

# Interschool

(300 points possible)

Mu Alpha Theta National Convention 2025

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## 1 Two Lies and a Truth v.2 (20 pts. total, 2 pts. each)

This is a throwback to the 2022 National Convention Interschool:  
“Welcome to Two Lies and a Truth. We’ve got a ton of random facts for you, but in each category two of the facts are completely made up, and one is true. All you have to do is choose the letter corresponding to the correct fact. Each question is worth 2 points, and **a wrong answer will cost you 1 point.** Happy educated guessing!” – 2022

### 1. Renaissance Art

- (a) Michelangelo’s *David* was originally intended as part of a large-scale fountain in Florence’s main square, with hidden channels that would make water flow from David’s outstretched arms.
- (b) Leonardo da Vinci’s *Last Supper* had its bottom portion removed in the 17th century to accommodate a new doorway in the convent’s dining hall, resulting in the partial loss of Jesus’s feet in the composition.
- (c) Raphael’s *School of Athens* was actually painted entirely by his apprentices, as Raphael traveled away from Florence during its final stages and left only basic sketches behind.

### 2. Japanese Festivals

- (a) The Sapporo Snow Festival started as a winter sports fair in the early 20th century, when local ice-skaters built small snow sculptures to help promote tourism in Hokkaido.
- (b) At Kanda Matsuri in Tokyo, an enormous fireworks display is launched from the rooftops of local shrines, synchronized to taiko drum performances that ward off evil spirits.
- (c) The Aomori Nebuta Festival features massive illuminated floats—often depicting historical warriors and mythical gods—paraded through the streets alongside dancers and musicians.

### 3. Oceanography

- (a) The “Milky Sea effect” is a rare bioluminescent phenomenon caused by luminous bacteria, capable of making huge patches of ocean surface glow bluish-white for multiple consecutive nights.
- (b) The Humboldt Abyss is the deepest known trench in the Atlantic Ocean, plunging over 12,500 meters below sea level and discovered by a German expedition in the early 1900s.
- (c) The Mariana Trench’s deepest waters remain near boiling due to volcanic vents, maintaining consistently high temperatures even in the Challenger Deep.

### 4. Ancient Civilizations

- (a) The Indus Valley Civilization, though advanced in urban layout, relied solely on sun-dried bricks and had no form of sewage or drainage systems in its major cities.
- (b) The Minoans of Crete built elaborate palaces like Knossos, showcasing advanced plumbing, multi-level architecture, and vibrant frescoes depicting both religious ceremonies and nature.
- (c) The Olmec civilization in Mesoamerica developed the earliest fully deciphered hieroglyphic script, preserving detailed day-to-day records of their rulers over several centuries.

## 5. Aviation History

- (a) Amelia Earhart was the first person to fly solo from Hawaii to the U.S. mainland, a route once thought to be more perilous than a transatlantic crossing.
- (b) The Wright brothers successfully demonstrated their first powered flight in 1900 at Central Park in New York, witnessed by thousands of onlookers who marveled at the 12-second flight.
- (c) Hot air balloons served as the principal method of commercial cross-continental travel in Europe through the mid-1800s, running on fixed schedules between major capitals.

## 6. Cryptids

- (a) Early Scottish folklore describes the Loch Ness Monster as a colossal amphibian with a horned head, said to protect hidden gold deposits beneath the lake's surface.
- (b) The Himalayan Yeti, or "Abominable Snowman," gained widespread Western attention when a 1921 British Mount Everest expedition reported discovering large footprints in the snow.
- (c) El Chupacabra sightings started in rural Argentina, where farmers claimed it feasted mostly on fruit crops, leaving behind mysterious circular patterns in the fields.

## 7. Famous Bridges

- (a) Upon opening in 1883, the Brooklyn Bridge was actually the shortest major bridge in New York, but it featured a little-known cable car system running beneath its roadway for a brief time.
- (b) The Golden Gate Bridge's color, officially "International Orange," was selected specifically to protect migrating bird species by making the structure visible in San Francisco's fog.
- (c) Florence's Ponte Vecchio has stood for centuries and was famously spared by German forces during World War II, even as they destroyed every other bridge in the city.

