2025 Precalculus Hustle MAO National Convention

1. $\frac{9}{2}$
2. 5π
3. $-\frac{1}{16}$
4. $512 + 512i\sqrt{3}$
5. $\frac{3}{4}$
6. 18
7. $\frac{1}{8}$
8. 4
9. 0
10. 2
11. $\frac{12}{5} + \frac{6}{5}i$
12. 6050
13. $\frac{33}{2}$
14. 8
15. $\frac{\sqrt{7}}{4}$
16. $32 + 4\pi$
17. $\frac{1}{3}$
18. 11
19. 1225
20. $36.53°$
21. $\frac{-\sqrt{2}-\sqrt{6}}{4}$
22. $\frac{-6\sqrt{6}-20}{35}$
23. 2
24. $\frac{37\sqrt{10}}{40}$
25. $\frac{11}{512}$

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1. $\tan(\theta) = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$

   $\tan(\theta) = \left|\frac{(2) - \left(-\frac{1}{4}\right)}{1 + \left(-\frac{1}{4}\right)(2)}\right|$

   $\tan(\theta) = \left|\frac{\left(\frac{9}{4}\right)}{\left(\frac{1}{2}\right)}\right|$

   $\tan(\theta) = \frac{9}{2}$

2. $r = 4\sin(\theta) + 2\cos(\theta)$

   $r^2 = 4r\sin(\theta) + 2r\cos(\theta)$

   $x^2 + y^2 = 4y + 2x$

   $x^2 - 2x + y^2 - 4y = 0$

   $(x-1)^2 + (y-2)^2 = 1 + 4$

   $(x-1)^2 + (y-2)^2 = 5$

   The graph is a circle. The area enclosed is $A = 5\pi$

3. $f(x) = \frac{x+2}{x^2-4}$

   $f(x) = \frac{1}{x-2}$

   $f(x) = (x-2)^{-1}$

   $f'(x) = -(x-2)^{-2}$

   $f'(6) = -(6-2)^{-2}$

   $f'(6) = -\frac{1}{16}$

4. $\prod_{n=0}^{4}\left(\sqrt{3} - i\right)^n = \left(\sqrt{3} - i\right)^{10}$

   $\prod_{n=0}^{4}\left(\sqrt{3} - i\right)^n = \left(2\left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)\right)^{10}$

   $\prod_{n=0}^{4}\left(\sqrt{3} - i\right)^n = 2^{10}\left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^{10}$

   $\prod_{n=0}^{4}\left(\sqrt{3} - i\right)^n = 2^{10}\left(cis(-30°)\right)^{10}$

$\prod_{n=0}^{4}(\sqrt{3}-i)^n = 1024cis(-300°)$

$\prod_{n=0}^{4}(\sqrt{3}-i)^n = 1024\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$

$\prod_{n=0}^{4}(\sqrt{3}-i)^n = 512+512\sqrt{3}i$

5. If n is divisible by 4, then the first term is divisible by 8 (if n is otherwise even, the second and third terms are odd). This is 25 values. If n is odd, then $n+1$ and $n+3$ are consecutive even numbers, at least one of which must therefore be divisible by 4. This is 50 more values. $\frac{75}{100}=\frac{3}{4}$.

6. There are 18 positive divisors $\{1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 32, 36, 48, 72, 96, 144, 288\}$

7. Recall $\cos\left(\frac{\pi}{2}-x\right)=\sin(x)$ and $\sin(\pi-x)=\sin(x)$

$\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)=\sin\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{\pi}{8}\right)$

$\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)=\sin\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\cos\left(\frac{3\pi}{8}\right)$

$\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)=\left[\frac{1}{2}\sin\left(\frac{\pi}{4}\right)\right]\left[\frac{1}{2}\sin\left(\frac{3\pi}{4}\right)\right]$

$\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)=\left[\frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)\right]\left[\frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)\right]$

$\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)=\left[\frac{\sqrt{2}}{4}\right]\left[\frac{\sqrt{2}}{4}\right]$

$\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)=\frac{2}{16}$

$\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)=\frac{1}{8}$

8. $202500 = 2^2 3^4 5^4$

$y = 4$

9. $\lim_{x\to 4}\left(\frac{\frac{1}{x+3}-\frac{1}{x+4}}{\frac{1}{x^2-16}}\right)=\lim_{x\to 4}\left(\frac{\frac{x^2-16}{x+3}-\frac{x^2-16}{x+4}}{1}\right)$

