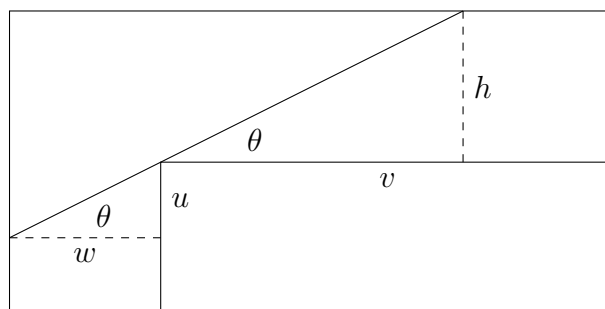


1	C	11	A	21	C
2	C	12	D	22	D
3	C	13	D	23	A
4	A	14	A	24	C
5	B	15	A	25	D
6	D	16	B	26	C
7	B	17	B	27	C
8	E	18	A	28	B
9	A	19	E	29	B
10	C	20	C	30	E

- 1) There are 7 letters, and 2 of them repeat. $\frac{7!}{2!} = \frac{5040}{2} = 2520$. C
- 2) If the distance I climb the ladder is d and my height above the ground is h , then by similar triangles, $\frac{d}{h} = \frac{17}{15}$. In other words, $d = \frac{17}{15}h$. Deriving, $d' = \frac{17}{15}h'$. Since $h' = 3$, $d' = \frac{17}{5}$. C
- 3) The height off the ground does not matter, and the velocity is maximized when the hole is at the bottom of the can, so $h = 8$. $\sqrt{2gh} = \sqrt{512} = 16\sqrt{2}$ feet per second. C
- 4) Vertically, the acceleration of the lemonade is -32 , the velocity is $-32t + 8\sin\theta$, and the position is $-16t^2 + 8\sin\theta t + 24$. The lemonade will land on the ground when this equals zero, so $-2t^2 + \sin\theta t + 3 = 0$. Using the quadratic formula, $t = \frac{-\sin\theta - \sqrt{\sin^2\theta + 24}}{-4}$. The horizontal velocity of the lemonade is a constant $8\cos\theta$, so the horizontal distance the lemonade travels is $2\cos\theta(\sin\theta + \sqrt{\sin^2\theta + 24})$. Noting monotonicity, set $\sin\theta = x$, so the distance is $2\sqrt{1-x^2}(x + \sqrt{x^2 + 24})$. The derivative of this is $\frac{-2x(x + \sqrt{x^2 + 24})}{\sqrt{1-x^2}} + 2\sqrt{1-x^2}\left(\frac{x}{\sqrt{x^2 + 24}} + 1\right) = \frac{(x + \sqrt{x^2 + 24})(x^2 + \sqrt{x^2 + 24}x - 1)}{\sqrt{-(x^2 - 1)(x^2 + 24)}}$. Observing values in the answer choices, $x = \frac{1}{\sqrt{26}}$ is a solution to this, so $\theta = \arcsin \frac{1}{\sqrt{26}}$. A
- 5) Consider the following diagram.



Let the slanted line have length L . Using similar triangles, $L = \frac{w}{\cos\theta} + \frac{h}{\sin\theta}$. Differentiating, $\frac{dL}{d\theta} = -\frac{w\sin\theta}{\cos^2\theta} - \frac{h\cos\theta}{\sin^2\theta}$. Setting this equal to 0, $w\sin^3\theta - h\cos^3\theta = 0$, so $\tan\theta = \left(\frac{h}{w}\right)^{\frac{1}{3}}$. $\cos\theta = \frac{w^{\frac{1}{3}}}{\sqrt{w^{\frac{2}{3}} + h^{\frac{2}{3}}}}$ and $\sin\theta = \frac{h^{\frac{1}{3}}}{\sqrt{w^{\frac{2}{3}} + h^{\frac{2}{3}}}}$, so $L = (w^{\frac{2}{3}} + h^{\frac{2}{3}})^{\frac{3}{2}}$. Plugging in values, $L = 24\sqrt{3}$ and $h = 8$, so $12 = 4 + w^{\frac{2}{3}}$ and $w = 8^{\frac{3}{2}} = 2^{\frac{9}{2}}$. $2 \cdot \frac{9}{2} = 9$. B