## 8. Space Missions

- (a) The Soviet Luna 1 achieved the first successful touchdown on the Moon in 1959, sending data from the lunar surface before contact was lost.
- (b) NASA's Apollo 13 mission failed to land on the Moon due to an oxygen tank explosion, yet managed to return the crew safely to Earth after a dramatic loop around the Moon.
- (c) China's Chang'e-4 program was canceled before launch, leading to Chang'e-5 becoming the first spacecraft to successfully touch down on the Moon's far side in 2019.

## 9. Famous Writers

- (a) Jane Austen published all her novels anonymously, and it was only after her death that her brother publicly revealed her authorship, crediting her for six major novels.
- (b) Charles Dickens embarked on tours across North America with an entourage of exotic parrots and monkeys, claiming they inspired his characters in *Oliver Twist* and *David Copperfield*.
- (c) Mark Twain, multilingual in English and French, drafted several chapters of *The Adventures of Tom Sawyer* in French to entertain his growing European readership.

## 10. Computer Programming

- (a) Fortran, introduced in the 1950s, was originally intended for real-time text processing rather than scientific or numerical computations.
- (b) Python was named after a rare giant snake discovered by Guido van Rossum in the 1980s, symbolizing the language's flexible yet powerful "grip" on data and memory.
- (c) COBOL, developed in 1959, was designed as a business-oriented language with English-like syntax, making it easier for government and corporate employees to learn.

**2 Spoiler Warning! v.2 (10 pts. total, 1 pt. each)**

This is a throwback to the 2022 National Convention Interschool:  
"Prepare for questions involving spoilers to all of those movies you've been meaning to get to. Is it worth a few measly points to ruin an entire watching experience for yourself?" – 2022

1. In *Interstellar* (2014), we discover that the mysterious "ghost" in Murph's bedroom is actually her father, Joseph Cooper, sending gravitational messages across time from inside the tesseract. Which object in Murph's room does Cooper use to communicate these messages?
2. In *The Martian* (2015), stranded astronaut Mark Watney finally reunites with the Ares III crew when he propels himself through space by piercing his pressure suit and using escaping air as thrust. Who is the mission commander that personally pulls him back aboard?
3. In *Suicide Squad* (2016), The Joker is presumed dead when his helicopter crashes, but in a surprise twist at the film's end, he reappears at Belle Reve prison. Who does he rescue at this point?
4. In *Wonder Woman* (2017), Diana believes her sword to be the fabled "Godkiller." Later, she discovers a startling truth during her battle with Ares. What does Diana learn is the real "Godkiller"?
5. In *Fantastic Beasts: The Crimes of Grindelwald* (2018), Queenie Goldstein ultimately joins forces with Gellert Grindelwald in the hopes of winning the freedom to marry Jacob Kowalski. What is Queenie's special magical ability, which allows her to read minds?
6. In *Jumanji: The Next Level* (2019), terminally ill Milo Walker chooses to remain in the game world rather than return to reality. Which new in-game form or avatar is the reason that Milo decides that staying in Jumanji is the right decision for him?
7. In *Shock Wave 2* (2020), during the final showdown, ex-bomb disposal officer Poon Sing-Fung races to stop a massive terrorist bomb, still pursued by the authorities. He successfully defuses the bomb but is fatally shot by the police, who still believe he is a terrorist. Where does this occur?
8. In *Sing 2* (2021), legendary rock legend Clay Calloway, distraught over his wife's death, finally returns to the stage with Ash to prove Buster Moon's show can succeed. Which iconic U2 song do Clay and Ash perform as the show-stopping duet?
9. In *Puss in Boots: The Last Wish* (2022), the sinister wolf, initially appearing as a bounty hunter, is ultimately revealed to be Death, fixated on taking Puss's final life. Which magical relic does Puss seek to restore his lost lives and avoid Death's pursuit?
10. In *Mission: Impossible – Dead Reckoning Part One* (2023), Gabriel kills Ilsa Faust on a Venetian rooftop, shattering Ethan Hunt's close ally. Who does he neutralize right before doing this?