$\lim_{x\to 4}\left(\frac{\frac{1}{x+3}-\frac{1}{x+4}}{\frac{1}{x^2-16}}\right)=\lim_{x\to 4}\left(\frac{x^2-16}{x+3}-\frac{x^2-16}{x+4}\right)$

$$\lim_{x\to 4}\left(\frac{\frac{1}{x+3}-\frac{1}{x+4}}{\frac{1}{x^2-16}}\right)=\lim_{x\to 4}\left(\frac{x^2-16}{x+3}-(x-4)\right)$$

$$\lim_{x\to 4}\left(\frac{\frac{1}{x+3}-\frac{1}{x+4}}{\frac{1}{x^2-16}}\right)=\lim_{x\to 4}\left(\frac{(16)^2-16}{(4)+3}-((4)-4)\right)$$

$$\lim_{x\to 4}\left(\frac{\frac{1}{x+3}-\frac{1}{x+4}}{\frac{1}{x^2-16}}\right)=0$$

10. $y=\frac{x^3-1}{2x^4+x^2-3}=\frac{(x-1)(x^2+x+1)}{(x^2-1)(2x^2+3)}$

$y=\frac{x^3-1}{2x^4+x^2-3}=\frac{(x-1)(x^2+x+1)}{(x-1)(x+1)(2x^2+3)}$

$y=\frac{x^3-1}{2x^4+x^2-3}=\frac{(x^2+x+1)}{(x+1)(2x^2+3)}$

There is one horizontal asymptote $y=0$ and one vertical asymptote $x=-1$

The total number of asymptotes is 2

11. $(2+i)+\left(\frac{1}{2-i}\right)=(2+i)+\left(\frac{1}{2-i}\right)\left(\frac{2+i}{2+i}\right)$

$(2+i)+\left(\frac{1}{2-i}\right)=(2+i)+\frac{2+i}{4-(-1)}$

$(2+i)+\left(\frac{1}{2-i}\right)=(2+i)+\frac{2}{5}+\frac{i}{5}$

$(2+i)+\left(\frac{1}{2-i}\right)=\frac{12}{5}+\frac{6}{5}i$

12. There are 55 terms and the common difference is 4

$S=\frac{55(2+218)}{2}$

$S=55(110)$

$S=6050$

13. $A=\pm\frac{1}{2}\begin{vmatrix}2 & 5 & 1\\ 5 & 8 & 1\\ 7 & -1 & 1\end{vmatrix}$

$A=\pm\frac{1}{2}\left[2\begin{vmatrix}8 & 1\\ -1 & 1\end{vmatrix}-5\begin{vmatrix}5 & 1\\ 7 & 1\end{vmatrix}+1\begin{vmatrix}5 & 8\\ 7 & -1\end{vmatrix}\right]$

$A=\pm\frac{1}{2}\left[2\big(8-(-1)\big)-5(5-7)+1(-5-56)\right]$

$A = \pm\frac{1}{2}[2(9) - 5(-2) + 1(-61)]$

$A = \pm\frac{1}{2}[18 - (-10) + (-61)]$

$A = \pm\frac{1}{2}[-33]$

$A = \frac{33}{2}$

14. $383 = 2^8 + 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2 + 1$

383 in base 10 is 101111111 in base 2 which has 8 ones.

15. $A = \frac{1}{2}ACsin(B)$

$36 = \frac{1}{2}(12)(8)\sin(B)$

$36 = 48\sin(B)$

$\sin(B) = \frac{36}{48}$

$\sin(B) = \frac{3}{4}$

$\cos(B) = \sqrt{1 - \sin^2(B)}$

$\cos(B) = \sqrt{1 - \frac{9}{16}}$

$\cos(B) = \frac{\sqrt{7}}{4}$

16. $A = \frac{1}{2}\theta r^2$

$32\pi = \frac{1}{2} \times \frac{\pi}{4} \times r^2$

$32\pi = \frac{\pi}{8} \times r^2$

$r = \sqrt{256}$

$r = 16$

$P = 2r + r\theta$

$P = 2(16) + (16)\left(\frac{\pi}{4}\right)$

$P = 32 + 4\pi$

17. The factors of 144 are $\{1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144\}$

The factors of 400 are $\{1, 2, 4, 5, 8, 10, 16, 20, 25, 40, 50, 80, 100, 200, 400\}$