- 6) The slope of the tangent line to $y = 9 - 15x + 36x^2 + 2x^3 - x^4$ is $y' = -15 + 72x + 6x^2 - 4x^3$. Attempting to maximize this requires taking another derivative, $y'' = 72 + 12x - 12x^2$. Critical points are where this equals zero, so $x^2 - x - 6 = 0$. The root with positive x is $x = 3$, which can be shown to be the absolute maximum slope. D
- 7) The total distance I move down is a geometric series with first term 16 and common ratio $\frac{1}{8}$, so its sum is $\frac{16}{1-\frac{1}{8}} = \frac{128}{7}$. However, there is also distance that I move up from my bounces. This distance contains all the same terms as the first geometric series except the initial 16-foot fall. $\frac{2}{1-\frac{1}{8}} = \frac{16}{7}$, so the total distance is $\frac{144}{7}$. $144 + 7 = 151$. B
- 8) If b is negative, neither step ever executes. None of the choices address this issue. E
- 9) $z_x = 2x + 2y + 4$ and $z_y = 2x + 4y + 3$. Creating a system where these are equal to 0, $2x + 2y = -4$ and $2x + 4y = -3$. Eliminating, $2y = 1$ so $y = \frac{1}{2}$ and $2x = -5$, so $x = -\frac{5}{2}$. Plugging these values into the function, $\frac{25}{4} - \frac{5}{2} + \frac{1}{2} - 10 + \frac{3}{2} + 5 = \frac{3}{4}$. $3 + 4 = 7$. A
- 10) $T(x, y) = 100 - x^2 - 4y^2 - 4xy$, so $T_x = -2x - 4y$ and $T_y = -4x - 8y$. $\nabla T(-4, 1) = \langle 4, 8 \rangle$. The direction vector is $\langle 12, 5 \rangle$, which has magnitude 13. $\langle 4, 8 \rangle \cdot \langle \frac{12}{13}, \frac{5}{13} \rangle = \frac{48}{13} + \frac{40}{13} = \frac{88}{13}$. $88 + 13 = 101$. C
- 11) Using the Chain Rule and Product Rule, $u_t = -\frac{1}{2t\sqrt{4\pi kt}}e^{x^2/nkt} - \frac{x^2/nk}{t^2\sqrt{4\pi kt}}e^{x^2/nkt} = -\frac{nkt+2x^2}{2nkt^2\sqrt{4\pi kt}}e^{x^2/nkt}$. Similarly, $u_x = \frac{2x}{nkt\sqrt{4\pi kt}}e^{x^2/nkt}$ and $u_{xx} = \frac{2}{nkt\sqrt{4\pi kt}}e^{x^2/nkt} + \frac{4x^2}{n^2k^2t^2\sqrt{4\pi kt}}e^{x^2/nkt} = \frac{2nkt+4x^2}{n^2k^2t^2\sqrt{4\pi kt}}e^{x^2/nkt}$. Setting $u_t = ku_{xx}$ and eliminating common terms, $-\frac{1}{2} = \frac{2k}{nk} = \frac{2}{n}$ and $n = -4$. A
- 12) Setting up the system of equations, $3 = 2\lambda x$, $4 = 2\lambda y$, and $5 = 2\lambda z$ in addition to $x^2 + y^2 + z^2 = 50$. $x = \frac{3}{2\lambda}$, $y = \frac{2}{\lambda}$, and $z = \frac{5}{2\lambda}$, so $\frac{9}{4\lambda^2} + \frac{4}{\lambda^2} + \frac{25}{4\lambda^2} = 50$. Solving this gives $\frac{50}{4\lambda^2} = 50$, so $\lambda = \frac{1}{2}$ and $\{x, y, z\} = \{3, 4, 5\}$. $3x + 4y + 5z = 50$. D
- 13) Using two multipliers now, the system of equations becomes $1 = 2\lambda x + \mu$, $1 = 2\lambda y + 2\mu$, and $1 = 2\lambda z - 3\mu$. $x = \frac{1-\mu}{2\lambda}$, $y = \frac{1-2\mu}{2\lambda}$, and $z = \frac{1+3\mu}{2\lambda}$. Using the plane, $(1-\mu) + 2(1-2\mu) - 3(1+3\mu) = 28\lambda$. This simplifies to $-14\mu = 28\lambda$, so $-\mu = 2\lambda$. Thus, $x = \frac{\mu-1}{\mu}$, $y = \frac{2\mu-1}{\mu}$, and $z = \frac{-3\mu-1}{\mu}$. Using the sphere, $\frac{\mu^2-2\mu+1}{\mu^2} + \frac{4\mu^2-4\mu+1}{\mu^2} + \frac{9\mu^2+6\mu+1}{\mu^2} = 80$, so $14\mu^2 + 3 = 80\mu^2$, so $\mu^2 = \frac{1}{22}$. $x + y + z = \frac{3}{2\lambda} = -\frac{3}{\mu}$, which has a maximum value of $3\sqrt{22}$. D
- 14) For a continuous probability distribution, the mode is the absolute maximum of the function. For $N(x) = \frac{1}{\sqrt{\pi}}e^{-x^2}$, this is where $-\frac{2x}{\sqrt{\pi}}e^{-x^2} = 0$, or $x = 0$. A
- 15) The probability a single discrete value from a continuous distribution is 0 because there are an uncountably infinite number of possible values to select. A
- 16) Using Integration by Parts with $a = 1 - \operatorname{erf}(x)$ and $db = dx$ gives $da = -\frac{2}{\sqrt{\pi}}e^{-x^2} dx$ and $b = x$. $\int_0^\infty (1 - \operatorname{erf}(x)) dx = (x - x\operatorname{erf}(x))|_0^\infty + \frac{2}{\sqrt{\pi}} \int_0^\infty xe^{-x^2} dx = \frac{1}{\sqrt{\pi}} \int_0^\infty e^{-u} du = \frac{1}{\sqrt{\pi}}$. B
- 17) $\int_0^{\ell_{\max}} \ell L d\ell = \int_0^{\ell_{\max}} \frac{3\ell^3}{\ell_{\max}^3} d\ell = \frac{3\ell_{\max}^4}{4\ell_{\max}^3} = \frac{3\ell_{\max}}{4}$. B
- 18) If the rod is cut into p pieces, each piece will have length $\frac{\ell_{\max}}{p}$. Since it takes $p - 1$ cuts (not $p!$ – but that's not relevant here), Mr. Roboto's profit is $\sqrt{\ell_{\max}p} - 2(p - 1)$. Deriving, $\frac{1}{2}\sqrt{\frac{\ell_{\max}}{p}} - 2 = 0$, so $p = \frac{\ell_{\max}}{16}$. A