### 3 Bash! Bash! Bash! (50 pts. total, 5 pts. each)

You know what bash is. Start bashing. Answers must be fully simplified.  
Note that the right column is a bit harder than the left column.

1. Find the sum of the factors of

$$277945762500.$$

6. Find the sum of the factors of

$$277945762500 \cdot 11^{11}.$$

2. Find

$$\binom{50}{25}.$$

7. Find

$$\binom{70}{35}.$$

3. Find

$$9354951841^{\frac{3}{4}}.$$

8. Find

$$6870484987921^{\frac{3}{4}}.$$

4. Find

$$\begin{vmatrix} 1 & 2 & 3 & 4 & 0 \\ 2 & 3 & 5 & 8 & 0 \\ 3 & 4 & 8 & 17 & 0 \\ 4 & 5 & 13 & 40 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix}.$$

9. Find

$$\begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 5 & 8 & 12 \\ 3 & 4 & 8 & 17 & 33 \\ 4 & 5 & 13 & 40 & 104 \\ 5 & 6 & 22 & 103 & 359 \end{vmatrix}.$$

5. Find

$$\frac{1}{4} \int_0^1 \frac{x^8(1-x)^8}{1+x^2} dx.$$

10. Find

$$\frac{1}{16} \int_0^1 \frac{x^{10}(1-x)^{10}}{1+x^2} dx.$$

## 4 Guess the Graph! (50 pts. total, 5 pts. each)

What follows is 10 images of graphs drawn in Desmos. The objective is to write down an equation ( $y = f(x)$ ,  $f(x, y) = 0$ , etc) or parameterization ( $(f(t), g(t))$ ) of each depicted graph. Responses should be in the form of a finite combination (with respect to addition, multiplication, and composition) of standard functions, i.e., polynomials, (inverse) trigonometric functions,  $\exp()$ ,  $\ln()$ , etc. Responses should NOT define custom functions or define functions piecewise. Responses do not need to be simplified.

Here are some additional functions you may use:

Let  $f, g: \mathbb{R} \rightarrow \mathbb{R}$ . Then, the absolute value function applied to  $f$  is

$$|f(x)| = \begin{cases} f(x) & f(x) \geq 0 \\ -f(x) & f(x) < 0 \end{cases}.$$

The floor function applied to  $f$  is

$$\lfloor f(x) \rfloor = \max\{n \in \mathbb{Z} \mid n \leq f(x)\}.$$

The modulo function applied to  $f$  with respect to  $g$  (where  $g \neq 0$ ) is

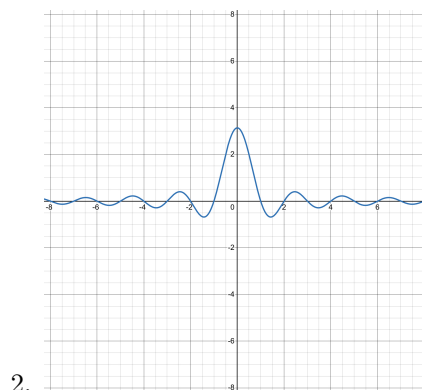
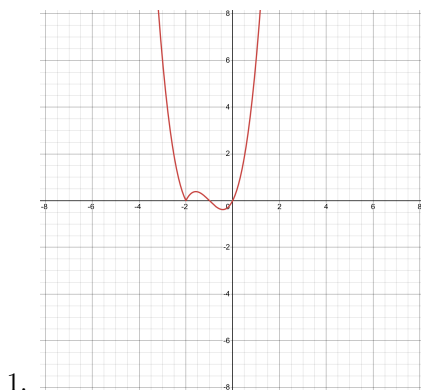
$$\text{mod}(f(x), g(x)) = f(x) - g(x) \left\lfloor \frac{f(x)}{g(x)} \right\rfloor.$$

Any answer will be counted as equivalent to the intended answer if it differs from the intended answer by at most a countable subset of  $\mathbb{R}$ . (Do not worry too much about this. This is imply saying that, for example, if an intended answer is  $f(x) = x$ , then  $g(x) = \frac{x^2}{x}$  would also be marked correct, even though  $g(x)$  is not defined at  $x = 0$  while  $f(x)$  is.)