Common factors are $\{1, 2, 4, 8, 16\}$ and the number 144 has 15 factors

$p = \frac{5}{15}$

$p = \frac{1}{3}$

18. $T = 5 + (-6) + 12$

$T = 11$

19. $V = |a \cdot (b \times c)|$

$a \cdot (b \times c) = \begin{vmatrix} 2 & 5 & -8 \\ 7 & 11 & 6 \\ -5 & 8 & 7 \end{vmatrix}$

$a \cdot (b \times c) = 2\begin{vmatrix} 11 & 6 \\ 8 & 7 \end{vmatrix} - 5\begin{vmatrix} 7 & 6 \\ -5 & 7 \end{vmatrix} + (-8)\begin{vmatrix} 7 & 11 \\ -5 & 8 \end{vmatrix}$

$a \cdot (b \times c) = 2(77 - 48) - 5\big(49 - (-30)\big) - 8\big(56 - (-55)\big)$

$a \cdot (b \times c) = 2(29) - 5(79) - 8(111)$

$a \cdot (b \times c) = 58 - 395 - 888$

$a \cdot (b \times c) = -1225$

$|a \cdot (b \times c)| = 1225$

20. $36°27'288" = 36° + \frac{27}{60}° + \frac{288}{3600}°$

$36°27'288" = 36° + \frac{9}{20}° + \frac{2}{25}°$

$36°27'288" = 36° + .45° + .08°$

$36°27'288" = 36.53°$

21. $\cos\left(\frac{11}{12}\pi\right) = \cos\left(\frac{2\pi}{3} + \frac{\pi}{4}\right)$

$\cos\left(\frac{11}{12}\pi\right) = \cos\left(\frac{2\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{2\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$

$\cos\left(\frac{11}{12}\pi\right) = \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\cos\left(\frac{11}{12}\pi\right) = \frac{-\sqrt{2}-\sqrt{6}}{4}$

22. $\tan(\alpha) = -\frac{4}{3} \rightarrow \sin(\alpha) = \frac{4}{5}$ and $\cos(\alpha) = -\frac{3}{5}$

$\cos(\beta) = \frac{5}{7} \rightarrow \sin(\beta) = -\frac{\sqrt{24}}{7}$

$\sin(-\beta - \alpha) = \sin(-\beta)\cos(\alpha) - \sin(\alpha)\cos(-\beta)$

$\sin(\beta) = -\sin(-\beta)$

$\cos(\beta) = \cos(-\beta)$

$\sin(-\beta - \alpha) = \left(\frac{\sqrt{24}}{7}\right)\left(-\frac{3}{5}\right) - \left(\frac{4}{5}\right)\left(\frac{5}{7}\right)$

$\sin(-\beta - \alpha) = \left(\frac{2\sqrt{6}}{7}\right)\left(-\frac{3}{5}\right) - \frac{20}{35}$

$\sin(-\beta - \alpha) = -\frac{6\sqrt{6}}{35} - \frac{20}{35}$

$\sin(-\beta - \alpha) = \frac{-6\sqrt{6}-20}{35}$

23. $2^{2x+5} + 4^x = 528$

$2^{2x+5} + 2^{2x} = 528$

$2^{2x}(2^5 + 1) = 528$

$2^{2x}(33) = 528$

$2^{2x} = 16$

$2^{2x} = 2^4$

$2x = 4$

$x = 2$

24. $d = \frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}$

$d = \frac{|(4)(6)+(12)(2)+(-11)|}{\sqrt{(4)^2+(12)^2}}$

$d = \frac{|24+24+(-11)|}{\sqrt{160}}$

$d = \frac{37}{4\sqrt{10}}$

$d = \frac{37\sqrt{10}}{40}$

25. $S_n = a_1 \frac{(1-r^n)}{(1-r)}$

$S_{10} = \left(\frac{1}{93}\right) \frac{\left(1-\left(\frac{1}{2}\right)^{10}\right)}{\left(1-\frac{1}{2}\right)}$

$S_{10} = \left(\frac{1}{93}\right) \frac{\left(1-\frac{1}{1024}\right)}{\frac{1}{2}}$

$S_{10} = \left(\frac{1}{93}\right)(2)\left(\frac{1023}{1024}\right)$

$S_{10} = \left(\frac{1}{93}\right)\left(\frac{1023}{512}\right)$

$S_{10} = \left(\frac{1}{93}\right)\left(\frac{93*11}{512}\right)$

$S_{10} = \frac{11}{512}$