- 19) Suppose Julio picks the real number x . Then the probability Cal picks a number greater than Julio's is $1 - x$. The distribution is geometric, so the expected number of picks until Cal is successful is $\frac{1}{1-x}$. $\int_0^1 \frac{dx}{1-x} = \ln|1-x|_0^1 \rightarrow +\infty$. [E]
- 20) On any round of the game, Victor wins with probability p and Randy wins with probability $(1-p)p$. If the coin flips Tails twice in a row, the game resets to the initial state, so Victor's probability of winning is $\frac{p}{p+(1-p)p} = \frac{1}{2-p}$. His probability of losing is $\frac{1-p}{2-p}$, so his expected profit per dollar is the difference of these, $\frac{p}{2-p}$. Integrating, $\int_0^1 \frac{p}{2-p} dp = \int_0^1 \left(-1 + \frac{2}{2-p}\right) dp = -1 + 2 \ln 2$. This is approximately 0.386, so $x = \$3.86$. [C]
- 21) Because all the increments from a particular time t have mean 0, the expected value of W_{t+u} for all $u > 0$ is W_t , so the expected value of W_{45} is $W_1 = 45$. [C]
- 22) $\int_2^4 \ln(3^x) dx = \ln 3 \int_2^4 x dx = \frac{x^2 \ln 3}{2} \Big|_2^4 = 6 \ln 3$. $\exp(6 \ln 3) = 3^6 = 729$. [D]
- 23) $\int_2^4 \ln(\sqrt[3]{x}) dx = \frac{1}{3} \int_2^4 \ln x dx = \frac{x \ln x - x}{3} \Big|_2^4 = \frac{4 \ln 4 - 4}{3} - \frac{2 \ln 2 - 2}{3} = 2 \ln 2 - \frac{2}{3}$. $\exp\left(2 \ln 2 - \frac{2}{3}\right) = 4e^{-2/3}$. $4 \cdot \left(-\frac{2}{3}\right) = -\frac{8}{3}$. [A]
- 24) $\int_2^4 \ln(e^{x^3}) dx = \int_2^4 x^3 dx = \frac{x^4}{4} \Big|_2^4 = 60$. $\exp(60) = e^{60}$. [C]
- 25) $2 \cos(2t) = 2(1 - 2 \sin^2 t) = 2(1 - 2x^2) = 2 - 4x^2$. This is a parabola on its domain. [D]
- 26) The radius of Ryan's head increases at a half inch per hour, so after 6 more hours, its radius is 18 inches. $S = 2\pi r^2$, since the great circle is not visible. Deriving implicitly, $\frac{dS}{dt} = 4\pi r \frac{dr}{dt}$. $r = 18$ and $\frac{dr}{dt} = \frac{1}{2}$, so $\frac{dS}{dt} = 36\pi$. [C]
- 27) There are 9 pages with 1 digit and 90 pages with 2 digits, for a total of $9 + 180 = 189$ digits by page 99. There are 1836 digits left, and all of them will be used on triple-digit pages. $\frac{1836}{3} = 612$. $99 + 612 = 711$. [C]
- 28) Note that $960 = 8 \cdot 120$, so the last operation can be 120×8 . 120 is close to 128, so the second-last operation can be $128 - 8$. $128 = 16 \cdot 8 = (8 + 8) \cdot 8$, so the sequence is $((8 + 8) \times 8 - 8) \times 8$. Replacing all the 8's with 3's gives $((3 + 3) \times 3 - 3) \times 3 = 45$. [B]
- 29) There is expected to be 6.75 B's on the test, so a total of 33.75 points will be earned putting B for all answers. This is 3.75 points more than leaving every question blank, which is $\frac{3.75}{30} = 0.125$ points per question. The reciprocal of this is 8. [B]
- 30) The diagram is drawn *slightly* incorrectly, but it's fine because it's labeled as not drawn to scale :) It's actually an isosceles right triangle with leg length 8; the missing side length is $IL = 8\sqrt{2} - 4$ and the missing angle is $\angle ILE = 180^\circ$. The area of this triangle is $\frac{1}{2} \cdot 8^2 = 32$. [E]