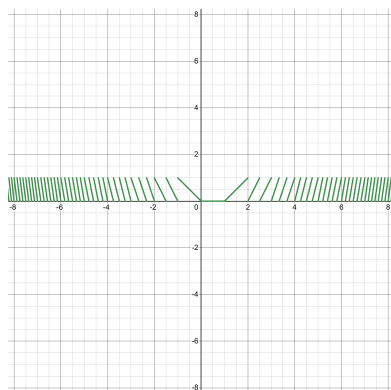
Note that none of the graphs make use of arbitrary constants unless their use is reasonably deducible from the provided image.

Note that any trigonometric functions used are graphed with radian inputs.

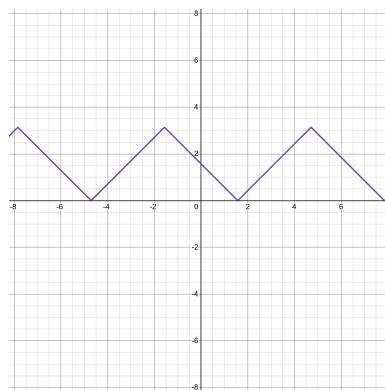
Note that some questions may award partial credit for responses that correctly identify key components of the intended answer, even if not fully correct.



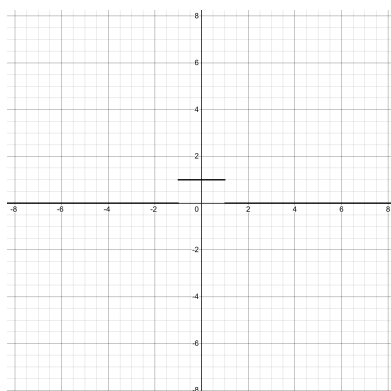
3.



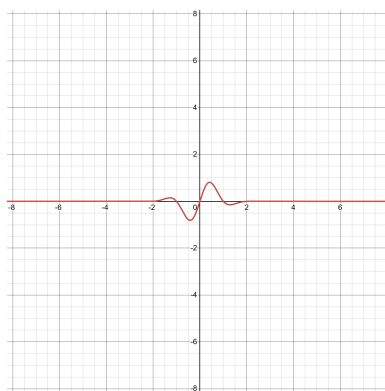
4.



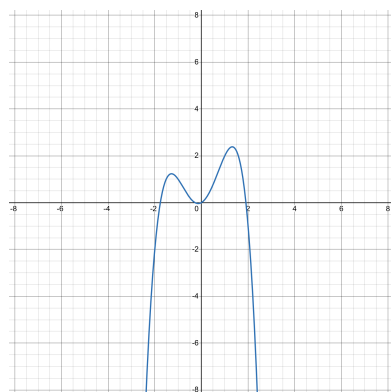
5.



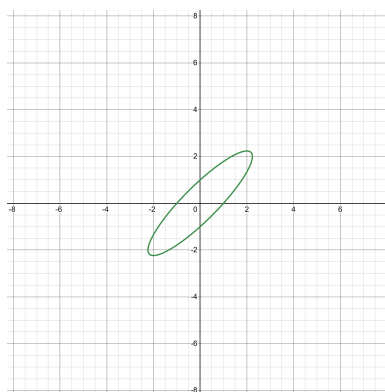
6.



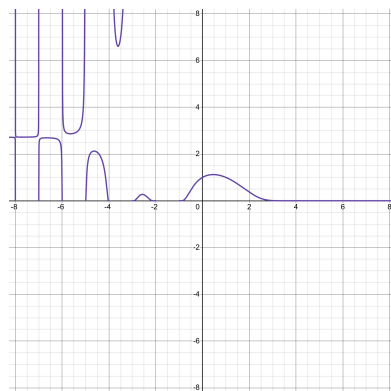
7.



