow (a + 4)(a + 2) = (a + 1)(a + 6) $a^2 + 6a + 8 = a^2 + 7a + 6$ 2 = a
25	14 ² / ₉ or 128/9	The areas of the 3 rings are: center = 4π , middle ring = $16\pi - 4\pi = 12\pi$, and outer ring = $36\pi - 16\pi = 20\pi$ The probability of hitting each ring is: center = $\frac{4\pi}{36\pi} = \frac{1}{9}$, middle ring = $\frac{12\pi}{36\pi} = \frac{1}{3}$, and outer ring = $\frac{20\pi}{36\pi} = \frac{5}{9}$ Expected value of a dart is the sum of the probability times the value of each ring. $EV = \frac{1}{9}(20) + \frac{1}{3}(16) + \frac{5}{9}(12) = \frac{20}{9} + \frac{16}{3} + \frac{60}{9} = \frac{128}{9} = 14\frac{2}{9}$