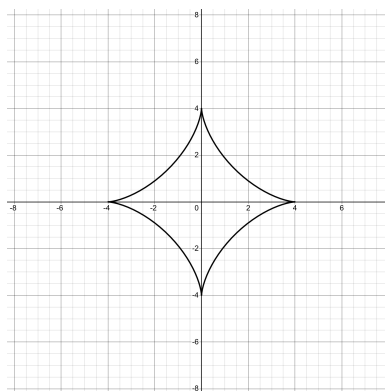
8.



9.



10.



## 5 True or False? (70 pts. total, 2 pts. each)

Each question is worth 2 points, and **a wrong answer will cost you 2 points.**

### 5.1 Discrete Geometry (30 pts.)

#### 5.1.1 Points, Unbounded

1. True or False: It is possible to put 3 points on lattice points in  $\mathbb{R}^2$  that define an equilateral triangle.
2. True or False: It is possible to put six points on  $\mathbb{R}^2$  such that the distance between any two of them is an integer, and no three are collinear.
3. True or False: It is possible to color all points in  $\mathbb{R}^2$  red, blue, or green such that no two at distance 1 have the same color.
4. True or False: It is possible to color all points in  $\mathbb{R}^2$  red, blue, green, or white such that no two at distance 1 have the same color.
5. True or False: It is possible to put 8 points in  $\mathbb{R}^2$  such that no five define a convex pentagon.

#### 5.1.2 Points, Bounded

6. True or False: It is possible to put 16 points on an  $8 \times 8$  lattice such that no three are collinear.
7. True or False: It is possible to put 6 points on a unit square such that no three define a triangle of area less than  $\frac{1}{8}$ . Note that triangles can have area 0.
8. True or False: It is possible to put 9 points on a unit square such that no three define a triangle of area at most  $\frac{1}{8}$ . Note that triangles can have area 0.
9. True or False: It is possible to put 19 points on a unit regular hexagon such that no two have distance at most  $\frac{\sqrt{3}}{3}$ .

#### 5.1.3 Packing

10. True or False: It is possible to pack 13 unit spheres in  $\mathbb{R}^3$  such that one is tangent to the other 12.
11. True or False: It is possible to pack 3 unit squares in a circle of radius  $\frac{\sqrt{79}}{8}$ .
12. True or False: It is possible to pack 3 unit squares in an equilateral triangle of side length 3.
13. True or False: It is possible to pack 6 unit squares in an equilateral triangle of side length  $2 + \frac{4}{\sqrt{3}}$ .
14. True or False: It is possible to pack 7 unit circles in an equilateral triangle of side length  $4\sqrt{3}$ .
15. True or False: It is possible to pack 12 unit circles in an equilateral triangle of side length  $4 + 4\sqrt{3}$ .



## 5.2 Number Theory (40 pts.)

### 5.2.1 Diophantine Equations

1. True or False:  $x^2 + y^2 = 3z^2$  has a solution in the positive integers.
2. True or False:  $x^3 + y^3 = z^3$  has a solution in the positive integers.
3. True or False:  $x^3 + y^4 = z^4$  has a solution in the positive integers.
4. True or False:  $x^2 + 17y^4 = z^4$  has a solution in the positive integers.
5. True or False:  $x^2 = 4y + 3$  has a solution in the positive integers.
6. True or False:  $x^2 + 2y^2 = 8z + 5$  has a solution in the positive integers.
7. True or False:  $x^2 + 11 = y^3$  has a solution in the positive integers.
8. True or False:  $x^2 = y^3 + 23$  has a solution in the positive integers.
9. True or False:  $5x^3 = 11y^3 + 13z^3$  has a solution in the positive integers.
10. True or False:  $x^w = y^z + 10$  has a solution in the positive integers.

### 5.2.2 Summations

11. Consider the twin prime numbers. True or False: The sum of the reciprocals of these integers converges. Note that this question was answered in 1919.
12. Consider the positive integers where every prime appearing in their prime factorization appears at least twice. True or False: The sum of the reciprocals of these integers converges to a value at most  $\frac{9}{5}$ .
13. Consider the distinct integers in the fifth cells of any row of Pascal's triangle starting with the five-term row 1 4 6 4 1. True or False: The sum of the reciprocals of these integers converges to  $\frac{3}{2}$ .
14. Consider the positive integers not containing the digit 1. True or False: The sum of the reciprocals of these integers converges.
15. Consider the positive integers of form  $q - 1$  where  $q$  is a perfect power (not 1) and omitting repetitions. True or False: The sum of the reciprocals of these integers converges to 1.

### 5.2.3 Decimal Coincidences

16. There are three integers between 99 and 9999 equal to the cube of the sum of their digits. True or False: The sum of the three integers is odd.
17. There are two positive integers (they are less than 99999) equal to the sum of the factorials of their digits. True or False: The sum of the two integers is odd.
18. There are four 3-digit integers equal to the sum of the cubes of their digits. True or False: The sum of the four integers is odd.
19. There is a 7-digit integer  $ABCDEFGH = A^1 + B^2 + C^3 + D^4 + E^5 + F^6 + G^7$ . True or False: The sum of its digits is odd.
20. There is a positive integer greater than 1 (it is less than 99999) equal to the sum of its digits raised to their own powers. True or False: The sum of its digits is odd. Note that  $0^0 = 1$ .

## 6 Logic (40 pts. total, 10 pts. each)

### 6.1 A Game of Coins

Dr. Radleigh Santos and Mr. Will Frazer play a game. In this game, Dr. Santos has 165 coins, and Mr. Frazer has four tokens, labeled D, D, N, N, as well as sufficiently many coins to not become bankrupt during the game. The game lasts four rounds. At the start of every round, Dr. Santos sets forward some of the coins he has. Mr. Frazer sees how many coins Dr. Santos sets forward and responds with one of his tokens. If Mr. Frazer responds D, then he returns those coins to Dr. Santos, plus the same amount ("double"). If Mr. Frazer responds N, then he takes those coins from Dr. Santos ("nothing"). Suppose Dr. Santos and Mr. Frazer are perfect logicians, and their goals are to maximize their respective individual gains from the game. How many coins does Dr. Santos end the game with?

### 6.2 Hats

A mother decides to play a game with her three daughters, Aley, Bailey, and Cayley.

She summons them to the living room and speaks:

Mother: I have placed onto your heads a hat with a positive integer scratched on.

Mother: You can see the numbers on each others' hats, but not your own.

Mother: It is necessary information that one of your hats' numbers is actually the sum of the numbers on the other two hats.

Mother: Now. I want you to discuss this situation amongst yourselves and figure out your hats' numbers.

Mother: However, when speaking, you are only allowed to say whether or not you know the number on your own hat.

The daughters look at each other in confusion as they begin to understand what is going on. Eventually, Aley breaks the silence, with the others following her (it is granted each daughter is a truthful, perfect logician):

Aley: The value of the number on my hat is unclear to me.

Bailey: The value of the number on my hat is unclear to me.

Cayley: The value of the number on my hat is unclear to me.

Aley: The value of the number on my hat is... Oh... It's 70!

Bailey: Then, we're done! Cayley and I should know our own numbers as well!

Cayley: Certainly.

What is the product of the non-zero digits of the three numbers?

### 6.3 Stardust Selection

Dr. Nova is cataloguing ten identical-looking meteor fragments in her lab. She is certain that a positive even number of the fragments are harmless “stardust rocks” while the remaining fragments (possibly none) are highly radioactive. Tragically, she no longer remembers how many of each there are.

On her workbench sits a device called the Dual-Chamber Geiger Gate (DCGG). The DCGG has two sample slots, one big lever, and a single display lamp. When Dr. Nova places one fragment in each slot and pulls the lever, the DCGG hums for two hours and then flashes green or red. A green flash indicates that at least one of the two tested fragments is harmless stardust, while a red flash indicates that neither fragment is harmless (both are radioactive).

Dr. Nova needs to ship two harmless fragments to a space-station museum, and she would like to do so as quickly as possible. In the worst case, how many times must she operate the DCGG to guarantee she can identify two harmless fragments?

### 6.4 The Perfect Polynomial

$f$  is a monic polynomial with integer coefficients such that the following questions have a unique correct answer.

1. The degree of  $f$  is

A) 0                      B) 1                      C) 2                      D) 3

2. The sum of the roots of  $f$  is

A) 2                      B) 4                      C) 6                      D) 8

3. The sum of the squares of the roots of  $f$  is

A) 12                      B) 18                      C) 24                      D) 30

4. The sum of the cubes of the roots of  $f$  is

A) 0                      B) 32                      C) 40                      D) 50

5. The product of the roots of  $f$  is

A)  $-10$                       B)  $-54$                       C)  $-60$                       D)  $-84$

What is  $f(1)$ ?

## 7 Puzzles (60 pts. total, 20 pts. each)

### 7.1 $36 = 12 \cdot 3 = 6^2$ Words

Bear	Oxygen
Ibis	Straw
Falcon	Ferrari
Kangaroo	Asparagus
Mustang	Lion
Jaguar	Pelican
Diamond	Black
Ruby	Cucumber
Lime	Wolf
Pomelo	Nitrogen
Copper	Tomato

Albatross

Porsche

Gold

Eagle

Owl

Orange

Silver

Sapphire

Hydrogen

Koala

Wallaby

Impala

Hawk

Goose

=====

A1 = A	.
.	.
.	.
.	.
.	.
.	F4 = L

=====

C1

F3

B2

C5

B6

A3

D2

E2

## 7.2 Let's Count to 1 Centillion!

five hundred fifty-one centillion eight hundred fifty-eight septenonagintillion six hundred eighteen octooc-togintillion three hundred thirty septemoctogintillion nine hundred forty-nine septenseptuagintillion two hundred sixty-six seseptuagintillion five hundred ten duoseptuagintillion three hundred one unseptuagin-tillion eight hundred ninety-seven octosexagintillion seven hundred thirty-nine quattuorsexagintillion five hundred seventy-one tresexagintillion four hundred sixty-two unsexagintillion four hundred fifty-five noven-quinquagintillion eight hundred eighty-one octoquinquagintillion four hundred twenty-two tresquinquagintil-lion one hundred seven unquinquagintillion six hundred two septenquadrageintillion seven hundred fourteen unquadrageintillion four hundred forty-three quintrigintillion seven hundred ninety-one quattuortrigintillion eight hundred thirty-nine untrigintillion three hundred seventy quattuorvigintillion five hundred forty-six quindecillion nine hundred fifty-eight duodecillion

Reference	
#	Name
$10^{33}$	Decillion
$10^{63}$	Vigintillion
$10^{93}$	Trigintillion
$10^{123}$	Quadrageintillion
$10^{153}$	Quinquagintillion
$10^{183}$	Sexagintillion
$10^{213}$	Septuagintillion
$10^{243}$	Octogintillion
$10^{273}$	Nonagintillion
$10^{303}$	Centillion

First Half	ABCDEFGHIJKLM
Second Half	NOPQRSTUVWXYZ

## 7.3 On the Flip Side

Craft before the fermentation stage

Cultivated root emblazoned with a protest slogan

Export of Ecuador blown

Fabric promoted for enhancing night-vision during WWII

Gemstone steeped by monks for centuries

Poetic line whose petals flavor Middle-Eastern sweets

Prized clusters propelled by fiery exhaust

Romantic emblem repeated for emphasis

Sphere that is slapstick-famous

String instrument tuned in fifths reinforced with copper rivets

Treatments labelled “alternative” in pharmacies prized in China

Western wear for miners that is high-pitched